(1)

If we take a look at just the data for Democrats and Republicans head to head in the general election for each district, we get

$$V_1 = \frac{56521}{56521 + 113967} \approx .332, \quad V_2 = \frac{162213}{162213 + 103019} \approx .612,$$
$$V_3 = \frac{137244}{137244 + 116823} \approx .540, \quad V_4 = \frac{101261}{101261 + 120501} \approx .457.$$

If we take the average of these, we get

$$\overline{V} = \frac{.332 + .612 + .540 + .457}{4} \approx .485.$$

There are two districts with vote shares above $\overline{V}$, so $m_\uparrow = 2$. The Republicans won 2 out of the four available seats, so $\overline{S} = 2/4 = .5$.

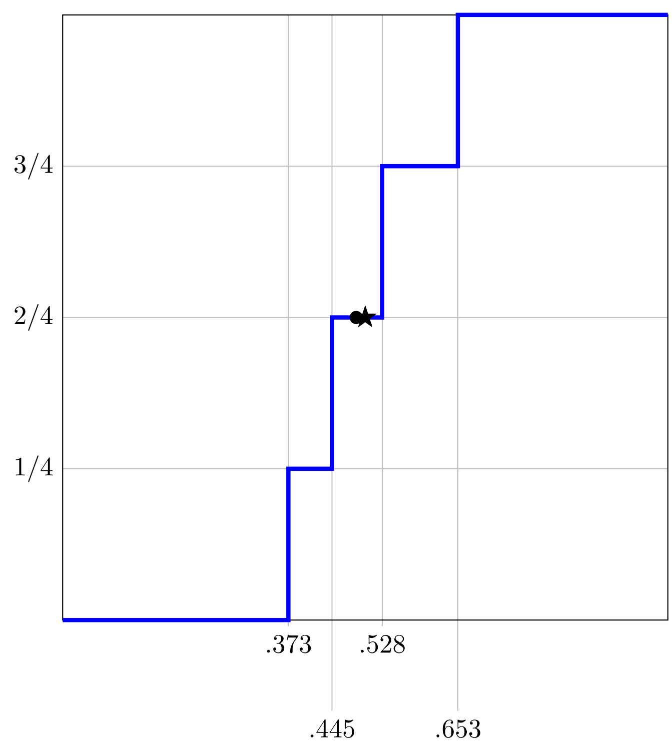
If we arrange the districts in order of increasing share of votes, the two middle districts are $V_4 \approx .421$ and $V_3 \approx .504$, so the median is the average of the two

$$V_{\text{med}} = \frac{.457 + .540}{2} \approx .499.$$

Theorem 1 tells us that the points $(1/2 + \overline{V} - V_{\text{med}}, 1/2)$ and $(1/2, m_\uparrow/m)$ will be on the SV curve. Those points turn out to be

$$\left(\frac{1}{2} + .485 - .499, \frac{1}{2}\right) = (.486, .5) \quad \text{and} \quad \left(\frac{1}{2}, \frac{2}{4}\right) = (.5, .5).$$

After drawing the graph, we can see that these two points are there.



(2)

If we follow the same procedure for Oregon in 2016, we get

$$V_1 = \frac{139756}{139756 + 225391} \approx .383, \quad V_2 = \frac{272952}{272952 + 106640} \approx .719, \quad V_3 = \frac{78154}{78154 + 274687} \approx .221,$$

$$V_4 = \frac{157743}{157743 + 220628} \approx .417, \quad V_5 = \frac{160443}{160443 + 199505} \approx .446$$

