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The Value of Stochastic Streamflow Models in Overyear Reservoir Design Applications

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The design of storage reservoirs using stochastic streamflow models and synthetic streamflow sequences has received considerable attention in the water resources literature. Fewer studies have addressed the sampling properties of estimates of the design capacity of a storage reservoir obtained using available historical records or using synthetic streamflow sequences generated with models whose parameters were estimated from such data. Our experiments document the bias and root-mean-square error of estimates of overyear required storage capacity distribution quantiles corresponding to fixed or to random demand levels. The results show that the use of stochastic streamflow models can lead to improvements in the precision of reservoir design capacity estimates. Estimates of the design capacity of a storage reservoir based upon relatively simple stochastic streamflow models have smaller root-mean-square errors than corresponding estimates based solely upon the historic record, even when the correct model form is not known a priori.

INTRODUCTION

Traditionally, water resource engineers have employed Rippl's mass curve approach [*Rippl*, 1883] or the sequent peak algorithm [*Thomas and Burden*, 1963] in conjunction with the historical streamflow sequence to obtain a single estimate of the design capacity of a storage reservoir. In some applications, more complex simulation models are employed, yet the character of the result remains the same. More recently, stochastic streamflow models have been recommended for use in deriving the probability distribution of the required capacity of a storage reservoir to maintain a specified release [e.g., *Fiering*, 1967; *Burges and Linsley*, 1971; *Wallis and Matalas*, 1972; *Lettenmaier and Burges*, 1977*a*; *Hoshi and Burges*, 1978; *Hirsch*, 1979; *Klemeš et al.*, 1981; *Stedinger and Taylor*, 1982*a*, *b*; *Stedinger et al.*, 1985*a*; *Vogel*, 1985; *Vogel and Stedinger*, 1987]. In practice, one must estimate the parameters of a stochastic streamflow model from available yet relatively short hydrologic records. The large sampling errors associated with typical stochastic streamflow model parameter estimates [*Loucks et al.*, 1981, Appendix 3*c*] introduces a degree of uncertainty into the derived storage-reliability-yield (S-R-Y) relationship. Some authors argue that this uncertainty should be incorporated into the analysis [*Vicens et al.*, 1975; *Wood*, 1978; *Moss and Dawdy*, 1980; *Stedinger and Taylor*, 1982*b*; *Stedinger et al.*, 1985*a*]. The question addressed here is whether required storage capacity estimates based upon synthetic streamflow sequences, with all their limitations, provide more precise estimates of desired required storage capacity volumes than those obtained by traditional drought-of-record analyses. For tractability, overyear reservoir storage design problems based upon annual streamflow sequences are considered.

Klemeš [1979] and *Klemeš et al.* [1981] examined the variability associated with estimates, based on synthetic sequences, of the annual reliability of a storage reservoir of fixed capacity; they found that an approximate 95% confidence interval for the reservoirs' annual reliability ranged from 0.80 to 1.00, when the mean and variance of an annual streamflow model were estimated from a 25-year record of annual streamflows with a coefficient of variation equal to 0.30. *Phatarford* [1977] used a first-order analysis to estimate the sampling variability of estimates of required storage capacity for independent gamma distributed annual inflows. His results indicate enormous bias and variance associated with estimates of the storage capacity even with a 50-year record; for a 50-year streamflow record with coefficient of variation equal to 0.4 and demand equal to 90% of the mean annual streamflow, the upward bias and standard error of the required storage capacity estimate are 69 and 56%, respectively, of the true reservoir capacity. These three studies document the instability of estimates of the S-R-Y relationship for independent inflows. The instability of S-R-Y estimates should be even greater if one considers autocorrelated streamflows and streamflow model uncertainty.

We consider two alternatives for calculating reservoir design-capacity estimates. Reservoir operations may be simulated using either the historical streamflow record, or a large number of synthetic streamflow sequences. The use of synthetic streamflow sequences is a relatively new approach for the practicing hydrologist. Traditionally, the design of a storage reservoir was based on the required capacity S equal to the minimum storage required over the n-year historical period which provides the target yield without shortages. This single estimate of required storage capacity $\hat{S}$ does not provide an estimate of the reliability of the performance of the storage reservoir. Nevertheless, one may interpret $\hat{S}$ as a nonparametric estimator of the median of the distribution of required storage capacity, based upon an n-year planning period, where n is the length of the historical streamflow record. Here we denote this traditional estimator by $\hat{S}_{50}{}^{h}$. The superscript h denotes an estimator based solely upon the his-

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toric record; the subscript 50 denotes the $p = 0.50$ percentile or median of the distribution of S.

Synthetic streamflow sequences generated using stochastic streamflow models provide an alternate approach for estimating S_{50}. Because a single historical sequence yields only a single value of S_{50} corresponding to the worst critical drawdown sequence in the record, the distribution of that single largest value is quite unstable, just as the largest flood to occur in an n-year period has a very large sampling variance. An idea behind synthetic hydrology is to develop a model of the marginal distribution of streamflows and their persistence so as to capture the joint distribution of n-year streamflow time series, and hence a smoothed estimate of the distribution of n-year required overyear reservoir storage capacity. This is analagous to the procedure of fitting a probability distribution to a 50-year flood record to obtain a better estimate of the 50-year flood than is obtained by simply using the largest observed flood peak. For normal or lognormal variates, the asymptotic relative efficiency of using a fitted distribution to estimate the pth quantile is $[\phi(\delta)]^2\ (1 + \delta^2/2)/[p(1-p)]$, where $\delta = \Phi^{-1}[p]$ and Φ and ϕ are the standard normal distribution's cummulative distribution function (CDF) and probability density function (pdf); the relative asymptotic efficiency ranges from 1.6 for $p = 0.5$, to 2.8 for $p = 0.02$.

Here generalized S-R-Y relationships developed by *Vogel and Stedinger* [1987] are employed to mimic the results of using synthetic annual or monthly stochastic streamflow sequences. An estimate of the pth quantile of the distribution of required reservoir storage capacity so obtained is denoted by $\hat{S}_p^s$.

