ME20
Project 02
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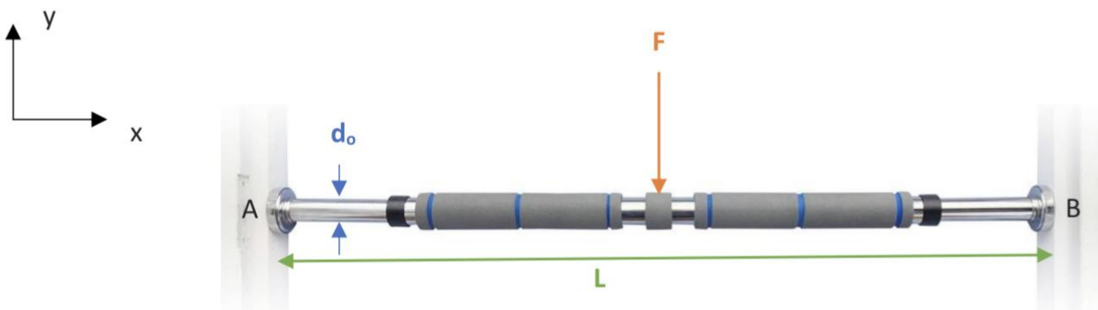
Pull-Up Bar in FEA

Objective:

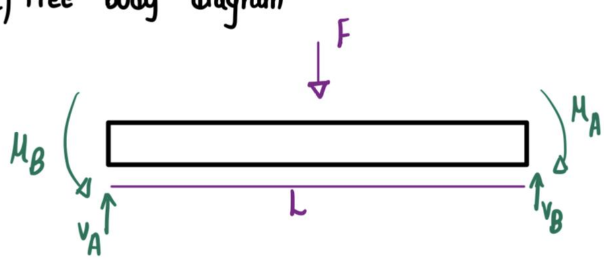
The goal of the following project is to initially determine using theoretical calculations the normal stress and deflection of a pull up bar being subjected to a concentrated load in the middle. Then, we compare the results determined to the results found by the FEA in ME10.



Part 1:
In this part, we determine the normal stress and deflection developed in the pull up bar.

a)

a) Free body diagram



$V_A = \frac{F}{2}$ $\quad M_A = \frac{FL}{8}$

$V_B = \frac{F}{2}$ $\quad M_B = \frac{FL}{8}$

$V_A = V_B = \frac{F}{2}$ $\quad M_A = M_B = \frac{FL}{8}$

b)

b) Write the equations

$$+\uparrow \Sigma F_y = 0 ; \ V_A + V_B - F = 0$$

$$V_A = V_B = \frac{F}{2}$$

$$\Sigma M_A = 0 ; \ M_A + V_B(L) - M_B - \frac{FL}{2} = 0$$

$$M_A = M_B = \frac{FL}{8}$$

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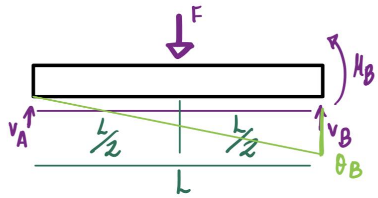
c)

c) What do we Know about the deflection of the fixed supports?

- We can say that the deflection of the fixed support should be zero.

d)

d) Method of surposition



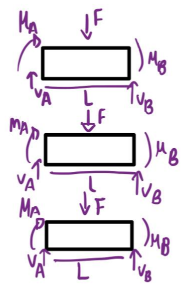
$$v_B + v_B' + v_B'' = 0$$

$$\theta_B + \theta_B' + \theta_B'' = 0$$

$$\theta = -\frac{FL^2}{8EI} \qquad v = -\frac{5L^2}{48EI}$$

$$\theta' = \frac{FL^2}{2EI} \qquad v = \frac{FL^3}{3EI}$$

$$\theta'' = \frac{M_B l}{EI} \qquad v = \frac{M_0 l^2}{2EI}$$

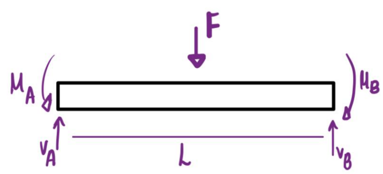


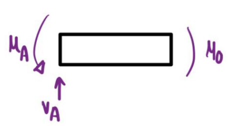
$$M_B = \frac{FL}{8}$$

Having determined the reaction forces and moments at the supports, we can find the normal stress and deflection developed at the bar.

