Experimental Validation of Building Vibration Propagation Using a Four Story Laboratory Model

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ABSTRACT

A 4-story scale model building was designed and constructed for the purpose of predicting vibration levels at each floor. Mathematical models of column, beam, and slab elements were developed and combined, forming a system model. Each component of the scale model building was tested in order to validate their corresponding mathematical model. Measured data from the scale model building was then compared to analytical predictions. All analytical models successfully validated their corresponding physical model by accurately predicting resonant amplifications and average vibration levels. Finally, recommendations for the modeling of full-scale structures are presented.

INTRODUCTION

In recent years floor vibrations have become a significant design consideration for engineers. The advancement of technology and building materials has increased a structure's susceptibility to floor vibrations. This is compounded by the growth of excitation sources, such as those from running trains or vehicular traffic, due to air rights development, the practice of constructing buildings over existing highways and railways. The use of an accurate predictive model is therefore necessitated to ensure human comfort as well as proper performance of vibration sensitive laboratory and manufacturing equipment in these structures.

A study by Yokoshima et al. (2005) determined vertical vibration levels are 10-15 dB higher than corresponding horizontal vibration levels for train-induced excitation. Examining the vertical propagation in buildings, Anderson (1994) found that at low frequencies (below 32 Hz) vibration levels between floors were amplified, whereas at high frequencies vibration levels were attenuated. Lee et al. (2000) successfully used finite element analysis (FEA) in conjunction with Guyan reduction to reduce mesh size, to model vertical vibration propagation in a building. However, this method had limited capability and is very sensitive to accurate selection of the appropriate master degrees of freedom.

In addition to "feelable" vibration, structurally radiated sound is also a concern. When studying audible vibrations the maximum frequency of interest is 20 kHz (Fausti, 1981). Thus, requiring extremely fine meshes, in order to capture the high frequency behavior of structures subjected to railway or traffic induced vibrations. For prediction purposes, FEA is an impractical technique as it is too computationally intensive for most full scale structures. Cremer et al. (1988) proposed modeling components as continuous, distributed elements, as opposed to discrete finite elements. This mesh-free approach is advantageous for modeling beams, girders, and slabs. Since using a FE mesh is not necessary, calculation speeds are increased while still capturing high frequency behavior. However, a simple and accurate predictive model for vibration propagation in a structure does not exist.

A series of full-scale measurements were conducted by Brett (2007) for validation of mathematical impedance models and prediction of vertical structure-borne sound and vibration propagation. The present research concerns a scale laboratory model for validation and improvement of Brett's mathematical models. The goals of this research are to develop, test, and verify impedance expressions for finite and infinite column, beam and slab components with varying boundary conditions. In addition, we design and construct a scale model building for validation of the attenuation predictions due to shaker induced vibrations.

THEORETICAL BACKGROUND

The mathematical model used to predict vibration propagation in the scale model building is a wave propagation-based transmission model of a single column connecting adjacent floors (Hughes, 2008). The complex dynamic behavior of the scale model building is idealized by making several key assumptions and simplifications. It is assumed that the response in adjacent columns, which are not directly excited and are present only to stabilize the structure, has minimal effect on the column under consideration. The predominant vibratory motion in the column is assumed to be axial, any bending or torsional motion is neglected. Floor slabs are modeled as thin isotropic rectangular plates with no discontinuities. Floor beams are assumed to be uniform and proportioned such that shear deformation is negligible. Transverse bending is assumed to be the predominate motion when modeling floors slabs and beams. Composite action between the floor slab and beams is not considered in this model. Only the beams and floor slabs which are framed directly into the column being studied are considered. Edge beams and columns are accounted for in the beams' and floor slabs' boundary conditions. Other structural complexities, such as partition walls, shear walls, or stairwells, are not considered.