In practice, a design engineer must use a single historical streamflow record to estimate $\hat{S}_{50}^h$ and $\hat{S}_{50}^s$. The Monte-Carlo studies use many possible "historical" streamflow records from a hypothesized population to compare the bias and root-mean-square error (rmse) of $\hat{S}_{50}^h$ and $\hat{S}_{50}^s$. Such Monte-Carlo experiments can be revealing. For example, *Vogel and Hellstrom* [1988] performed a similar study of Boston's water supply system. They found that a 99% confidence interval for the system "safe yield" ranged from 232 million gallons/day (mgd) to 370 mgd, though the average safe yield was 300 mgd. Similarly, Staschus and Kelman (unpublished manuscript, 1988) compared the rmse associated with reliability-based dependable electric generating capacity levels for California's Central Valley Project. They found no significant advantage to fitting a stochastic streamflow model when compared to the strict use of the historic streamflows.

The importance of choosing the correct stochastic streamflow model for use in estimating the S-R-Y relationship has been a continuing concern discussed by *Fiering* [1967], *Askew et al.* [1971], *Wallis and Matalas* [1972], *Wallis and O'Connell* [1973], *Hirsch* [1979], *Klemeš et al.* [1981], *Stedinger and Taylor* [1982*a*], and others. *Stedinger and Taylor* [1982*a*] showed for their example that the impact of incorporating parameter uncertainty into a relatively simple stochastic streamflow model was much greater than the impact of model choice. Our experiments also consider the sampling properties of estimates of S_p obtained when a hypothetical hydrologist fits the wrong, but a reasonable model, to the flows available at a gaged site.

Estimation of the Overyear Storage-Reliability-Yield Relationship

Vogel and Stedinger [1987] report general S-R-Y relationships for the case when annual inflows are characterized by a two-parameter lognormal distribution and follow a first-order Markov process. These analytical approximations of the pth quantile of the distribution of overyear storage capacity can be expressed as

$$S_p^s = f(m, C_v, \rho_1, N, \alpha, p) \tag{1}$$

with

$$m = (1-\alpha)\frac{\mu}{\sigma} = \frac{1-\alpha}{C_v}$$

where

- m standardized inflow;
- μ mean of annual streamflows;
- σ standard deviation of annual streamflows;
- C_v coefficient of variation of annual streamflows;
- ρ_1 autocorrelation of annual streamflows;
- α demand as a fraction of μ;
- N planning period length.

Equation (1) is employed to mimic the annual inflow analysis performed by a hydrologist who fits the first-order autoregressive model

$$X_{t+1} = \mu_x + \rho_1(x)(X_t - \mu_x) + \varepsilon_t\sigma_x(1 - \rho_1^2(x))^{1/2} \tag{2}$$

to the transformed annual flows $X_t = \ln[Q_t]$. The Q_t are annual streamflows, and the ε_t are independent normal disturbances with zero mean and unit variance; μ_x, σ_x^2, and $\rho_1(x)$ are the mean, variance, and serial correlation of the log-transformed streamflows. The model in (2) will be referred to as the autoregressive (AR)(1) lognormal model.

After fitting (2) to an observed sequence of annual streamflows, the hydrologist could generate M sets of N-year streamflow sequences; each trace could then be processed with the sequent peak algorithm [*Thomas and Burden*, 1963; *Loucks et al.*, 1981, p. 235] to obtain M estimates of the required capacity S_i. A distribution could be fit to the sample $\{S_i, i = 1, \ldots, M\}$ to obtain an estimate $\hat{S}_p^s$ of S_p. The function in (1) provides a much quicker but analogous approach.

The mean, variance, and lag-one autocorrelation of the annual flows Q required in (2) were estimated from the corresponding statistics calculated from the log-transformed values $X_t = \ln[Q_t]$ of the generated flows and the appropriate transformations [*Loucks et al.*, 1981, p. 285]. *Stedinger* [1980, 1981] showed that these estimators generally have lower rmse than the method-of-moments estimators. Substitution of $\hat{\mu}$ and $\hat{\sigma}$, a fixed α, $\hat{\rho}_1$, p, and C_v, and an assumed value for N into the expression in (1) leads to the estimator $\hat{S}_p^s$ [see *Vogel*, 1985; *Vogel and Stedinger*, 1987].

Monte Carlo Experiments

Experimental Design

All of the experiments follow the same general procedure. First 10,000 sets of n-year annual streamflow traces are generated from one of four stochastic streamflow models. Our hypothetical hydrologist then fits an AR (1) lognormal (LN) model to each streamflow sequence. Each fitted model yields a value of $\hat{S}_p^s$ via (1).

In this study, an AR(1) LN model is always the hydrologist's choice. It is a hydrologically reasonable model and general S-R-Y relationships are available. *Markovic* [1965] showed that annual streamflow volumes in the western United States are well-approximated by the two-parameter lognormal distribution at 398 out of 446 sites considered using a 5% significance level hypothesis test.

In practice, having selected an AR(1) LN, or some other model a hydrologist would then proceed to generate a large number of synthetic streamflow sequences so as to be able to obtain $\hat{S}_p{}^s$. However, for the AR(1) LN model, the functional relationship between that model's parameters and quantiles of the required overyear storage capacity distribution are available in *Vogel and Stedinger* [1987], as indicated in (1). Thus our hydrologist used (1) in lieu of actually generating synthetic sequences with the fitted model. The regression equations for $S_p{}^s$ developed in *Vogel* [1985] and *Vogel and Stedinger* [1987] are not completely general. They are valid for $0.1 \leq m \leq 1.0$, $0.1 \leq C_v \leq 0.5$, $0.0 \leq \rho_1 \leq 0.5$ and $20 \leq N \leq 100$. In the following Monte-Carlo experiments, samples are screened to assure that $m \geq 0.1$; estimates of m below 0.1 result in unreasonably large storage capacities and the regression equations are not accurate in that range. Samples with $m < 0.1$ are rejected and additional samples are generated until 10,000 acceptable samples are obtained. Similarly samples with estimates of C_v outside the range $0.1 \leq C_v \leq 0.5$ were rejected. It is not uncommon for the stochastic hydrologist to obtain $\hat{\rho}_1 < 0$, however most annual flow records exhibit $\hat{\rho}_1$ in the range 0.0–0.5 [see *Matalas*, 1963; *Yevjevich*, 1964]. Because most hydrologists would probably use an AR(0) model in situations when $\hat{\rho}_1 < 0$, $\hat{\rho}_1$ was set to zero in those situations. *Vogel and Stedinger*'s [1987] equations provide physically realistic estimates of S_p even for $\rho_1 > 0.5$ and $m > 1.0$; thus samples with $\hat{\rho}_1 > 0.5$ or $\hat{m} > 1.0$ are accepted. The constraints placed on accepted samples serve to reduce the variability of both $\hat{S}_{50}{}^h$ and $\hat{S}_{50}{}^s$ by excluding unrealistic design situations.