e)

e)





$$\Sigma M_0 = 0 \; ; \; M_0 - V_A x_1 + M_A = 0$$

$$M_0 = \frac{F}{2} x_1 - \frac{FL}{8}$$

$$0 \leq x_1 \leq L/2$$

$$M_0 = \frac{F}{2} x_1 - \frac{FL}{8}$$

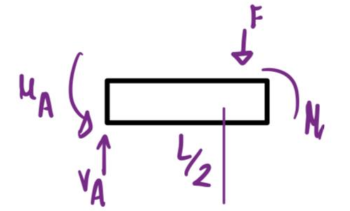
$$\rightarrow EI \frac{d^2v}{dx^2} = \frac{F}{2} x - \frac{FL}{8}$$

$$\rightarrow EI \frac{dv}{dx} = \frac{Fx^2}{4} - \frac{FL}{8} + C_1$$

$$\rightarrow EI \; v = \frac{F}{12} x^3 - \frac{FL}{16} x^2 + C_1 x + C_2$$

$$x = 0 \qquad C_1 = 0$$

$$\frac{dv}{dx} = 0 \qquad C_2 = 0$$



$$\Sigma M_1 = 0;\ M_A + F\left(x_2 - \frac{L}{2}\right) - V_A x_2 + M_1 = 0$$

$$M_1 = -\frac{F}{2} x_2 + \frac{3}{8} FL$$

$$\rightarrow EI \frac{d^2v}{dx^2} = -\frac{F}{2} x_2 + \frac{3FL}{8}$$

$$\rightarrow EI \frac{dv}{dx} = -\frac{F}{4} x^2 + \frac{3FL}{8} x + C_3$$

$$\rightarrow EI\ v = -\frac{F}{12} x^3 + \frac{3}{16} FL\, x^2 + C_3 x + C_4$$

$x = L$

$\frac{dv}{dx} = 0$

$$0 = -\frac{FL^2}{4} + \frac{3FL^2}{8} + C_3 \longrightarrow C_3 = -\frac{1}{8} FL^2$$

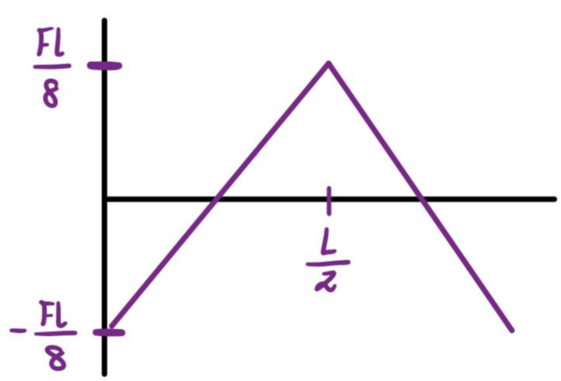
$x = L$

$v = 0$

$$0 = -\frac{FL^3}{12} + \frac{3FL^3}{16} + C_3 x + C_4 \longrightarrow C_4 = -\frac{1FL^3}{48}$$

In the following diagram, we draw the moment diagram along the bar:

Moment Diagram



In the following steps, we will be determing our maximum stress and deflection.

