

Voting Handout

Election from the text, p. 47 (for use during class)

3	2	1	1	2	1	1	1	1	2
<i>A</i>	<i>B</i>	<i>B</i>	<i>B</i>	<i>D</i>	<i>D</i>	<i>E</i>	<i>B</i>	<i>B</i>	<i>E</i>
<i>C</i>	<i>C</i>	<i>A</i>	<i>C</i>	<i>A</i>	<i>A</i>	<i>C</i>	<i>E</i>	<i>E</i>	<i>C</i>
<i>D</i>	<i>D</i>	<i>C</i>	<i>A</i>	<i>B</i>	<i>E</i>	<i>D</i>	<i>C</i>	<i>C</i>	<i>A</i>
<i>E</i>	<i>E</i>	<i>E</i>	<i>E</i>	<i>E</i>	<i>B</i>	<i>B</i>	<i>A</i>	<i>D</i>	<i>B</i>
<i>B</i>	<i>A</i>	<i>D</i>	<i>D</i>	<i>C</i>	<i>C</i>	<i>A</i>	<i>D</i>	<i>A</i>	<i>D</i>

1. Use the following preference schedule to show elimination and Coombs can result in the same winner, while the runoff winner will not be the same.

3	3	5	4	3
<i>A</i>	<i>A</i>	<i>R</i>	<i>L</i>	<i>D</i>
<i>R</i>	<i>L</i>	<i>L</i>	<i>R</i>	<i>L</i>
<i>L</i>	<i>R</i>	<i>A</i>	<i>A</i>	<i>R</i>
<i>D</i>	<i>D</i>	<i>D</i>	<i>D</i>	<i>A</i>

2. Use the following preference schedule to show elimination and Coombs can result in different winners.

5	2	1	4	3
<i>F</i>	<i>F</i>	<i>S</i>	<i>S</i>	<i>D</i>
<i>X</i>	<i>S</i>	<i>X</i>	<i>D</i>	<i>X</i>
<i>S</i>	<i>X</i>	<i>D</i>	<i>X</i>	<i>S</i>
<i>D</i>	<i>D</i>	<i>F</i>	<i>F</i>	<i>F</i>

3. Use the preference schedule from problem 1 to show:

- (a) Plurality is not Condorcet-fair.
- (b) Plurality is not Smith-fair.
- (c) Runoff is not Condorcet-fair.
- (d) Runoff is not Smith-fair.
- (e) Runoff is not monotonic.

4. Consider the preference schedule below:

- (a) Show elimination is not monotonic.
- (b) Find the winner using beatpath.

12	10	7	4
<i>T</i>	<i>K</i>	<i>B</i>	<i>H</i>
<i>B</i>	<i>T</i>	<i>H</i>	<i>B</i>
<i>K</i>	<i>H</i>	<i>K</i>	<i>T</i>
<i>H</i>	<i>B</i>	<i>T</i>	<i>K</i>

5. Use the preference schedule from problem 2 to show that Borda is not Smith-fair.
6. Use the following preference schedule to show that plurality allows both losing spoilers and weak spoilers.

4	3	3	2
A	B	C	D
B	C	A	C
C	D	D	B
D	A	B	A

7. Consider the following preference schedule (from Mathematical Excursions, p. 18):

2	6	4	1	1	4	4
A	B	B	C	C	D	E
D	A	A	B	D	A	C
C	C	D	A	A	E	D
B	D	E	D	B	C	B
E	E	C	E	E	B	A

With the following one-to-one comparisons:

$A : B$	7 : 15	$B : D$	11 : 11
$A : C$	16 : 6	$B : E$	14 : 8
$A : D$	13 : 9	$C : D$	12 : 10
$A : E$	18 : 4	$C : E$	10 : 12
$B : C$	10 : 12	$D : E$	18 : 4

- (a) Show pairwise comparison allows losing spoilers.
- (b) Find the beatpath winner(s).
8. Consider the following preference schedule:

12	6	4	4	3	4
T	K	K	B	H	H
B	T	T	H	K	B
K	H	B	K	T	T
H	B	H	T	B	K

With the following one-to-one comparisons:

$T : B$	25 : 8	$T : H$	22 : 11	$B : H$	20 : 13
$T : K$	16 : 17	$B : K$	20 : 13	$K : H$	22 : 11

- (a) Show the plurality winner is different than the a priori Smith fair plurality winner.
- (b) Show the elimination winner is different than the a priori Smith fair elimination winner.