Stochastic Streamflow Models

The four stochastic streamflow models employed in the experiments are described below. The models were selected to represent a range of flow distributions and autocorrelation structures.

AR(1) normal (N) model. The AR(1) N synthetic flow records were generated using (2) with $X_t = Q_t$, rather than using a log transformation.

AR(1) lognormal model. The AR(1) LN synthetic flow records were generated using (2).

AR(1) gamma (G) model. The AR(1) G synthetic flow records were generated using

$$Q_{t+1} = \mu + \rho_1(Q_t - \mu) + v_t\sigma(1 - \rho_1{}^2)^{1/2} \tag{3}$$

where the v_t are specially distributed random disturbances described by *Lawrance* [1978]. *Obeysekera and Yevjevich* [1985, 1986] reviewed this procedure in more detail.

Autoregressive moving average (ARMA)(1,1) normal model. ARMA time series models have received considerable attention in the continuing debate as to which stochastic model best represents annual streamflow series. *O'Connell* [1974] suggested that ARMA(1,1) models can approximate fractional Gaussian noise. *Fiering* [1967] and *Salas and Smith* [1981] derive an ARMA(1,1) model of annual streamflows from a conceptual watershed model driven by an independent precipitation process and a linear groundwater storage process. Here an ARMA(1,1) model is employed to examine the impact of different persistence structures.

Although an ARMA(1,1) normal model is not used in these experiments, it is described here to clarify the development of the ARMA(1,1) lognormal model actually employed. Normal annual streamflows which follow a first-order autoregressive moving average model can be generated via

$$Q_{t+1} = \mu + \phi(Q_t - \mu) + v_{t+1} - \theta v_t \tag{4}$$

where the v_t are normally distributed random disturbances with zero mean and variance $\sigma^2(1 - \phi^2)/(1 - 2\theta\phi + \theta^2)$. The parameters of an ARMA(1,1) model may be determined from the estimated moments of the streamflows [see *Box and Jenkins*, 1976]. In general, the correlation structure of an ARMA(1,1) model is

$$\rho_1 = \frac{(1 - \phi\theta)(\phi - \theta)}{1 + \theta^2 - 2\phi\theta} \qquad k = 1 \tag{5a}$$

$$\rho_k = \rho_1\phi^{k-1} \qquad k \geq 2 \tag{5b}$$

ARMA(1,1) LN model. ARMA(1,1) LN streamflows may be conveniently generated using (4) to generate the logarithms of the flows. *Lettenmaier and Burges* [1977*a*, *b*] describe the ARMA(1,1) LN model in more detail. *Stedinger et al.* [1985*b*] derive a simple and general expression for generating "start-up" values for univariate and multivariate ARMA(1,1) series which we employed.

Description of Long-Term Persistence

Several studies have examined the S-R-Y relationships which result from flows with different autocorrelation structures. For example, *Wallis and Matalas* [1972] compared the storage requirements resulting from the use of an AR(1) annual flow model with those resulting from the use of fractional gaussian noise. They concluded that an AR(1) model is an operationally useful model for $\alpha < 0.8$, even if the real world is more accurately described by a fractional Gaussian noise process. *Lettenmaier and Burges* [1977*a*] showed that ARMA(1,1) models approximate fractional Gaussian noise models if one's purpose is to estimate the CDF of required reservoir storage volumes. *Klemeš et al.* [1981] conclude that in view of the inescapable socioeconomic and hydrologic uncertainties associated with typical reservoir planning and design applications "the replacement of short-memory models with long-memory models in reservoir analysis cannot be objectively justified." What has not been examined are the sampling properties of estimators of quantiles of the CDF of required storage when a short-memory model is fit to streamflow sequences which exhibit greater persistence.

The ARMA(1,1) model structure can exhibit a range of persistence structures. Two cases are considered corresponding to ARMA(1,1) models which exhibit relatively short-term persistence (stp) and relatively long-term persistence (1tp) structures. Long-term persistence corresponds to large values of ϕ (see equation (5)). Table 1 reports the values of θ_x, ϕ_x, and $\rho_1(x)$ used in (4) to generate ARMA(1,1) LN synthetic flow sequences which exhibit a lag-one autocorrelation ρ_1 equal to 0.3 and greater persistence than obtained with the AR(1) models.

Details of the Experiment

The four stochastic streamflow models are used to generate 10,000 streamflow sequences which yielded an equal number of required storage capacities S_i ($i = 1, \ldots, 10{,}000$). The true values of the quantiles of the distribution of S corresponding to each flow model were determined from the empirical distribution.

In these experiments the autocorrelation of nature's annual streamflow model was always 0.3. This value is representative of many basins in the United States. For example, *Matalas*

TABLE 1. Parameters of ARMA(1,1) Lognormal Model

Case	σ	σ_x	$\rho_1(x)$	ϕ_x	θ_x
Short-term persistence	0.25	0.2462	0.3064	0.8015	0.5695
Short-term persistence	0.40	0.3853	0.3159	0.8037	0.5658
Long-term persistence	0.25	0.2462	0.3064	0.9504	0.7983
Long-term persistence	0.40	0.3853	0.3159	0.9511	0.7958

Here, μ = 1 and ρ_1 = 0.3.

[1963] and *Yevjevich* [1964] report values of ρ_1 in the range 0.0–0.5 throughout the United States. To allow a comparison of the use of stochastic streamflow models ($\hat{S}_{50}{}^s$) with the strict use of the historic record ($\hat{S}_{50}{}^h$), most of our examples use a planning period N equal to the gaged record length n.

