MATH 19-02: HW 6

TUFTS UNIVERSITY DEPARTMENT OF MATHEMATICS
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(1) Here's an initial ballot in which P is a Pareto candidate (everyone's first choice).

$$
\begin{array}{ccccc}
\#1 & \#2 & \#3 & \#4 & \#5 \\
P & P & P & P & P \\
M & M & M & M & M \\
N & N & N & N & N \\
O & O & O & O & O
\end{array}
$$

For any voting method of your choice, you can follow the strategy from the proof of Müller-Satterthwaite and move O into first place column by column. First you change a column $PMNO \to POMN$ moving O into second place, then $POMN \to OPMN$ moving O into first. If that doesn't flip the outcome, you move to the next column. By the end of this process, O is a Pareto candidate. That means at some point the winner has to switch from P to O.

Let k be the column where a switch first occurs from P winning to O winning. (So that $W = P$ before you switch O to first in column k, and $W = O$ after you switch in that column.) What is k for (a) Plurality with alphabetical tiebreaker? (b) Beatpath with alphabetical tiebreaker? (c) Borda with alphabetical tiebreaker?

(2) Consider this preference schedule:

$$
\begin{array}{ccc}
\times 3 & \times 2 & \times 3 \\
C & B & A \\
B & A & C \\
A & C & B
\end{array}
\mapsto
\begin{array}{cccc}
\times 3 & \times 1 & \times 3 & \times 1 \\
C & B & A & A \\
B & A & C & B \\
A & C & B & C
\end{array}
$$

Use this to show that Borda count is not strategy proof. Make sure you explain which voter was being "strategic"!

(3) The Gibbard-Satterthwaite Theorem states that *any Pareto-efficient, strategy-proof, single-winner voting system with $n \geq 3$ candidates must be Dictatorship.*

(a) Explain why Gibbard-Satterthwaite ensures that the Sequential system must be vulnerable to strategic voting.

(b) Consider a Sequential (O, C, S, M) election with the preference schedule below. Who wins? (Note: you do NOT need the full PWCG to answer this.)

×8	×12	×10	×10	×4
M	C	O	S	S
O	S	M	M	O
S	O	C	C	C
C	M	S	O	M

(c) Suppose that you are one of the voters in the first column. What would the outcome be if you voted the *opposite* of your true preferences instead? (That is, you reverse your $MOSC$ ballot to a $CSOM$ ballot.) Is this a successful strategic vote?

(d) Was that change ($MOSC \rightarrow CSOM$) a move favorable to candidate C? Why or why not?