9. Consider the following preference schedule (from Mathematical Excursions, p. 13):

93	44	10	30	42	81
<i>A</i>	<i>B</i>	<i>C</i>	<i>C</i>	<i>D</i>	<i>E</i>
<i>B</i>	<i>D</i>	<i>A</i>	<i>E</i>	<i>C</i>	<i>D</i>
<i>C</i>	<i>E</i>	<i>E</i>	<i>B</i>	<i>E</i>	<i>C</i>
<i>D</i>	<i>C</i>	<i>B</i>	<i>A</i>	<i>A</i>	<i>B</i>
<i>E</i>	<i>A</i>	<i>D</i>	<i>D</i>	<i>B</i>	<i>A</i>

With the following one-to-one comparisons:

<i>A</i> : <i>B</i> 145 : 155	<i>A</i> : <i>D</i> 133 : 167	<i>B</i> : <i>C</i> 137 : 163	<i>B</i> : <i>E</i> 137 : 163	<i>C</i> : <i>E</i> 175 : 125
<i>A</i> : <i>C</i> 93 : 207	<i>A</i> : <i>E</i> 103 : 197	<i>B</i> : <i>D</i> 177 : 123	<i>C</i> : <i>D</i> 133 : 167	<i>D</i> : <i>E</i> 179 : 121

Find the winner(s) using:

- (a) Plurality
 - (b) a priori Smith fair plurality
 - (c) a posteriori Smith fair plurality
 - (d) elimination
 - (e) pairwise
 - (f) beatpath
10. For the preference schedule in problem 2, find the one-shot and recursive rankings for plurality, Borda, pairwise comparison, and elimination.
11. Consider the following preference schedule:

5	2	2	4	3	4
<i>A</i>	<i>A</i>	<i>B</i>	<i>B</i>	<i>C</i>	<i>D</i>
<i>B</i>	<i>C</i>	<i>D</i>	<i>C</i>	<i>D</i>	<i>B</i>
<i>C</i>	<i>D</i>	<i>C</i>	<i>D</i>	<i>B</i>	<i>A</i>
<i>D</i>	<i>B</i>	<i>A</i>	<i>A</i>	<i>A</i>	<i>C</i>

With the following one-to-one comparisons:

<i>A</i> : <i>B</i> 7 : 13	<i>A</i> : <i>D</i> 7 : 13	<i>B</i> : <i>D</i> 11 : 9
<i>A</i> : <i>C</i> 11 : 9	<i>B</i> : <i>C</i> 15 : 5	<i>C</i> : <i>D</i> 14 : 6

- (a) Draw the graph with margins, and find the Smith set.
- (b) Find the winner(s) using runoff and beatpath.
- (c) Find the 1-shot ranking using plurality.
- (d) Find the 1-shot ranking using Coombs.
- (e) Find the 1-shot ranking using elimination.
- (f) Find the 1-shot ranking using Borda.
- (g) Find the 1-shot ranking using pairwise comparison.
- (h) Find the recursive ranking using plurality.

- (i) Find the recursive ranking using Coombs.
- (j) Find the recursive ranking using elimination.
- (k) Find the recursive ranking using Borda.
- (l) Find the recursive ranking using pairwise comparison.
- (m) What is the set of winners for any Smith fair method?
- (n) Show that C is a losing, weak spoiler using plurality or runoff.
- (o) Show that B is a losing spoiler using plurality.
- (p) Show that C is not a spoiler using Borda.
- (q) Show that elimination violates monotonicity in this election (we did this in class).

12. Consider the following preference schedule:

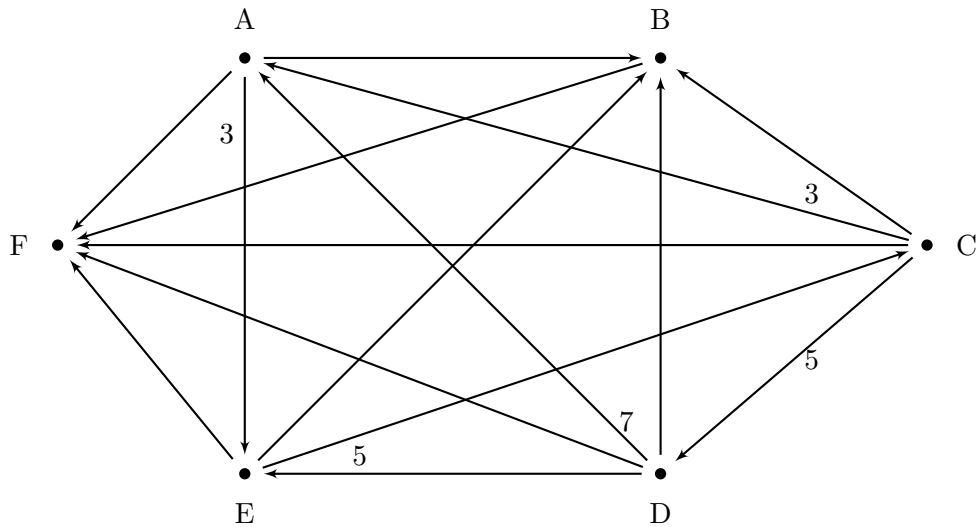
	5	4	2	2	3	1	3
<i>A</i>	<i>B</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>D</i>	<i>D</i>	<i>E</i>
<i>C</i>	<i>D</i>	<i>E</i>	<i>A</i>	<i>E</i>	<i>A</i>	<i>A</i>	<i>D</i>
<i>B</i>	<i>A</i>	<i>D</i>	<i>E</i>	<i>A</i>	<i>E</i>	<i>E</i>	<i>A</i>
<i>E</i>	<i>C</i>	<i>A</i>	<i>D</i>	<i>C</i>	<i>C</i>	<i>C</i>	<i>C</i>
<i>D</i>	<i>E</i>	<i>C</i>	<i>B</i>	<i>B</i>	<i>B</i>	<i>B</i>	<i>B</i>