RESULTS

Bias and Variance of Storage Capacity Estimators

In the first set of experiments $n = N = 40$ or 80; $\alpha = 0.8$ or 0.9; and $C_v = 0.25$ or 0.40. This range of values of C_v is typical of basins in the eastern United States. For example, an estimate of C_v equal to 0.34 was obtained for the Quabbin Reservoir watershed in Massachusetts [*Vogel and Hellstrom*, 1988]. Figure 1 and Tables 2 and 3 document the bias and rmse of $\hat{S}_{50}{}^h$ and $\hat{S}_{50}{}^s$ when the streamflows arise from AR(1) LN, AR(1) N, AR(1) G, ARMA(1,1)-LN-stp and ARMA(1,1)-LN-ltp models.

When streamflows arise from an AR(1) N, AR(1) LN, or AR(1) G model, the rmse associated with $\hat{S}_{50}{}^h$ is always in excess of 36% of S_{50}. When streamflows arise from an ARMA(1,1) LN model, the rmse associated with $\hat{S}_{50}{}^h$ always exceeded 45% of S_{50}. In the cases examined, $\hat{S}_{50}{}^h$ exhibits upward bias. The bias is to be expected because the bias is computed as $(E[\hat{S}_{50}{}^h] - S_{50})$, where $E[\hat{S}_{50}{}^h] = E[S] = \mu_S$. The distribution of S is generally positively skewed and is often well-approximated by a three-parameter lognormal or a Gumbel extreme value type I distribution [*Vogel and Stedinger*, 1987; *Burges and Linsley*, 1971; *Hoshi et al.*, 1978] and hence its mean μ_s is greater than its median S_{50}.

The results with $\hat{S}_{50}{}^h$ and $\hat{S}_{50}{}^s$ in Tables 2 and 3 and in Figure 1 provide a comparison of the use of the AR(1) LN model and the use of no model at all. In general, Table 3 and Figure 1 shows that use of an AR(1) LN model always leads to more precise (lower rmse) estimates of S_{50} even in situations when an AR(1) LN model is not the correct model.

Table 2 documents a downward bias (which increases with α) associated with the estimator $\hat{S}_{50}{}^s$ when an AR(1) LN model is fit to streamflow sequences with an ARMA(1,1) LN parent. This result was to be expected: models which exhibit long-term persistence result in larger storage requirements than a simple AR(1) model would indicate, particularly for $\alpha \geq 0.8$, as is the case here (see *Wallis and Matalas* [1972], for additional examples).

If an AR(1) LN model is the correct model, an increase in the gaged record length n, and planning period N, leads to substantial reductions in both the bias and rmse of $\hat{S}_{50}{}^s$. This is not the case when an AR(1) LN model is fit to flows which originate from an ARMA(1,1) LN model. Table 2 shows that as $n = N$ increases from 40 to 80 the upward bias associated with the estimator $\hat{S}_{50}{}^s$ increases dramatically, particularly for the long-term persistence ARMA(1,1) LN case. However, for small samples (i.e., $n \leq 40$) fitting the short-memory AR(1) LN model leads to approximately the same sampling properties for $\hat{S}_{50}{}^s$ as if nature were just AR(1); on the other hand, the rmse of $\hat{S}_{50}{}^h$ increased dramatically.

In general, fitting long-memory models to flow sequences which arise from long-memory models would lead to greater variability in estimates of S_p than if short-memory models are fit to flow sequences which arise from short-memory models. A short-memory AR(1) model has one less parameter than a long-memory ARMA(1,1) model and parameter estimates are more reliable when the flows are less persistent. Thus given the small contribution of the bias to the rmse, if we had fit an ARMA(1,1) LN model to annual flow sequences so as to estimate S_{50}, its rmse would almost surely have been larger than that of the AR(1) LN model.

Tables 2 and 3 and Figure 1 also illustrate the impact of fitting an AR(1) LN model when flows arise from AR(1) N and AR(1) G models. In these cases, a lognormal distribution closely resembles a gamma distribution in the important low-flow portion of their distributions. Hence one would expect the AR(1) LN model to perform rather well when the flows arise from an AR(1) G model, as is the case in Tables 2 and 3. Table 3 reveals that fitting an AR(1) LN model when flows are AR(1) N leads to an increase in the rmse of $\hat{S}_{50}{}^s$ in comparison with the case when flows are AR(1) LN. However, the increased rmse may be as much due to the normal distributions thicker left-hand tail (which would effect the distribution of the moments of the logarithms of the flows) as to use of the wrong streamflow model.

On the basis of Tables 2 and 3 and Figure 1, the AR(1) LN model appears to provide an extremely robust procedure for estimating S_{50}, particularly for the small samples ($n = 40$) so frequently encountered in practical situations. For larger samples and planning period lengths ($n = N = 80$), the AR(1) LN model still performs well, moreover, in these situations one is

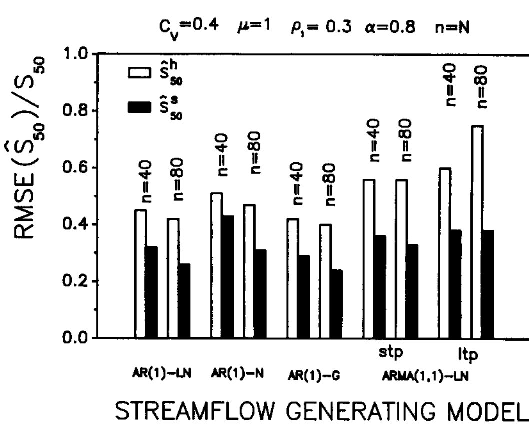

Fig. 1. Root-mean-square error of $\hat{S}_{50}{}^s$ when an AR(1) LN model is fit to flow sequences from four different parents compared with the rmse of $\hat{S}_{50}{}^h$ corresponding to each of those four parents.