Columns are modeled as two-node finite wave propagating rods and are one-dimensional elements that connect the floors above and below as a result of the propagation of axial waves in the column. Analytically, the top and bottom of the column are related using a 2-by-2 transmission matrix. The amplitudes of the force and velocity at floor slabs and beams are represented by their input impedance characteristics at the location where they are connected to the columns. Floor slabs and beams are modeled as either finite or infinite. Energy dissipation in all components is modeled using a complex modulus of elasticity, where the imaginary component is proportional to the damping loss factor, η.

System Modeling

The mathematical system model of a single story is idealized as shown in Figure 1. The floor slab and beams are modeled as lumped impedance, Z, attached at node 2 of the column. The single story configuration shown in Figure 1 is represented analytically by Equation (1).

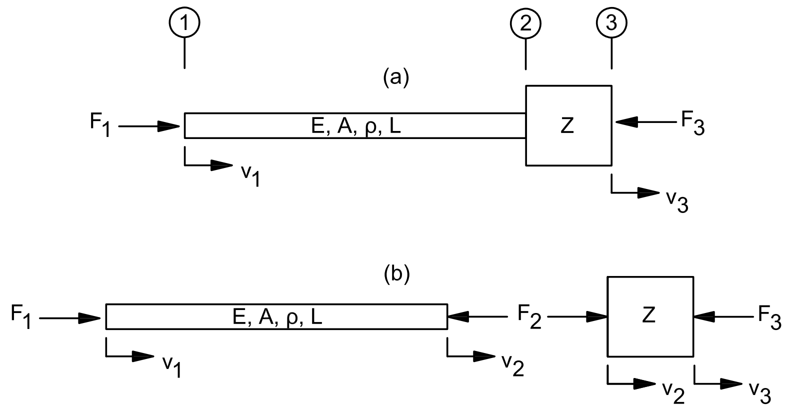

FIGURE 1
IDEALIZED SCHEMATIC OF A SINGLE FLOOR USING A WAVE PROPAGATING ROD AND LUMPED IMPEDANCE

$$\begin{Bmatrix} v_3 \\ F_3 \end{Bmatrix} = \begin{bmatrix} \cos(k_c L) & \dfrac{-i}{Z_{Inf}} \sin(k_c L) \\ -Z\cos(k_c l) - iZ_{Inf}\sin(kL) & \dfrac{i\,Z}{Z_{Inf}} \sin(k_c L) + \cos(k_c L) \end{bmatrix} \begin{Bmatrix} v_1 \\ F_1 \end{Bmatrix} \tag{1}$$

where Z_{inf} is the input impedance for a semi-infinite column under axial deformation; k_c is the wave number for axially propagating waves (Hughes, 2008); *E, A, ρ, L* represent the modulus of elasticity, cross-sectional area, mass density, and length of the column, respectively.

These sub-systems are cascaded in order to model each story of the structure. Thus, for a column subjected to an input force, F_1, and input velocity, v_1, the force and velocity in the column at each floor is known (Hughes, 2008).

DESIGN OF A SCALE MODEL BUILDING

While axial wave speed is independent of frequency, bending waves are proportional to frequency. Scaling full scale structures for vibration testing is accomplished by preserving the ratio of various component impedances from the full scale structure to the scale model, thereby simulating the full-scale transmission characteristics. The primary transmission path is column-to-slab, as beam impedance is relatively insignificant at low frequencies. Thus, assuming infinite components, the column-to-slab impedance ratio is given by:

$$Z_{ratio} = \frac{Z_{Column}}{Z_{Slab}} = \frac{A_c\sqrt{E_c\rho_c}}{8\sqrt{D_s}\sqrt{\rho_s t}} \tag{2}$$

where A is cross sectional area, E is Young's Modulus, ρ is mass density, D_s is the flexural rigidity of the slab, and t is slab thickness. Subscripts 'c' and 's' denote column and slab properties, respectively.

Impedance Matching

For a full scale structure, a W14x90 structural steel column supporting a 120 mm (4.75 in) thick reinforced concrete floor slab is assumed. Utilizing Equation (2), these components correspond to an impedance ratio of 0.42.

