

Midterm Practice Problems

Mathematics of Social Choice
Duchin, Spring 2021



The midterm will have the same format as quizzes, mostly multiple choice. Here are some problems to be sure you're up on the voting theory material.

1. True/False: if a 3-candidate election has two Smith candidates out of three, then there must be a tie in the pairwise comparison graph.
2. Find an example (system and preference schedule) for which there is a spoiler who is a Smith candidate. In your example, are they a winning spoiler? Losing spoiler? Weak spoiler?
3. Suppose there are $n = 10$ candidates and $N = 100$ voters in a particular preference schedule. How many consolidations do you have to consider to run each of these methods?: plurality, runoff, elimination, Coombs, Borda, Smith, Smithified plurality, pairwise comparison, sequential, and dictatorship. Example: Runoff requires you to identify the top two vote-getters and compare them head-to-head, so you do **one** consolidation (down to those two candidates).

In case it is helpful: a pairwise comparison graph with ten vertices has 45 edges.

4. Suppose again there are $n = 10$ candidates and $N = 100$ voters in an election. Can you tell from the pairwise comparison graph alone whether an election has a Condorcet candidate? Pareto candidate? majority candidate? unanimous preference of X over Y ?
5. Build a preference schedule each candidate has under 40% of the first-place votes, but there is some consolidation which produces a majority candidate.
6. True/False: the runoff method is unanimity-fair.
7. Is it possible for a move to be favorable to one candidate and neutral to another candidate? If so, give an example.
8. Give an example of a preference schedule with five candidates so that all of them are involved in a Condorcet cycle.
9. Make up an example of a voting system that is unanimity-fair but not Pareto efficient, or explain why this is impossible.