Worksheet 8
Mathematics of Social Choice
Duchin, Spring 2021

Problem 1.

1. Suppose there are $m = 11$ seats on a governing council, and the districts in the town (districts B, P, and R) have populations $M_B = 54, M_P = 243$, and $M_R = 703$. Apportion the seats by Hamilton's method. (There's no constitutional requirement that everybody gets a seat on the council.)

2. In the following election cycle, the populations have grown a bit to ${M_B}' = 56, {M_P}' = 255$, and ${M_R}' = 789$. Reapportion.

3. What's the "paradox" here?

Problem 2.

1. Next suppose there are $m = 10$ seats on the governing council, and the districts in the town (districts B, P, and R) have populations $M_B = 54$, $M_P = 243$, and $M_R = 703$. Apportion the seats by Hamilton's method.

2. In the previous problem, you already worked out how this changes when the number of seats goes up to $m = 11$. What's the "paradox" here?

Problem 3.

1. Back to the original scenario ($m = 11, M_B = 54, M_P = 243, M_R = 703$): suppose that a new neighborhood is annexed to the town, with population $M_J = 580$. Explain why it would be reasonable to give the new district $m_J = 6$ seats, increasing the size of the council to $m = 17$.

2. Reapportion with $m = 17$. What's the "paradox" here?

Problem 4. The Huntington-Hill method tells you to do the initial apportionment by geometric mean rounding and then adjust ideal district size until geometric mean rounding gives out the right number of seats. Do this by hand for the scenario above ($m = 11, M_B = 54, M_P = 243, M_R = 703$).