Worksheet 4

Mathematics of Social Choice
Duchin, Spring 2021

Problem 1. Given any preference schedule, you can find the Smith set $\mathcal{S}$ directly from those election results, no matter the winner selection method. A dominating set is *subset of candidates with all arrows pointing out*. The Smith set is the smallest domset. From now on, we will call a candidate "strong" if they are in the Smith set.

For the following preference schedule, find $\mathcal{D}_A, \mathcal{D}_B, \mathcal{D}_C$ and $\mathcal{S}$.

$\times 3$	$\times 2$	$\times 2$	$\times 1$	$\times 1$
A	B	C	A	B
B	A	B	C	C
C	C	A	B	A

Problem 2. Suppose $|\mathcal{S}| = 1$. What can you say about the strong candidate in that case?

Problem 3. The winner of a sequential tournament, no matter what order it was done in, is always in the Smith set. For the preference schedule below, make the PWC graph and show work for a sequential tournament to find a strong candidate. Draw a Venn-style diagram showing the candidate strength tiers in $\mathcal{C}$.

$\times 3$	$\times 2$	$\times 2$
A	A	D
B	E	A
D	D	C
C	B	E
E	C	B