

## Final Exam Practice Problems

Mathematics of Social Choice  
Duchin, Spring 2021



The final will have the same format as quizzes and midterm, mostly multiple choice. Here are some problems that go over material from both halves of the semester.

1. An election is held with candidates A, B, and C. Candidate B has over 50% of the vote. Name two winner selection methods that guarantee that B is the sole winner, and two winner selection methods that do not. Show an example of a preference schedule that fits this description, and give a winner selection method for which  $B \notin \mathcal{W}$ .
2. Suppose there are  $m = 15$  seats on a governing council, and the zones in the town have populations  $M_H = 86$ ,  $M_R = 241$ ,  $M_T = 332$ , and  $M_B = 706$ . Use the Huntington Hill method to apportion seats on the council to these zones. Because not every zone will get exactly its quota, some will be moderately over-represented and others moderately under-represented. Which is the most over-represented?
3. Draw a realistic  $SV$  curve where partisan bias shows a small Republican advantage but mean-median shows a large Republican advantage. Also, explain why it's impossible for  $PB$  and  $MM$  to disagree about which party is favored.
4. Suppose a  $10 \times 10$  square is to be divided into ten districts of equal area, and District 1 is required to be a rectangle. What's the best and worst PoPo score it can possibly have?
5. Recall that  $EG$  is the difference in wasted votes achieved by two parties, divided by the total number of votes. And  $EG'$  is a closely related formula that just uses seats and votes. For the following election, find  $EG$  and  $EG'$ .

District	Party X votes	Party Y votes
1	1372	1168
2	1622	1030
3	565	1139
4	1012	1205

Explain why they are not exactly equal. Do they agree on which party was given an advantage by the districting plan?

6. (Bonus) Recall the simplified formulas for  $MM$  and  $PB$  from the [notes](#): if you take the vote share by district,  $MM$  is the median minus the mean, and  $PB$  is the share of seats where the party did better than average, minus one-half. For instance, if there are four districts and the vote shares are  $(.3, .4, .6, .8)$ , then  $MM = .5 - .525 = -.025$  and  $PB = .5 - .5 = 0$ . (That's because the vote shares have a median of  $.5$ , a mean of  $\bar{V} = .525$ , and the point-of-view party won  $1/2$  of the districts.)

Consider a state where Republicans have  $\bar{V} = 0.6$ . If there are 5 districts, come up with vote shares for each District that get you a  $MM$  score of 0. What is the  $PB$  for your scenario? What are all possible  $PB$  scores holding  $MM = 0$ ?