Worksheet 10

Mathematics of Social Choice
Duchin, Spring 2021

Problem 1. In this problem, you'll be drawing a *seats-votes curve* (or "SV curve") for an election.

Go to the wikipedia page United States House of Representatives elections in Nevada, 2020 and get the results from the four congressional races.

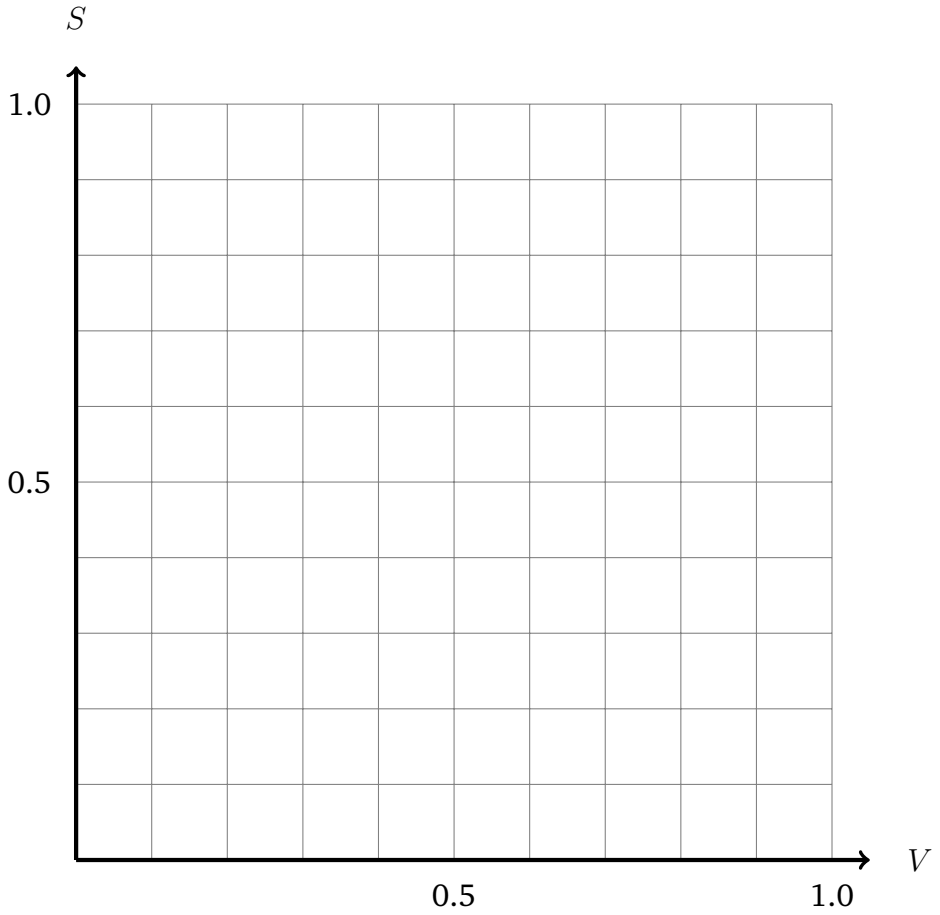
(a) Find the Republican vote shares V_1, V_2, V_3, V_4.

	District 1	District 2	District 3	District 4
R votes				
D votes				
R share				

(b) Find the mean Republican vote share $\overline{V}$, which is the average of these district shares V_1, V_2, V_3, V_4. Check that it is a bit different from the statewide share of Republican votes.

(c) Find the Republican seat share $\overline{S}$, which is the share of seats that were won by Republican candidates in this race.

(d) Now we can start drawing the SV curve. The Republican vote share will be on the x-axis, and the share of seats won by Republicans will be on the y-axis. (Note that both of these range from 0 to 1.) Using the previous two parts of this question, plot the point $(\overline{V}, \overline{S})$ on your graph.



(e) To find the rest of the curve, we will be using the assumption of *uniform partisan swing*–that means that we consider what would happen if we add or subtract the same amount from the vote share in each district. By how much do you have to increase the Republican vote share in each district to increase the number of Republican seats by 1? Add this to your plot as a step.

(f) Continue this process of uniformly changing the Republican vote share in each district to plot the rest of the SV curve. Use this space for scratch work.

(g) Plot the point $(0.5, 0.5)$ on your graph. We define the *mean-median* score (or MM) to be the horizontal distance of the curve from the center point, and the *partisan bias* (or PB) to be the vertical distance of the curve from the center point. Find MM and PB for this election. Interpret them—from the partisan symmetry point of view, is this districting plan biased in favor of Democrats, Republicans, or neither?