TABLE 2. Bias of $\hat{S}_{50}{}^s$ and $\hat{S}_{50}{}^h$ When the True Flow Models are AR(1) Lognormal, AR(1) Normal, AR(1) Gamma, and ARMA(1,1) Lognormal but the Hydrologist Employs an AR(1) Lognormal Model, $[E[w] - S_{50}]/S_{50}$

			AR(1) Lognormal		AR(1) Normal		AR(1) Gamma		ARMA(1,1)-LN-stp		ARMA(1)-LN-ltp	
			None	AR(1) LN	None	AR(1) LN	None	AR(1) LN	None	AR(1) LN	None	AR(1) LN
C_v	α	$n = N$	$w = \hat{S}_{50}{}^h$	$w = \hat{S}_{50}{}^s$	$w = \hat{S}_{50}{}^h$	$w = \hat{S}_{50}{}^s$	$w = \hat{S}_{50}{}^h$	$w = \hat{S}_{50}{}^s$	$w = \hat{S}_{50}{}^h$	$w = \hat{S}_{50}{}^s$	$w = \hat{S}_{50}{}^h$	$w = \hat{S}_{50}{}^s$
0.25	0.8	40	0.09	−0.08	0.10	−0.17	0.09	−0.13	0.12	−0.10	0.08	−0.11
0.25	0.8	80	0.07	−0.04	0.09	−0.15	0.07	−0.10	0.13	−0.11	0.16	−0.08
0.25	0.9	40	0.04	−0.08	0.06	−0.01	0.05	−0.05	0.06	−0.15	0.02	−0.16
0.25	0.9	80	0.04	−0.02	0.07	−0.00	0.05	−0.04	0.05	−0.23	0.07	−0.30
0.40	0.8	40	0.03	−0.07	0.22	−0.18	0.05	−0.06	0.05	−0.17	0.01	−0.16
0.40	0.8	80	0.06	−0.03	0.23	−0.17	0.07	−0.03	0.08	−0.22	0.12	−0.25
0.40	0.9	40	0.00	−0.07	0.25	−0.07	0.03	0.01	−0.02	−0.22	−0.04	−0.24
0.40	0.9	80	0.02	−0.03	0.29	−0.04	0.06	0.03	0.00	−0.28	−0.04	−0.38

This table is based upon 10,000 replicate experiments. Here, $\mu = 1$ and $\rho_1 = 0.3$; stp, short-term persistence; ltp, long-term persistence.

less apt to choose the wrong model, since larger samples allow for a more reliable determination of the autocorrelation structure of annual and monthly streamflow series.

Alternative Design Problems

The Monte-Carlo experiments summarized in Tables 2 and 3 and Figure 1 assume that α, the level of development, is fixed. This corresponds to situations in which one wishes to regulate a predetermined fraction of the total available streamflow in a basin. For example, a particular region's water plan may anticipate development of a fixed percentage of its water resources for irrigation, water supply, and/or hydropower. Of course, there is no guarantee that the design capacity which results would satisfy other environmental, recreational, or structural feasibility constraints. When the level of development is fixed, the estimated regulated outflow or demand D becomes a random variable, since one must use an estimate $\hat{\alpha}$ of α to obtain $\hat{D} = \hat{\alpha}\mu$.

For high-value water uses, such as municipal and industrial activities, it may be more reasonable to consider the reservoir design problem as one in which the demand D is essentially fixed by the projected levels for those activities, and the problem is to determine the size of the reservoir necessary to meet the projected demand levels.

Then, the estimated level of development becomes a random variable: $\hat{\alpha} = D/\hat{\mu}$. Thus two general design situations can be considered: (1) α is fixed and the corresponding design demand $\hat{D}$ is a random variable or (2) D is fixed and the estimated level of development $\hat{\alpha}$ is a random variable. In general, reality will lie somewhere between these simple extremes. When the level of development is unknown and the demand is fixed, it is possible to generate streamflow sequences for which the mean $\hat{\mu}$ is less than the demand D. This is particularly true in situations when the record length n is small and α is near unity.

Experiments were performed to examine the sampling variability of $\hat{S}_{50}{}^h$ and $\hat{S}_{50}{}^s$ when the demand is fixed. Table 4 summarizes the bias and rmse of these estimators for the cases when α is held constant (and demand is variable as in Tables 2 and 3) and when it is a random variable (and demand is given); in both cases the hydrologist employs the correct stochastic streamflow model: AR(1) LN. Fixing demand leads to larger rmse's of both $\hat{S}_{50}{}^h$ and $\hat{S}_{50}{}^s$. Thus Table 3 understates the variability in design applications in which the demand is fixed. As in Table 3 and Figure 1, the rmse of $\hat{S}_{50}{}^s$ is substantially lower than that of $\hat{S}_{50}{}^h$.

In practice, the design of a storage reservoir is more complex than indicated by this study. Multiple and competing objectives related to water supply, hydropower, recreation, irrigation and flood control complicate the design problem as do seasonal fluctuations in streamflow series and projected demands. The choice of a reservoir design capacity is often coupled with the choice of a treatment plant capacity, hydropower plant capacity, or irrigation network. *Jettmar and Young* [1975], *Vicens et al.* [1975], and *Moss and Dawdy* [1980] have considered economic ramifications associated

TABLE 3. rmse of $\hat{S}_{50}{}^s$ and $\hat{S}_{50}{}^h$ When the True Flow Models are AR(1) Lognormal, AR(1) Normal, AR(1) Gamma, and ARMA(1,1) Lognormal But the Hydrologist Employs an AR(1) Lognormal Model, rmse $[w]/S_{50}$

			AR(1) Lognormal		AR(1) Normal		AR(1) Gamma		ARMA(1,1)-LN-stp		ARMA(1)-LN-ltp	
			None	AR(1) LN	None	AR(1) LN	None	AR(1) LN	None	AR(1) LN	None	AR(1) LN
C_v	α	$n = N$	$w = \hat{S}_{50}{}^h$	$w = \hat{S}_{50}{}^s$	$w = \hat{S}_{50}{}^h$	$w = \hat{S}_{50}{}^s$	$w = \hat{S}_{50}{}^h$	$w = \hat{S}_{50}{}^s$	$w = \hat{S}_{50}{}^h$	$w = \hat{S}_{50}{}^s$	$w = \hat{S}_{50}{}^h$	$w = \hat{S}_{50}{}^s$
0.25	0.8	40	0.55	0.35	0.55	0.40	0.55	0.35	0.70	0.39	0.65	0.38
0.25	0.8	80	0.45	0.27	0.46	0.28	0.45	0.26	0.63	0.30	0.79	0.36
0.25	0.9	40	0.43	0.31	0.44	0.36	0.44	0.30	0.55	0.35	0.57	0.34
0.25	0.9	80	0.39	0.24	0.40	0.29	0.38	0.24	0.49	0.32	0.65	0.39
0.40	0.8	40	0.45	0.32	0.51	0.43	0.42	0.29	0.56	0.36	0.60	0.38
0.40	0.8	80	0.42	0.26	0.47	0.31	0.40	0.24	0.56	0.33	0.75	0.38
0.40	0.9	40	0.39	0.29	0.51	0.35	0.37	0.27	0.46	0.34	0.49	0.37
0.40	0.9	80	0.37	0.23	0.51	0.30	0.36	0.23	0.43	0.35	0.52	0.44