Data:

- $F = 1300 N$
- $L = 770 mm$
- $d_o = 25 mm$
- $d_i = 17 mm$
- $I_2 = \frac{\pi(d_o^4 - d_i^4)}{64}$
- $E = 210 GPa$

$$c = \frac{d_o}{2}$$
$$c = \frac{25}{2}$$
$$c = 12.5 mm$$

$$I = \frac{\pi(d_o^4 - d_i^4)}{64}$$
$$I = \frac{\pi(25^4 - 17^4)}{64}$$
$$I = 15074.93 mm^4$$

$$M_{max} = \frac{FL}{8}$$
$$M_{max} = \frac{1300 \times (770)}{8}$$
$$M_{max} = 125125 N.mm$$

Maximum Stress:

$$\sigma_{max} = \frac{M_{max} \cdot c}{I} = \frac{125125 \times 12.5}{15074.93}$$
$$= 103.752 MPa$$

Deflection:

$$V_{max} = \frac{F}{8}x^3 - \frac{FL}{16}x^2 = \frac{\frac{1300}{8} \times (385)^3 - \left(\frac{1300 \times 770}{8}\right)(385)^2}{210.10^6 \times \left(\frac{\pi(25^2 - 17^2)}{64}\right)}$$

$$V_{max} = -0.976 mm$$

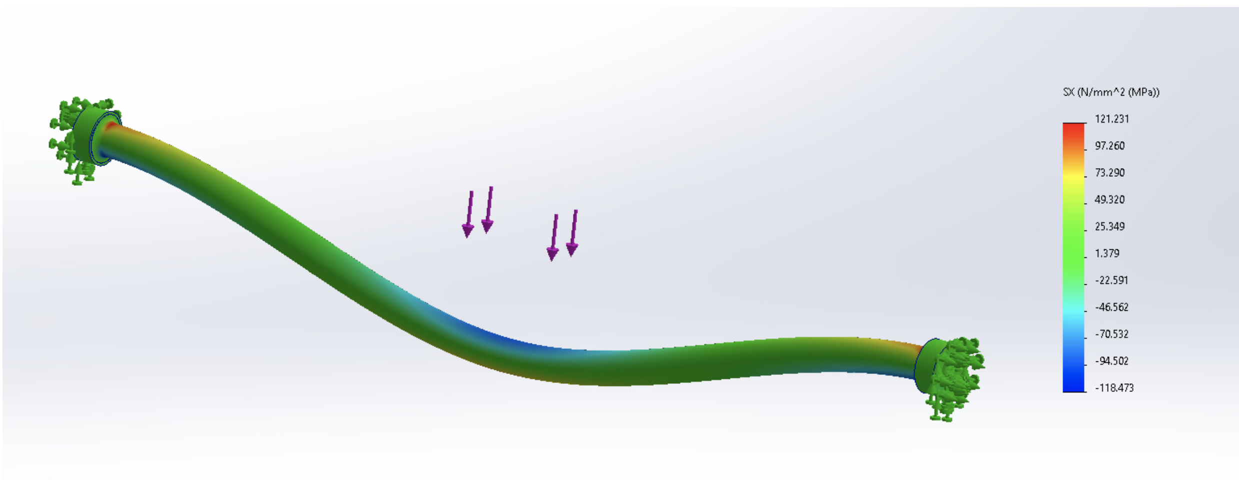
Part 2:
In this part, we will compare the peak normal stress and deflection with the ones from the FEA.