. When we compute the average, we get

$$\overline{V} = \frac{.383 + .719 + .221 + .417 + .446}{5} \approx .437.$$

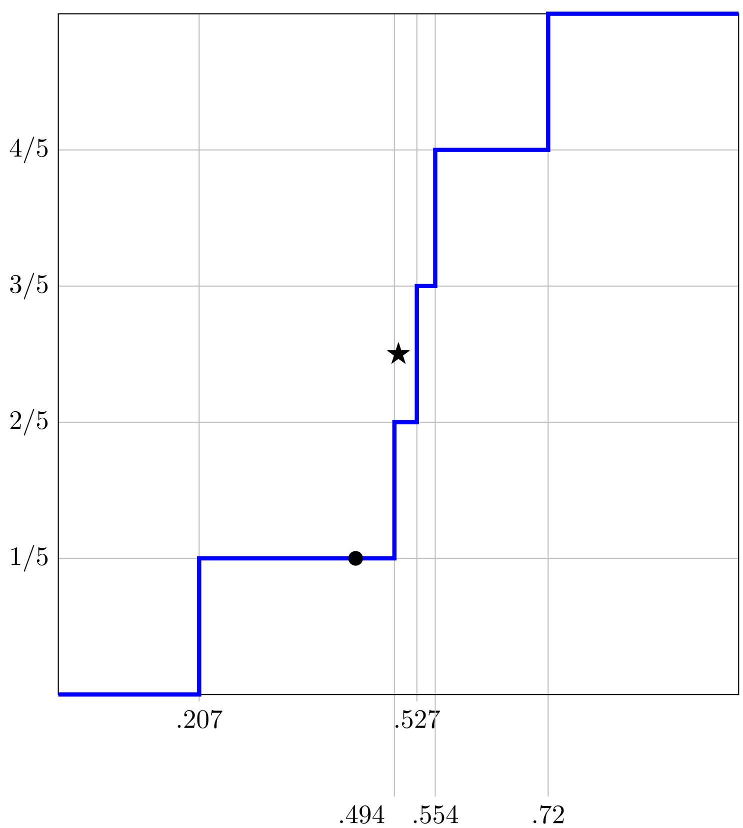
Despite the Republicans winning one out of the five seats ($\overline{S} = 1/5 = .2$), 2 of the districts are above the average .437, so $m_{\uparrow} = 2$.

Because there are only five districts, we don't have to take an average to find the median, it is just the middle vote share:

$$.221 \leq .383 \leq .417 \leq .446 \leq .719.$$

So, $V_{\text{med}} = .417$.

By looking at the SV curve or using the results from Corollary 1, we can see that $MM = .52 - .5 = .437 - .417 = .02$ and $PB = .5 - .4 = .1$.



(3)

Let's assume that each district in Oregon had a turnout of exactly 1000 people. (We have a standing assumption of equal turnout, and EG is done by proportion so it doesn't matter what turnout we assume for the districts.) So in district 1, where the vote share $V_1 = .383$, we will assume 383 people in that district voted for the Republican candidate. To get the number of Democratic votes per district we will take 1000 and subtract the number of Republican votes.

i	R votes	D votes	W_i^R	W_i^D
1	383	617	383	117
2	719	281	219	281
3	221	779	221	279
4	417	583	417	83
5	446	554	446	54
total	2186	2814	1686	814

If we calculate the efficiency gap as $EG = \frac{W^R - W^D}{Tot}$, we get

$$EG = \frac{1686 - 814}{5000} \approx .1744.$$

If we calculate the efficiency gap as $EG = 2\overline{V} - \overline{S} - \frac{1}{2}$, we get

$$EG = 2(.4372) - .2 - .5 = .1744.$$

The numbers match!
The EG magnitude is well above .08, so the authors of EG would conclude that OR was likely gerrymandered—and in favor of Democrats, since it is positive.

(4)

If we set $EG = 0$, we get $0 = 2\overline{V} - \overline{S} - \frac{1}{2}$, if we add $\overline{S}$ to both sides, we get

$$\overline{S} = 2\overline{V} - \frac{1}{2}.$$

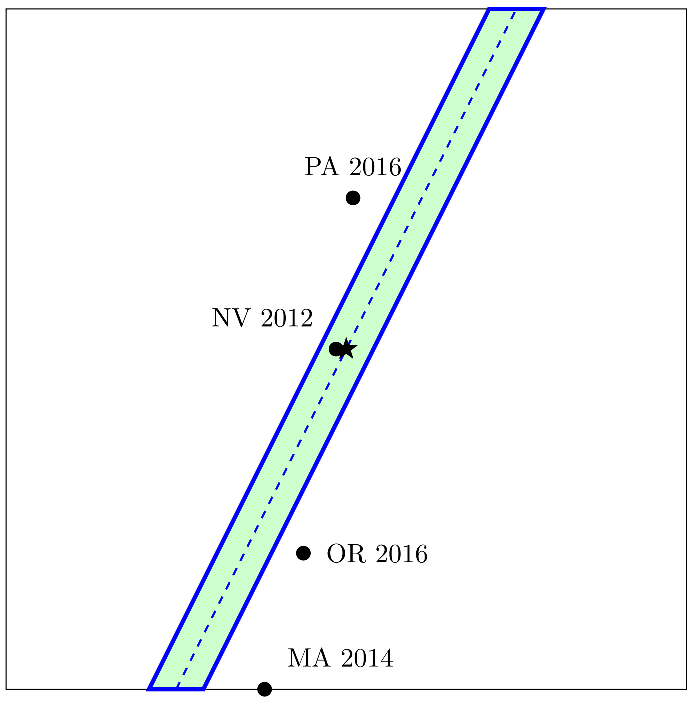
We can repeat this same process when we set $EG = -.08$ and $EG = .08$, except that we will add .08 to both sides when $EG = -.08$ and subtract .08 from both sides when $EG = .08$. When $EG = -.08$, we get

$$\overline{S} = 2\overline{V} - .42.$$

When $EG = .08$, we get

$$\overline{S} = 2\overline{V} - .58.$$

We can see that all three lines are parallel, having a slope of 2. We can now graph these lines on the SV plot.