Aluminum columns used in the scale model building were obtained from 80/20 Incorporated who stocks a large inventory of 6105-T5 aluminum profiles and accessories. The selected column profile is model number 25-2525 which provides the instrumentation and connection versatility needed for scale model construction and testing. Additionally, for the purpose of efficiency, all beams and columns used in the scale model building are the same. The MDF floor slab in the scale model measures 1,524 mm x 1,524 mm x 19 mm (60 in x 60 in x 0.75 in). From Equation (2), these components correspond to an impedance ratio of 0.44 which is similar to a full-scale structure.

Building Configuration

Column, beam and slab sizes determined using impedance matching are approximately one-tenth the size of their counterpart in a full scale structure (Hughes, 2008). The scale model is a two-bay by two-bay, doubly symmetric, 4 story building. With component sizes on the order of a one-tenth scale, the scale model building is sized to the same degree, as shown in Figure 2.

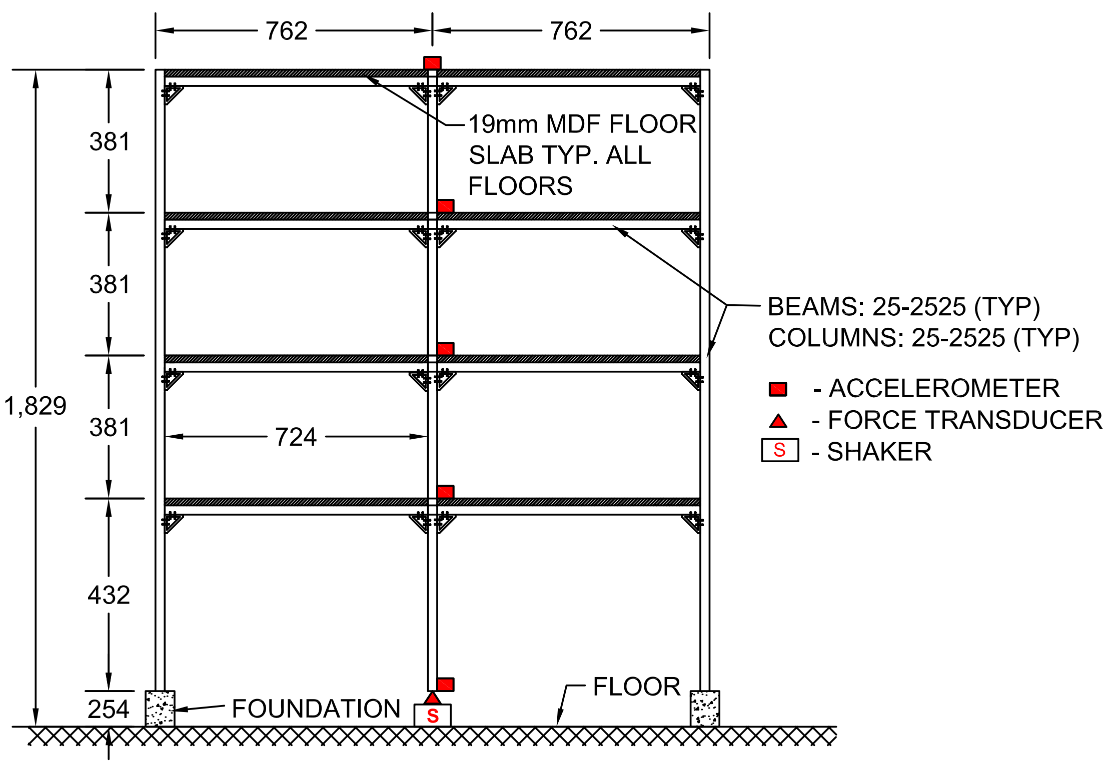

FIGURE 2
SCALE MODEL BUILDING WITH INSTRUMENTATION LOCATIONS AND DIMENSIONS (MM)

Beams are attached to the slab using 38 mm (1.5 in) 10-32 machine screws spaced at 190 mm (7.5 in). All columns are founded in five gallon sand buckets. Composite shims, which the columns bear on, sit on 51 mm (2 in) of sand and an additional 254 mm (10 in) of sand is placed around the column. This simulates the termination at a typical column foundation. For this research, the center column is studied, which is the only column not founded in a five gallon sand bucket. The center column is attached to a Brüel & Kjær Type 4808 electrodynamic shaker. The shaker to column connection is made through a threaded rod intended to drive the column axially. The shaker sits on four neoprene base isolators used to minimize excitation of the other columns by way of the lab floor. The assembled scale model building is shown in Figure 3.

