Worksheet 5

Mathematics of Social Choice
Duchin, Spring 2021

Problem 1. You can "*Smithify*" a winner selection method by (1) finding the Smith set $\mathcal{S}$, (2) *consolidating* the preference schedule to those strong candidates, then (3) running the method. How does Smithified plurality compare to regular plurality for the following preference schedule?

What if you just took the strong candidate with the most first-place votes? Does this pick the same winner?

$\times 1$	$\times 2$	$\times 1$	$\times 1$	$\times 2$	$\times 2$
D	D	D	A	B	C
A	B	C	B	C	A
B	C	A	C	A	B
C	A	B	D	D	D

Problem 2. Like pairwise comparison, the *beatpath* winner can be computed just from the PWC graph. But this time you need the margins as well as the arrow directions.

We say that a *beatpath* between two candidates is any sequence you can follow in the graph from one to the other. The *strength* of a beatpath is the lowest margin of victory of any step along the way. We say that candidate X eliminates candidate Y (denoted $X \triangleright Y$) if there's a beatpath from X to Y that is stronger than any from Y to X. The winner(s) of the beatpath method are all candidates left standing after all possible eliminations are made.

Check that, even though both are based on head-to-heads, the beatpath winner set is different from the PWC winner set for this election.

$\times 3$	$\times 2$	$\times 4$
A	C	B
D	A	C
B	D	A
C	B	D

Problem 3. A **weak spoiler** is a spoiler who is weak (i.e., not in $\mathcal{S}$). For beatpath and one other method of your choice, check for weak and strong spoilers in the following election.

$\times 2$	$\times 1$	$\times 3$	$\times 2$	$\times 2$
A	D	C	B	A
D	C	A	C	D
C	A	D	A	B
B	B	B	D	C