MATH 19-02: HW 8, PART 1

Let's define the Polsby-Popper score of a shape S to be the ratio $\mathsf{PoPo}(S) = 4\pi A/P^2$, where P is the perimeter and A is the area of the shape.

(1) (a) Verify that the Polsby-Popper score of a circle of radius 10 is the same as for a circle of radius 3. Going further, verify that PoPo of a circle of radius r does not depend on r.

(b) Verify that PoPo of a square with a side of length s does not depend on s.

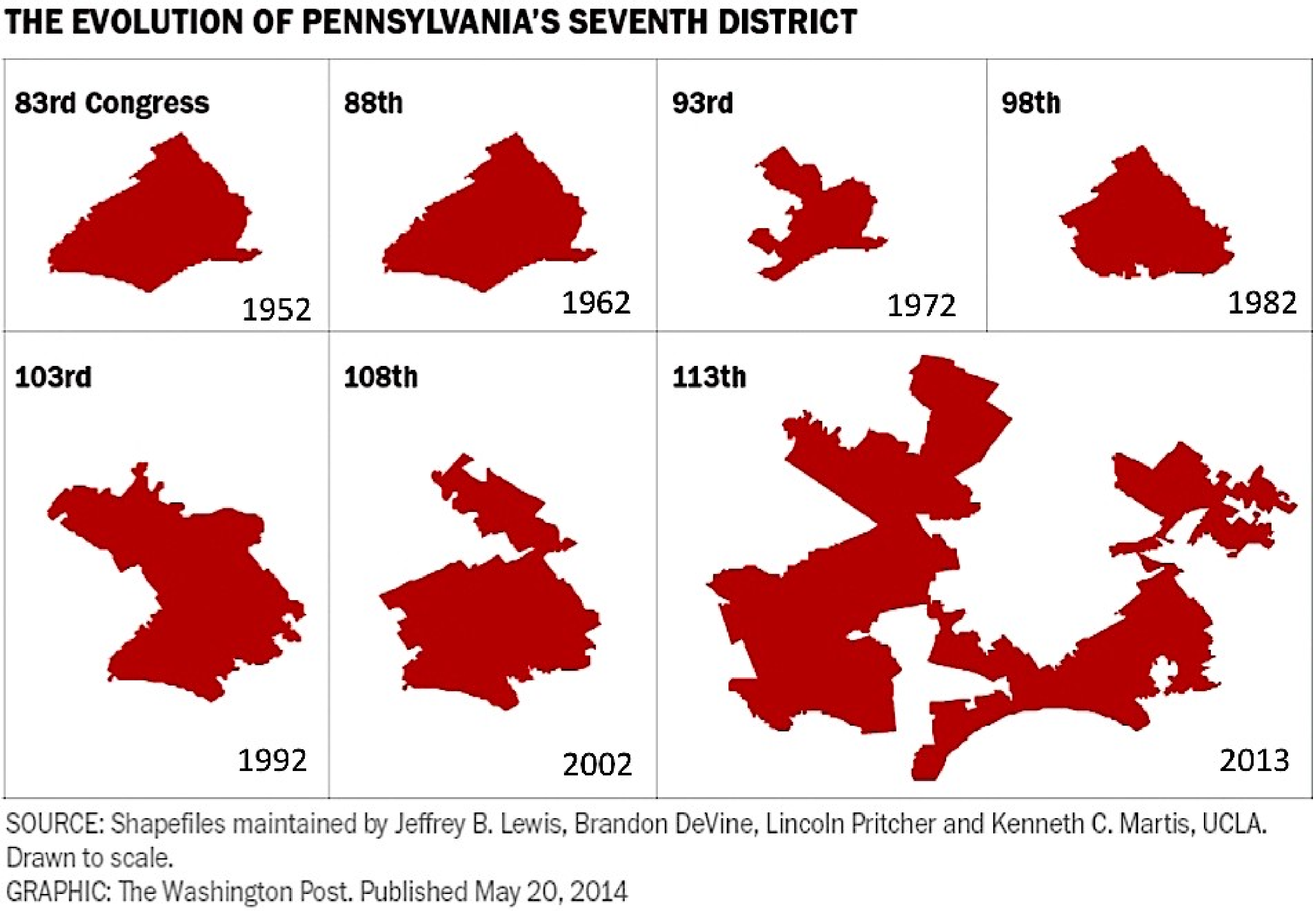
(c) Suppose that a rectangle has length ℓ and width w. Show that the PoPo score of the rectangle only depends on the ratio ℓ/w.

(2) Derive a formula for *the ratio of the area of a shape S to the area of a circle with the same perimeter*. Compare this formula to the formula for PoPo.

(3) Let H be a regular hexagon, and let H' be a hexagon with vertices $(1,0),(1,1),(0,1),(-1,0),(-1,-1),(0,-1)$. Let O a regular octagon and O' an octagon with vertices $(2,1),(1,2),(-1,2),(-2,1),(-2,-1),(-1,-2)$, $(1,-2),(2,-1)$. Sketch these shapes, find their PoPo scores, and make a conjecture about which polygons are the most "compact."

(4) (a) The original gerrymander! Right here in Massachusetts. This is a famous political cartoon from 1812 objecting to the shape of the South Essex district in the MA legislature, designed to favor Governor Gerry's favored candidates. Estimate its compactness score and explain how you do so.

(b) Same for Pennsylvania's recent 7th district.



PART 2

You have a 10×10 grid with 40 orange squares (lighter gray on printout) and 60 pink (darker gray).

Here are some redistricting agendas you might adopt:
(1) proportional representation (4 orange seats), as compact as possible
(2) max orange representation (6 orange seats), as compact as possible
(3) competitiveness (seek districts that are 6-4 or 5-5), as compact as possible
(4) safe seats (seek 8-2, 9-1, 10-0), as compact as possible
(5) simply as compact as possible

Below, let the *bounding rectangle* mean the smallest rectangle with NSEW sides that contains a district. Here are some compactness metrics you might consider:
(A) "isosquarimetric": $16A/P^2$ for each district
(B) "square Reock": area of district divided by area of bounding rectangle
(C) "box score": sum of the square Reock score and the skew (short side over long side) of the bounding rectangle
(D) convex hull score: area of district divided by area of convex hull
(E) total perimeter of all ten districts
Note that for scores A-D, you'll have to figure out how to turn the scores for individual districts into a total score for the plan! The leading options are: (i) average them, or (ii) average their reciprocals.

Your assignment: pick two or more redistricting agendas and pick two or more compactness metrics. Score your plans with the metrics.

We'll compile the most extreme results from the whole class to investigate the efficacy of compactness metrics at detecting gerrymandering!