COMPONENT TESTING

Analytical component models are based on idealized boundary conditions and the above assumptions. Component testing was done to determine how well the analytical models predict component behavior. Column, beam, and slab components are tested in the frequency range of 10 Hz to 5 kHz, corresponding to a full scale frequency range of 1-500 Hz. Additionally, damping in the mathematical models is accounted for using the structural loss factor, η. In general, η, which is a material property, is not a mathematically derived quantity, rather it must be measured. The structural loss factor characterizes hysteretic damping and is approximately equivalent to 2ζ, the viscous damping ratio (for small levels of damping). In order to determine damping values, model curve fitting to measurements has shown to be accurate (Zhu et al., 1989) and is employed in this research. The damping measurements determined by component testing are summarized in Table 1 (Hughes, 2008). The column element did not posses any significant damping in addition to the material damping of aluminum ($\eta=0.002$). However, the boundary conditions for beam and slab elements significantly increase damping as a result of friction or other dissipation at the boundaries. These damping values provided a good fit to component measurements; therefore they are used in the mathematical modeling of the scale model building.

Boundary Conditions	Material	η
Free-Free Beam	Aluminum	0.002
Free-Free Beam	MDF	0.02
Free-Free Column	Aluminum	0.002
Fixed-Free Beam	Aluminum	0.05
Pinned-Free Beam	Aluminum	0.04
Single Story Slab Model	MDF/Aluminum	0.04

TABLE 1
DAMPING VALUES DETERMINED BY COMPONENT TESTING

The effects of boundary conditions were studied in two beam configurations. Changing boundary conditions does not increase or decrease the impedance of components. Rather, altering boundary conditions changes the termination impedance. Therefore, a single story model is created with edge beams but without cross beams. In analytically modeling this setup, the slab is idealized as being simply supported along all edges. This is a simplification, but comparisons

FIGURE 3
THE CONSTRUCTED AND INSTRUMENTED SCALE MODEL BUILDING

with measurements showed a good match to the analytical model. The analytical model slightly under predicts resonance frequencies, as expected. Thus, making use of idealized boundary conditions is a sufficient approximation when modeling complex edge conditions. Moreover, measurements showed that above 1 kHz, the slab effectively acts like an infinite plate, representing a condition independent of the boundary conditions.

SCALE MODEL TESTING

The mathematical model is a system of column, beam, and slab components linked together by their driving point impedance characteristics. For the case of the scale model building, the column of interest is divided into four interconnected subassemblies, where each subassembly consists of a column segment and floor slab along with the optional inclusion of four beam elements, if necessary. In this section two sets of predictions are made assuming either finite floor slabs and beams or infinite floor slabs and beams. Based on results from component testing finite floors slabs are modeled as simply supported along all edges. As a result of symmetry, there is no rotation at the center of the slab. Finite beams are thus modeled as guided-fixed, where the guided end is that which frames into the center column, allowing translation but no rotation. Damping values used in the predictive model are determined via component testing, as shown in Table 1.

The center column in the scale model is driven by the electrodynamic shaker. Velocity measurements are taken at each floor as well as the at the column base using accelerometers. The scale model is excited between 10 Hz and 5 kHz using linear sine sweeps. Measurements are displayed in terms of velocity ratios, where the velocity at each floor is normalized with respect to the velocity input at the column base. Expressing data in this form is advantageous as velocity ratios are a measure of attenuation, or amplification, in the system.