The interpretation is that the strip you see here is the zone in the SV plot where EG declares the districts to be permissible– anything outside the strip is claimed to be gerrymandered. You no longer have to create a whole SV curve. If you buy this, then Nevada was not a gerrymander, but Oregon, Massachusetts, and Pennsylvania were. Which side of the strip you're on tells you which party did the presumed gerrymandering!

(5)

(a)

The Democrats would waste all of their votes if they lose! In this case, there would be anywhere from 0 to 49,999 votes for the Democratic candidate out of 100,000.

(b)

We can solve this by algebra or by trial and error. Here's an algebraic solution!
Out of the whole vote, Republicans get proportion V and Democrats get $1-V$.
If Rs win, then $W^D = 1-V$ (Ds waste all their votes) and $W^R = V - 1/2$ (Rs waste their votes over half). Setting them equal, I get $1-V = V-1/2$, which is equivalent to $2V = 3/2$, or $V = 3/4$. Applying that share to the total, Republicans would need $3/4$ of the $100,000$ votes, or $75,000$ votes. In that case Rs and Ds would each waste $25,000$.
It's similar if Ds win, because then we solve $V = 1/2 - V$ and find $V = 1/4$, so Rs get $25,000$ votes and waste them all.

(c)

Similar solution! I want to solve for $W^R = 2W^D$. Plugging in the expressions above, I get $V-1/2 = 2(1-V)$ if Rs win, and $V = 2(\frac{1}{2} - V)$ if Ds win. I can solve those and find that Republicans need $V = 5/6$ of the votes if they win and $V = 1/3$ if they lose.
So, rounding to the nearest whole number of votes, Republicans can achieve this if they secure either 33,333 or 83,333 votes!

(6)

The statewide share of R votes in that race was

$$\overline{V} = \frac{789378}{1285736 + 789378} \approx .380$$

The share of R seats is $\overline{S} = 0$. Using this we can calculate our efficiency gap as

$$EG = 2(.38) - 0 - .5 = .26.$$

EG wants to tell you that Massachusetts is massively gerrymandered! But there are other points of view. If you believe that Republicans are extremely uniformly distributed across the state, then it's very likely that each district will have somewhere near .38 fraction of R votes, and in that case Rs would not win any seats without any gerrymandering to blame....

(7)

(a)

Let's assume that Republicans have 38% of the statewide vote in MA and we are drawing district lines for 9 Congressional districts. We can set the population to be any convenient number, because everything just comes down to proportions. So let's imagine that the statewide voting turnout is 9000, so that there are just 1000 voters in each district. In that case, the total number of R votes is 38% of 9000, which is 3420 voters.

In order to barely win a district, then, Rs need 501 votes in that district. Since 3240/501 is about 6.46..., this means they can barely win six districts, but there's no way for them to get a seventh district. We also have $3420 - 6(501) = 414$, so one possible way for them to get six seats is if the voting numbers are

$$(0, 0, 414, 501, 501, 501, 501, 501, 501).$$

On the other hand, if every district has 38% R votes, then they won't win any at all. Perfectly equal distribution would be

$$(380, 380, 380, 380, 380, 380, 380, 380, 380).$$

So we can see that Rs can get anywhere from zero to six seats with just 38% of the vote!

(b)

Let's try to get good scores for PB, MM, and EG, which would mean making them near zero in each case. We need to come up with vote shares $(V_1, V_2, \ldots, V_9)$ that average to $\overline{V} = .38$.
There are lots of ways to do this! Here's one possibility:

$$(.283, .283, .288, .288, .38, .384, .497, .51, .51)$$

We can see that Republicans get two seats here, which means $EG = 2\overline{V} - \overline{S} - \frac{1}{2} = .76 - \frac{2}{9} - \frac{1}{2} \approx .04$. That's not bad, and it's comfortably less than the .08 threshold for a gerrymander.
Also, these numbers are rigged so that the median (the middle value) is .38, which is the same as the mean. That means that $MM = 0$. Since there is no horizontal distance from the curve to $(1/2, 1/2)$, that can only mean that the curve *passes through* $(1/2, 1/2)$, so the partisan bias score is also zero! (That is, you get a slightly better result than the one guaranteed by Theorem 1 from your handout.
So this is a really good-looking districting plan: it has no partisan bias at all, no mean-median gap, and a low efficiency gap!