With the following one-to-one comparisons:

<i>A</i> : <i>B</i>	13 : 7	<i>A</i> : <i>D</i>	7 : 13	<i>B</i> : <i>C</i>	6 : 14	<i>B</i> : <i>E</i>	11 : 9	<i>C</i> : <i>E</i>	11 : 9
<i>A</i> : <i>C</i>	16 : 4	<i>A</i> : <i>E</i>	12 : 8	<i>B</i> : <i>D</i>	11 : 9	<i>C</i> : <i>D</i>	7 : 13	<i>D</i> : <i>E</i>	8 : 12

- (a) Draw the graph with margins, find the sequential winner, and the Smith set.
- (b) Find the winner(s) using runoff and beatpath.
- (c) Find the 1-shot ranking using plurality.
- (d) Find the 1-shot ranking using Coombs.
- (e) Find the 1-shot ranking using elimination.
- (f) Find the 1-shot ranking using Borda.
- (g) Find the 1-shot ranking using pairwise comparison.
- (h) Find the recursive ranking using plurality.
- (i) Find the recursive ranking using Coombs.
- (j) Find the recursive ranking using elimination.
- (k) Find the recursive ranking using Borda.
- (l) Find the recursive ranking using pairwise comparison.
- (m) Is B a polarizing candidate in this election?
- (n) Show that elimination violates monotonicity in this election.
- (o) Why are there no weak spoilers in this election, no matter what voting method is used?
- (p) Show that A is a losing spoiler using plurality.
- (q) Show that B and E are both losing spoilers using pairwise comparison.

13. Consider this graph of a preference schedule:



Note that the margin from A to E is 3, the margin from C to A is 3, the margin from D to A is 7, and the margin from E to C is 5, and all the other margins are 1.

- (a) Find the sequential comparison winner, assuming alphabetical order. Show some work.
- (b) Find the Smith Set.
- (c) Find the beatpath winner(s). Show disqualifications with strengths as we have in class.
- (d) Find D_B .
- (e) Find D_F .
- (f) Find the strength of the strongest beatpath from C to A.
- (g) Find the strength of the strongest beatpath from A to D.
- (h) Find the strength of the strongest beatpath from D to E.

14. Consider the following preference schedule:

	2	6	6	1	4	4	1	2
<i>C</i>	<i>C</i>	<i>A</i>	<i>C</i>	<i>E</i>	<i>D</i>	<i>D</i>	<i>D</i>	<i>D</i>
<i>A</i>	<i>B</i>	<i>E</i>	<i>E</i>	<i>C</i>	<i>A</i>	<i>C</i>	<i>A</i>	<i>A</i>
<i>B</i>	<i>A</i>	<i>D</i>	<i>D</i>	<i>D</i>	<i>C</i>	<i>A</i>	<i>E</i>	<i>E</i>
<i>D</i>	<i>D</i>	<i>B</i>	<i>B</i>	<i>B</i>	<i>E</i>	<i>E</i>	<i>E</i>	<i>B</i>
<i>E</i>	<i>E</i>	<i>C</i>	<i>A</i>	<i>A</i>	<i>B</i>	<i>B</i>	<i>B</i>	<i>C</i>

With the following one-to-one comparisons:

<i>A</i> : <i>B</i>	15 : 11	<i>A</i> : <i>D</i>	14 : 12	<i>B</i> : <i>C</i>	8 : 18	<i>B</i> : <i>E</i>	8 : 18	<i>C</i> : <i>E</i>	14 : 12
<i>A</i> : <i>C</i>	12 : 14	<i>A</i> : <i>E</i>	21 : 5	<i>B</i> : <i>D</i>	8 : 18	<i>C</i> : <i>D</i>	13 : 13	<i>D</i> : <i>E</i>	15 : 11

- (a) Draw the graph.
- (b) Find the sequential comparison winner, assuming alphabetical order. Show some work.
- (c) Find the Smith Set.

- (d) Find the plurality winner(s)
- (e) Find the runoff winner(s).
- (f) Find the elimination winner(s).
- (g) Find the pairwise comparison winner(s).
- (h) Find the Borda winner(s).
- (i) Find the Coombs winner(s).
- (j) Find the a priori Smith fair Borda winner(s).
- (k) Show that A is a losing spoiler using Pairwise comparison.
- (l) Show that A is a losing spoiler using Plurality.
- (m) Show that A is a losing spoiler using beatpath.
- (n) Show that C is a winning spoiler using elimination.