Scale Model Building Without Cross Beams

Aside from individual components, the simplest mathematical model is that of cascaded column and floor systems, with cross beams not attached. Figure 4 compares the measured velocity ratios to analytical predictions. The analytical model is configured using either an infinite or finite floor slab. Below 400 Hz there is little attenuation between floors, a behavior that is predicted by the mathematical model. The finite model accurately predicts the magnitude of the velocity ratio at resonance; however it tends to under predict the resonance frequencies. This is consistent with behavior in the floor slab component test.

The most significant attenuation is observed in the high frequency region, above 1 kHz. Below 3.5 kHz, the model predictions match measurements on all floors. Measured resonance peaks continue to match the finite model well. As these peaks become less pronounced, the structure behaves more like a building with infinite floors. On floor 3, at 3 kHz there is a 10 dB loss compared to floor 2, this large attenuation is also captured by the mathematical model. The amplification at floor 4 relative to floor 3 is the result of the free end at the top of the column. For all floors at higher frequencies the fluctuation of the response corresponds to modal behavior of the column.

Scale Model Building With Cross Beams

The mathematical model predictions provided a sufficient fit to the scale model building without cross beams. In order to increase the complexity of the scale model building and to mimic a typical building, cross beams are added to the structure. Velocity ratios in the scale model building with cross beams are compared to mathematical model predictions in Figure 5. As in the case of the scale model without cross beams, below 400 Hz there is minimal attenuation at any of the floors. However at higher frequencies, on floors 2 through 4 there is significantly more attenuation than the case without cross beams (Figure 4). At higher frequencies, the impedance of the cross beams become comparable to the impedance of the floor slab. This phenomenon results in significantly higher attenuation at higher frequencies compared to lower frequencies. This also caused noticeable deviation from the mathematical model which was not seen in the preceding section where cross beams were not attached. Through additional testing (Hughes, 2008), it was determined that this deviation was the result of beam/slab composite action, which was not considered in the present work.

In general, the predictions tend to slightly under estimate the measured attenuation at each floor. Unlike the measurements without cross beams, here the finite model of floors 3 and 4 do not exhibit infinite-like behavior; rather there are sharp and noticeable peaks and valleys, indicative of resonances.