This table is based upon 10,000 replicate experiments. Here, $\mu = 1$ and $\rho_1 = 0.3$; stp, short-term persistence; ltp, long-term persistence.

TABLE 4. Comparison of the Bias and rmse of $\hat{S}_{50}^s$ and $\hat{S}_{50}^h$ When a Fixed Level of Development α and the Estimate $\hat{\alpha}$ are Used to Estimate m When Inflows are AR(1) Lognormal and the Hydrologist Employs an AR(1) Lognormal Model

			$[E[w] - S_{50}]/S_{50}$				$\mathrm{rmse}[w]/S_{50}$			
			α		$\hat{\alpha}$		α		$\hat{\alpha}$	
C_v	α	$n = N$	$w = \hat{S}_{50}^h$	$w = \hat{S}_{50}^s$	$w = \hat{S}_{50}^h$	$w = \hat{S}_{50}^s$	$w = \hat{S}_{50}^h$	$w = \hat{S}_{50}^s$	$w = \hat{S}_{50}^h$	$w = \hat{S}_{50}^s$
0.25	0.8	40	0.09	−0.08	0.14	−0.02	0.55	0.35	0.67	0.46
0.25	0.8	80	0.07	−0.04	0.10	−0.01	0.45	0.27	0.50	0.32
0.25	0.9	40	0.04	−0.08	0.14	0.02	0.43	0.31	0.68	0.43
0.25	0.9	80	0.04	−0.02	0.11	0.02	0.39	0.24	0.51	0.35
0.40	0.8	40	0.03	−0.07	0.13	0.01	0.45	0.32	0.62	0.44
0.40	0.8	80	0.06	−0.03	0.09	0.01	0.42	0.26	0.48	0.34
0.40	0.9	40	0.00	−0.07	0.22	0.01	0.39	0.29	0.85	0.37
0.40	0.9	80	0.02	−0.03	0.14	0.01	0.37	0.23	0.64	0.32

This table is based upon 10,000 replicate experiments. Here, $\mu = 1$ and $\rho_1 = 0.3$

with the design of a storage reservoir in the context of streamflow model choice and streamflow model parameter uncertainty.

Likely Range of System Reliabilities

Another situation is also of interest. Given that a reservoir of capacity S_p is built, and long-term contracts are signed for water D, what is the systems' actual reliability? To answer that question, this section examines the distributions of $\hat{S}_{50}^h$, $\hat{S}_{50}^s$, and $\hat{S}_{90}^s$ generated in each of the previous Monte-Carlo experiments. Normal (N), two-parameter lognormal (LN2), and three-parameter lognormal (LN3) distributions were fit to the 10,000 values of $\hat{S}_{50}^h$, $\hat{S}_{50}^s$, and $\hat{S}_{90}^s$ generated in each of the previous Monte-Carlo experiments. Filliben's probability plot correlation coefficient [*Filliben*, 1975; *Vogel*, 1986] was used as a goodness-of-fit statistic. The LN3 distribution was the only distribution which yielded probability plot correlation coefficients in excess of 0.99 for all the cases considered. It was chosen to approximate their pdfs.

To evaluate the range of values of $\hat{S}_{50}^h$, $\hat{S}_{50}^s$, and $\hat{S}_{90}^s$ one is likely to obtain in practice, we obtained the qth quantile of the distribution of the estimators $\hat{S}_{50}^h(q)$, $\hat{S}_{50}^s(q)$, and $\hat{S}_{90}^s(q)$ from the fitted three-parameter lognormal distributions. The likely range of values using these estimators was represented by the interval between the 2.5th percentile and the 97.5th percentile of their distributions. That is, the likely range (or the 2.5–97.5% range) is that interval in which values of these estimators will fall 95% of the time. Figure 2 illustrates the likely range for $\hat{S}_{50}^h$ and $\hat{S}_{50}^s$ when inflows are AR(1) LN. The true values of S_{50} are depicted by diamonds.

The likely range for the traditional estimator $\hat{S}_{50}^h$ is truly alarming; even with a sample of length $n = 80$, one only obtains an "order-of-magnitude" estimate of the design capacity using the historical record. Use of $\hat{S}_{50}^s$ reduces this design interval considerably.

Figure 3 compares the likely ranges for $\hat{S}_{50}^s$ and $\hat{S}_{90}^s$ with $\mu = 1$, $C_v = 0.25$, and $\alpha = 0.80$. Since Figure 3 does not include $\hat{S}_{50}^h$, cases could be considered for which the gaged record length n does not equal the length of the planning period N. Figure 3 also documents the impact of different ρ_1. The cases with $\rho_1 = 0.3$ and $n = 20$ lead to a wider range of likely design capacities, for the same planning period N, than cases with $\rho_1 = 0.0$ or $n = 60$. Also, the range for $\hat{S}_{90}^s$ is wider than for $\hat{S}_{50}^s$. However, for any given sample, $\hat{S}_{90}^s$ will be larger than $\hat{S}_{50}^s$.