Problem 2. Go to the wikipedia page United States House of Representatives elections in Oregon, 2020 and get the results from the five congressional races.

(a) Compute V_1, V_2, V_3, V_4, V_5, $\overline{V}$, and $\overline{S}$ for this election.

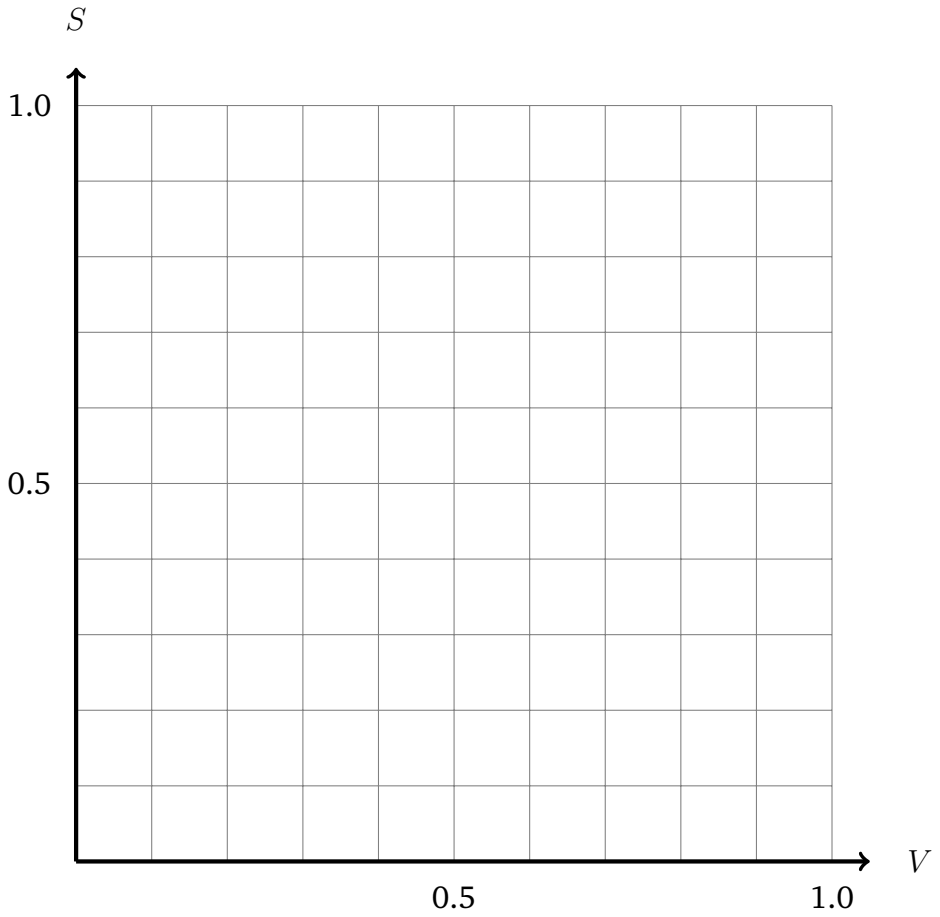
	District 1	District 2	District 3	District 4	District 5
R votes					
D votes					
R share					

(b) Let's say that a *wasted vote* is any winning vote in excess of half the major-party votes in a district, or any losing vote at all. Find the Democratic and Republican wasted votes in each district, and add them up to find W^D and W^R overall.

(c) There are two ways to define the *efficiency gap* for a districting plan. One is the difference in wasted votes as a share of the total votes: $EG = \frac{W^R - W^D}{Tot}$. The other can be computed just by comparing seat share to vote share: $EG' = 2\overline{V} - \overline{S} - \frac{1}{2}$.

Compute EG and EG' for this election. (If the districts had equal numbers of votes cast, these would be equal.)

(d) Plot $(\overline{V}, \overline{S})$ and draw the SV curve for this election.



(e) Let's consider the "simplified" efficiency gap $EG' = 2\overline{V} - \overline{S} - \frac{1}{2}$. Solve for $EG' = 0$ algebraically (find $\overline{S}$ as a function of $\overline{V}$) and plot that line on the SV plane above.

Next, solve for $EG' = -.08$ and for $EG' = .08$ and plot those lines. Using those, shade the region that corresponds to $-.08 \leq EG' \leq .08$ on your seats-votes plot.

Interpret this—from the efficiency gap point of view, is this districting plan biased in favor of Democrats, Republicans, or neither?