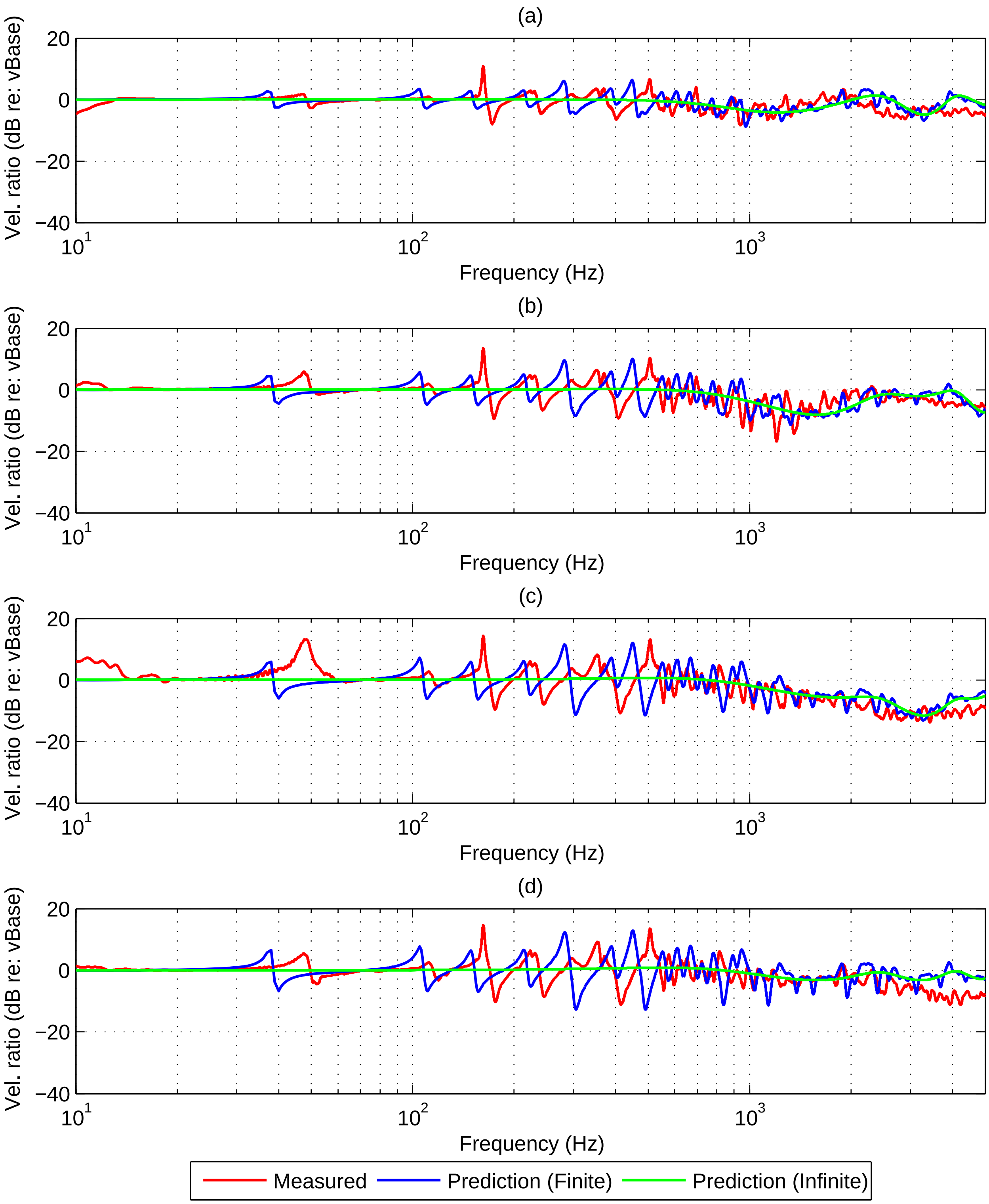

FIGURE 4
MEASURED VELOCITY RATIOS OF THE SCALE MODEL BUILDING WITHOUT CROSS BEAMS COMPARED TO PREDICTIONS: (a) 1ST FLOOR, (b) 2ND FLOOR, (c) 3RD FLOOR, (d) 4TH FLOOR