The range of design capacities in Figures 2 and 3 may be transformed into the corresponding range of system reliabilities or nonexceedance probabilities p associated with N-year failure-free reservoir operation. Here p the probability a reservoir with design capacity S_{50} when operated to supply $\hat{D} = \alpha\hat{\mu}$ will operate without failure over an N-year planning period. To accomplish this transformation, let the function $p[\hat{S}_{50}^s(q)]$ be the nonexceedance probability associated with N-year failure-free reservoir operation with reservoir capacity $\hat{S}_{50}^s(q)$. This function is approximated using the S-R-Y relationships in the work by *Vogel and Stedinger* [1987]. However, those

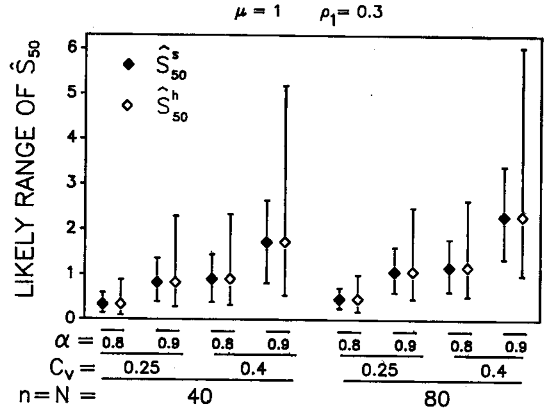

Fig. 2. Likely range of $\hat{S}_{50}^s$ when an AR(1) LN model is fit to flow sequences which arise from an AR(1) LN parent compared with the likely range of $\hat{S}_{50}^h$. True value S_{50} is denoted using diamonds with error bars depicting the 2.5th and the 97.5th percentiles of each estimator.

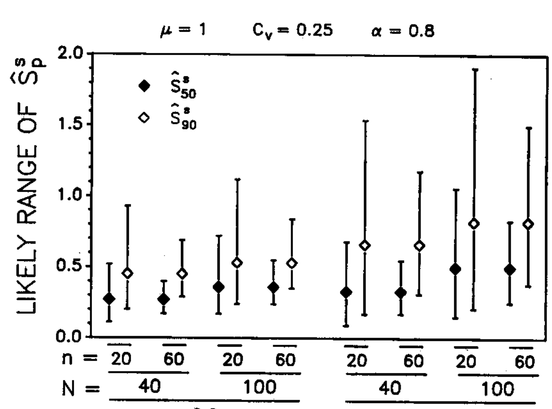

Fig. 3. Likely range of $\hat{S}_{50}^s$ and $\hat{S}_{90}^s$ when an AR(1) LN model is fit to flow sequences which arise from an AR(1) LN parent. True values S_{50} and S_{90} are denoted using diamonds with error bars depicting the 2.5th and the 97.5th percentiles of each estimator.

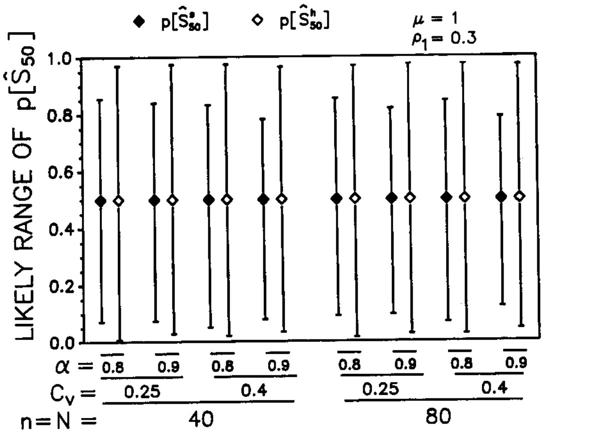

Fig. 4. Likely range of the estimated system reliability $p[\hat{S}_{50}{}^s]$ when an AR(1) LN model is fit to flow sequences which arise from an AR(1) LN parent as compared with the likely range of the estimated system reliability $p[\hat{S}_{50}{}^h]$. True system reliability $p[\hat{S}_{50}]$ is denoted using diamonds with error bars depicting the likely range of each estimate.

S-R-Y relationships generate biased estimates of S_p for $p < 0.05$ and $p > 0.95$; hence the values of $p[\hat{S}_p(q)]$ reported here are slightly biased.

Figures 4 and 5 display approximate values of $p[\hat{S}_{50}{}^h(q)]$, $p[\hat{S}_{50}{}^s(q)]$, and $p[\hat{S}_{90}{}^s(q)]$ for $q = 0.025$ and $q = 0.975$. The variability associated with the probability of N-year failure-free reservoir operation when one uses $\hat{S}_{50}{}^h$, $\hat{S}_{50}{}^s$, or $\hat{S}_{90}{}^s$ is astonishing, even with gaged record lengths equal to 80 years. *Stedinger et al.* [1983, p. 1392] show that the no-failure system reliability p associated with N-year failure-free reservoir operation when demand is fixed (rather than α) is a uniform random variable distributed between zero and one; therefore a 95% confidence interval for $p[\hat{S}_{50}{}^s]$ is [0.025, 0.975]. The reliabilities and their likely ranges, reported here, correspond to systems dominated by overyear storage requirements. *Vogel* [1987] and *Stedinger et al.* [1983] provide simplified relations between N-year failure-free reliability and annual reliability for systems dominated by within-year storage requirements.

Detailed Evaluation of the Sampling Properties of $\hat{S}_p{}^s$

Table 5 illustrates the sampling properties of $\hat{S}_{50}{}^s$ and $\hat{S}_{90}{}^s$ for a wider range of ρ_1, n, N, and p values than those in Tables 2 and 3. All of the results assume the hydrologist has chosen the AR(1) LN stochastic streamflow model, which is the correct choice.

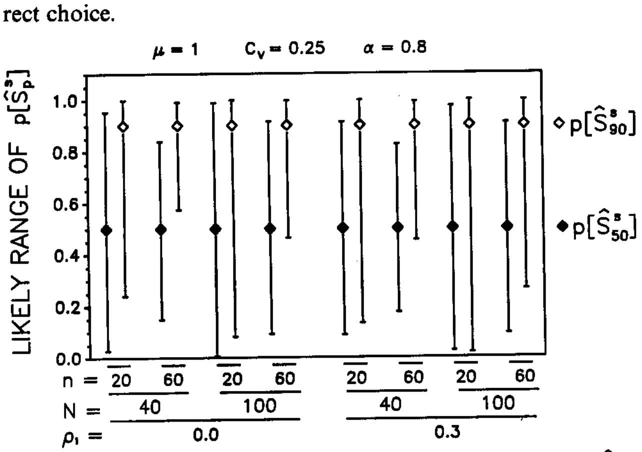

Fig. 5. Likely range of the estimated system reliabilities $p[\hat{S}_{50}{}^s]$ and $p[\hat{S}_{90}{}^s]$ when an AR(1) LN model is fit to flow sequences which arise from an AR(1) LN parent. True system reliabilities $p[\hat{S}_{50}]$ and $p[\hat{S}_{90}]$ are denoted using diamonds with error bars depicting the likely range of each estimate.

