

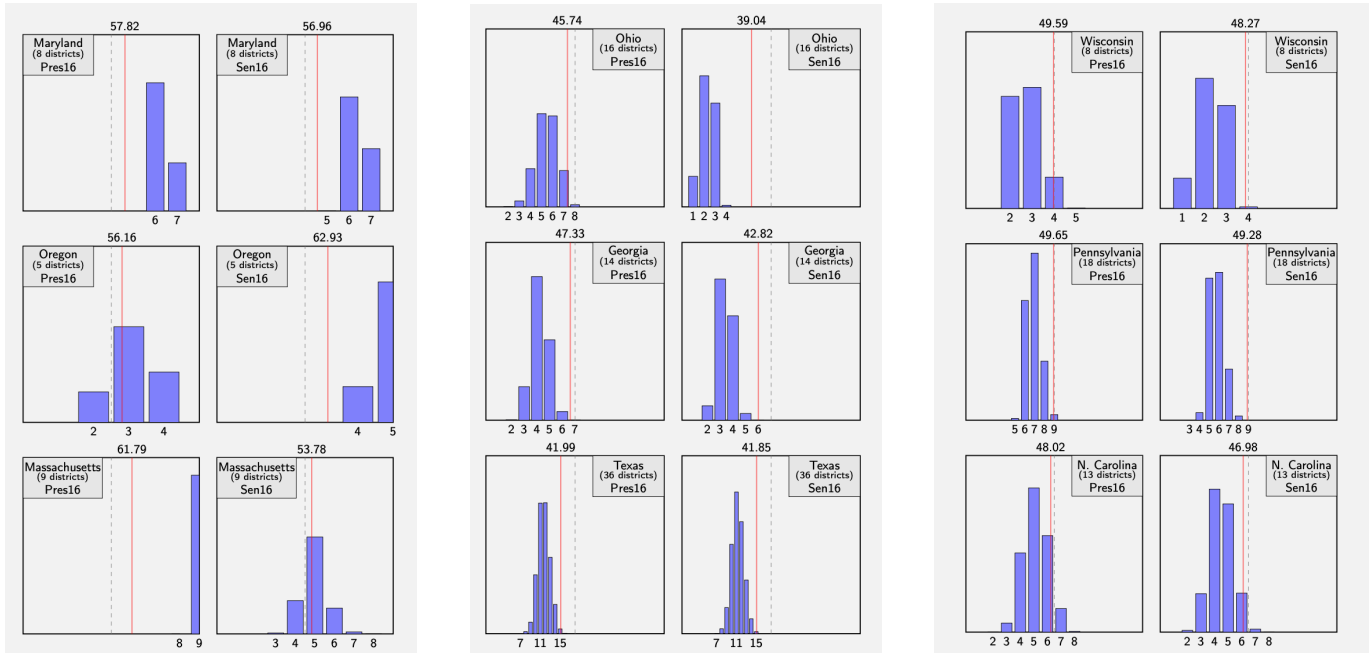
**Worksheet 11**  
Mathematics of Social Choice  
Duchin, Spring 2021



**Problem 1.** The seats/votes formula for efficiency gap is  $EG' = 2\bar{V} - \bar{S} - \frac{1}{2}$ . The idea of the efficiency gap is that a fair election would have efficiency gap zero. Suppose Republicans have 40%, 50%, 53%, 58% or 75% of the vote. From the point of view of the efficiency gap, what would be their fair share of the seats in each case? Explain the pattern, and discuss whether you believe that this is a reasonable picture of fairness.

**Problem 2.** It is hard to estimate the Democratic vs Republican voting in 2020 Massachusetts congressional elections, because four out of nine of them had no Republican candidate! However, in the 2020 Senate race, the Democrat (Markey) got 2,357,809 votes and the Republican (O'Connor) got 1,177,765. Let's suppose that congressional preference by party is similar. All nine seats are currently all occupied by Democrats. Compute the efficiency gap for that election using the seats/votes formula. What can you conclude about gerrymandering in MA?

# Discuss!

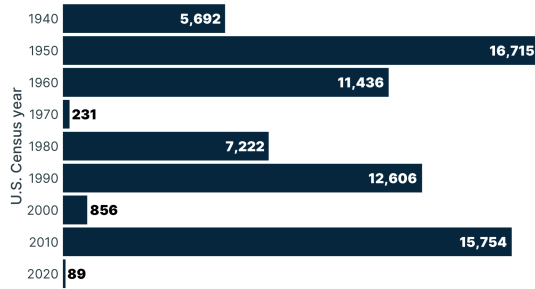


1. The plots above are made from the Democratic party point of view. In each election, the red line shows the proportional share of seats for Democrats, and the histogram shows how many seats have a Democratic majority in 20,000 random districting plans. What patterns and trends do you see?
2. How does looking at these bell curves inform your idea about the “fair share” of seats for Democrats in each state?
3. Even if partisan metrics (like mean-median and efficiency gap) don’t always tell the *right* story, is still important to have some metrics on hand so that courts can treat gerrymandering as objective rather than subjective? Or is a flawed metric worse than nothing?
4. Partisan symmetry metrics require that you have a way to model what the election outcome *would have been* if the vote preferences swung toward one party or the other. The simple models use uniform swing. Sketch out some other possible models of how votes swing.
5. Most American cities are heavily Democratic-favoring. (In fact, of the top 40 cities in the country by population, only two preferred Trump to Clinton: Colorado Springs, CO and Mesa, AZ.) On one hand, city voters can argue that they have a lot in common in terms of their identity and needs so they should be kept together in a district. On the other hand, this could lead to packing of Democratic voters. How should we balance these issues?

6. Apportionment totals were just released! New York lost a congressional seat, dropping from 27 to 26. Minnesota was expected to lose a seat but didn't, remaining steady at 8 seats. If the Census had just counted 89 more people in New York, they would have kept that seat and Minnesota would have lost one. This is the closest margin since at least 1940. Is this fair?

### MN kept 8th House seat by record-thin margin

Number of extra people needed for runner-up state to receive the final seat in Congress. For example, if New York had 89 more people in the 2020 Census, it would have received the last seat over Minnesota.



Source: U.S. Census Bureau. Graph by David H. Montgomery | MPR News

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