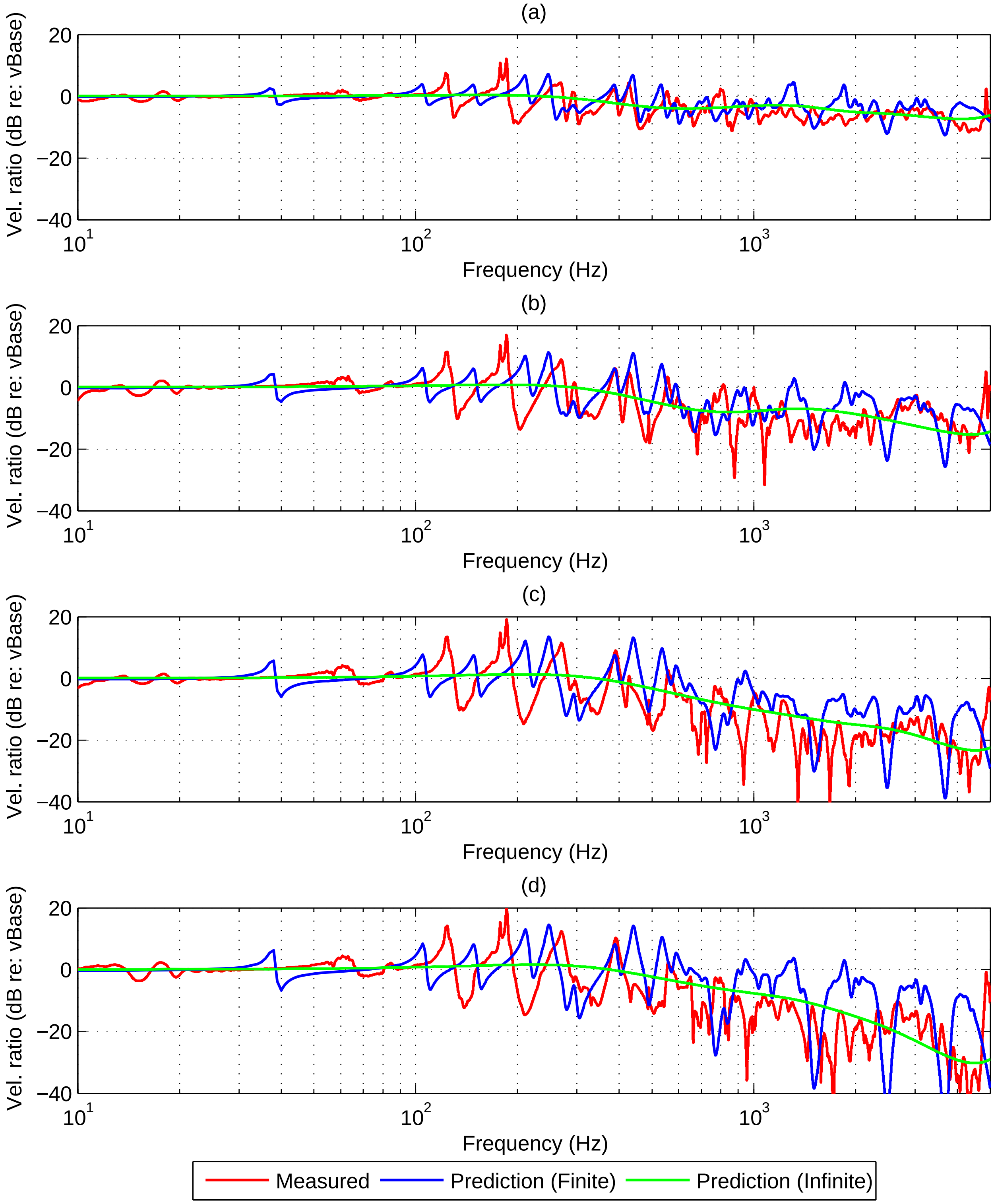

FIGURE 5

MEASURED VELOCITY RATIOS OF THE SCALE MODEL BUILDING WITH CROSS BEAMS COMPARED TO PREDICTIONS: (a) 1ST FLOOR, (b) 2ND FLOOR, (c) 3RD FLOOR, (d) 4TH FLOOR

SUMMARY AND CONCLUSIONS

The mathematical model used here was validated in the scale model building as vibrations levels were accurately predicted between 10 Hz and 5 kHz. Thus, the simplifications and assumptions made for modeling purposes were accurate and confirmed by measurements. The infinite mathematical model accurately predicts the average levels of attenuation from floor to floor but does not capture modal behavior. The finite mathematical model also accurately predicts average behavior, and also predicts the presence of resonant and anti-resonant peaks. However, it is difficult to predict the exact frequency at which the resonances and anti-resonances occur.

The extensive experimental and analytical work presented here led to a number of insights into the dynamic behavior of structural components and the approach for the modeling of full-scale structures. First, we find that boundary conditions of finite models only shift resonance frequencies and material damping is relatively insensitive to boundary conditions. Modeling complex boundary conditions using idealizations and simplifications will therefore provide an accurate measure of the amplification (or attenuation) due to modal behavior. Second, representing components as infinite provide an average impedance magnitude. However, these infinite models do not account for modal behavior of the component. Third, beam elements at higher frequencies have a significant impact on the reduction of vibration levels. Moreover, composite behavior, though difficult to model, further increases floor-to-floor attenuation. Therefore, composite construction is an effective tool for mitigating vibrations.

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