TABLE 5. Bias and rmse of $\hat{S}_p{}^s$ When the Inflows are AR(1) Lognormal

					$[E[\hat{S}_p{}^s - S_p]/S_p$		$[\text{rmse}[\hat{S}_p{}^s]]/S_p$	
ρ_1	C_v	α	N	n	$p = 0.5$	$p = 0.9$	$p = 0.5$	$p = 0.9$
0.0	0.25	0.8	40	20	−0.01	0.04	0.39	0.42
0.0	0.25	0.8	40	60	0.01	0.04	0.22	0.23
0.0	0.25	0.8	100	20	0.02	0.05	0.38	0.43
0.0	0.25	0.8	100	60	0.03	0.04	0.22	0.24
0.0	0.25	0.9	40	20	0.02	0.05	0.33	0.39
0.0	0.25	0.9	40	60	0.03	0.05	0.19	0.22
0.0	0.25	0.9	100	20	0.04	0.06	0.35	0.41
0.0	0.25	0.9	100	60	0.04	0.06	0.21	0.24
0.0	0.40	0.8	40	20	−0.03	0.00	0.33	0.37
0.0	0.40	0.8	40	60	0.02	0.04	0.21	0.23
0.0	0.40	0.8	100	20	−0.01	0.01	0.35	0.39
0.0	0.40	0.8	100	60	0.04	0.05	0.23	0.25
0.0	0.40	0.9	40	20	−0.02	0.00	0.28	0.33
0.0	0.40	0.9	40	60	0.02	0.04	0.19	0.21
0.0	0.40	0.9	100	20	0.00	0.02	0.32	0.37
0.0	0.40	0.9	100	60	0.04	0.05	0.20	0.23
0.3	0.25	0.8	40	20	−0.12	−0.10	0.48	0.57
0.3	0.25	0.8	40	60	−0.05	−0.04	0.29	0.34
0.3	0.25	0.8	100	20	−0.11	−0.10	0.49	0.58
0.3	0.25	0.8	100	60	−0.04	−0.03	0.31	0.36
0.3	0.25	0.9	40	20	−0.13	−0.11	0.41	0.51
0.3	0.25	0.9	40	60	−0.05	−0.04	0.25	0.31
0.3	0.25	0.9	100	20	−0.12	−0.11	0.45	0.54
0.3	0.25	0.9	100	60	−0.04	−0.04	0.28	0.33
0.3	0.40	0.8	40	20	−0.18	−0.17	0.41	0.48
0.3	0.40	0.8	40	60	−0.05	−0.04	0.28	0.32
0.3	0.40	0.8	100	20	−0.18	−0.17	0.42	0.49
0.3	0.40	0.8	100	60	−0.05	−0.05	0.29	0.34
0.3	0.40	0.9	40	20	−0.17	−0.17	0.36	0.43
0.3	0.40	0.9	40	60	−0.05	−0.05	0.24	0.29
0.3	0.40	0.9	100	20	−0.17	−0.17	0.40	0.46
0.3	0.40	0.9	100	60	−0.05	−0.05	0.27	0.31

Table 5 contains the bias and rmse of $\hat{S}_{50}{}^s$ and $\hat{S}_{90}{}^s$ when an AR(1) LN model is fit to flow sequences which originate from an AR(1) LN model. In general, the rmse of each estimator is due primarily to the estimator's variance. The most important conclusions which may be drawn from Table 5 is that both $\hat{S}_{50}{}^s$ and $\hat{S}_{90}{}^s$ are extremely variable. With a gaged record of length $n = 60$ years, the rmse of these estimators can be as much as $\pm 34\%$ of the true value of S_p. The use of stochastic hydrology when the true model is known results in relatively low bias, yet still imprecise (high rmse) estimates of S_p, even with gaged records of length $n = 60$.

Conclusions

This study illustrates the variability of required reservoir storage capacity estimates based on 20–80 year streamflow records. In our experiments, an AR(1) lognormal model was "fit" to "historical" flow sequences generated with four different stochastic streamflow models: AR(1) lognormal, AR(1) normal, AR(1) gamma, and an ARMA(1,1) lognormal model. These experiments document the sampling variabilities of estimators of required capacity quantiles S_p derived with stochastic streamflow models ($\hat{S}_p{}^s$), as well as from use of the historical streamflow record alone ($\hat{S}_{50}{}^h$).

In general, fitting an AR(1) lognormal model leads to more precise estimates of annual storage requirements S_{50} than if only the historical flows are employed, even in situations when the flows were not generated with an AR(1) lognormal model. However, even estimates of S_p obtained by fitting stochastic annual streamflow models to 80-year samples can be highly variable.

Recognition of the variability of reservoir storage capacity, yield, and reliability estimates is important within the context of typical reservoir system design applications. Such a realization may lead to the incorporation of uncertainty into the analysis as recommended by *Stedinger et al.* [1985*a*] and others. Arguments over which stochastic streamflow model structure to employ in a given application appear to be moot within the context of the overall problem of estimating the reservoir system storage-reliability-yield relationship.

There are other uses for annual and monthly stochastic streamflow models. Not only are they useful for estimating required reservoir capacities associated with various reliabilities, and the reliabilities associated with specified capacities, they are also useful for generating the long and rich multisite streamflow sequences to help refine estimates of the distribution of a whole set of system performance indices that may be of interest in reservoir system planning or operating studies. Such models are particularly valuable in the evaluation of alternative multireservoir operating strategies when the number of policy variables can easily overwhelm the variety of circumstances and challenges presented by a single historical streamflow record.

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