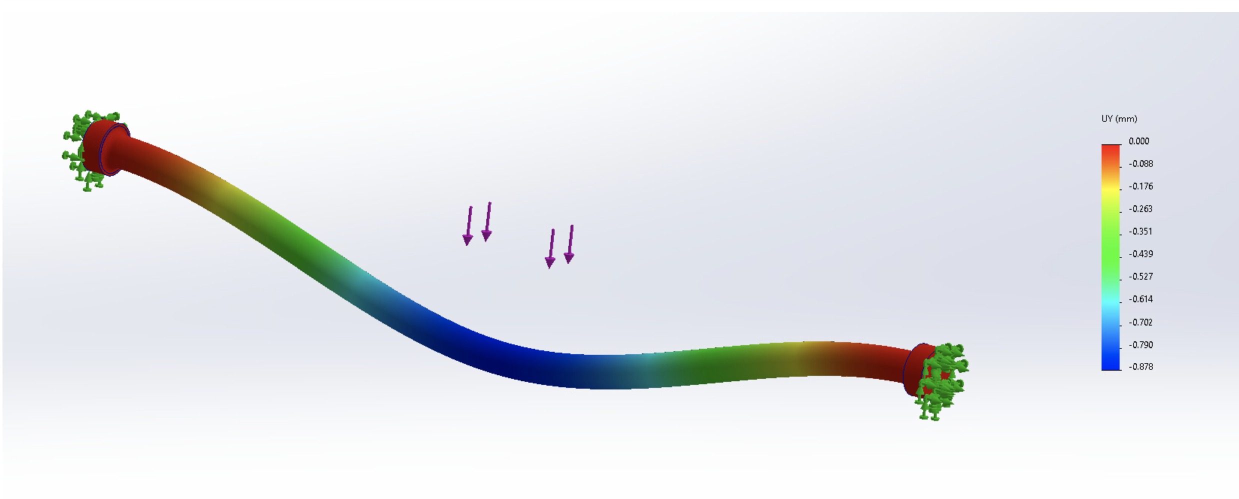
The following data will be used:

- F = 1300N
- L = 770 mm
- Do = 25mm
- Di = 17mm
- Iz = π (do^4 - di^4) / 64
- E = 210 GPa

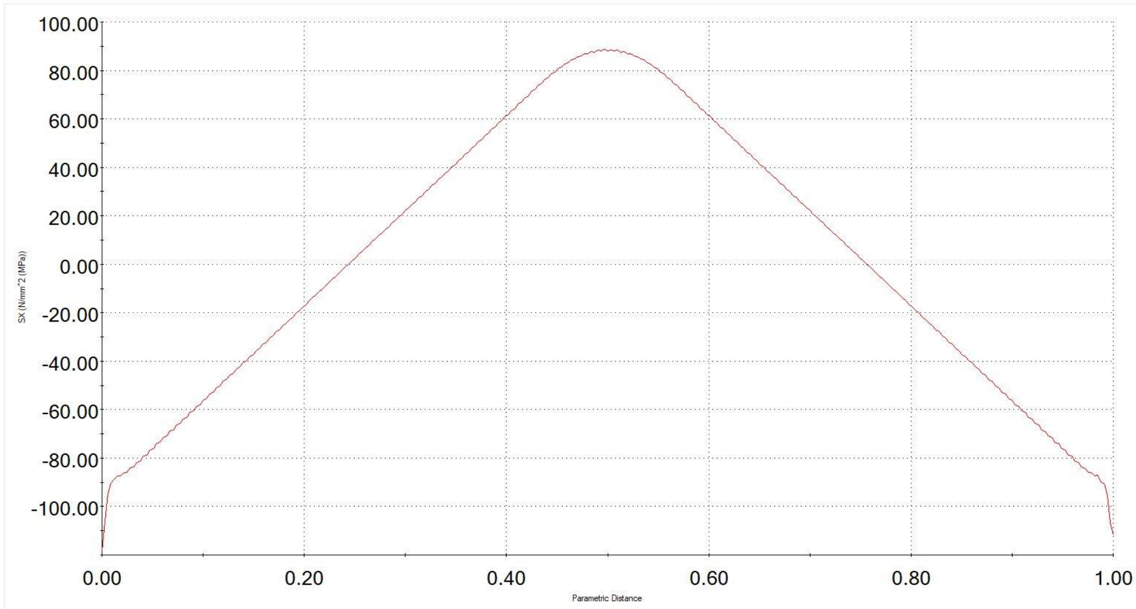
What we found in the FEA:

- ***Deliverables 1&2, Full Bar:***





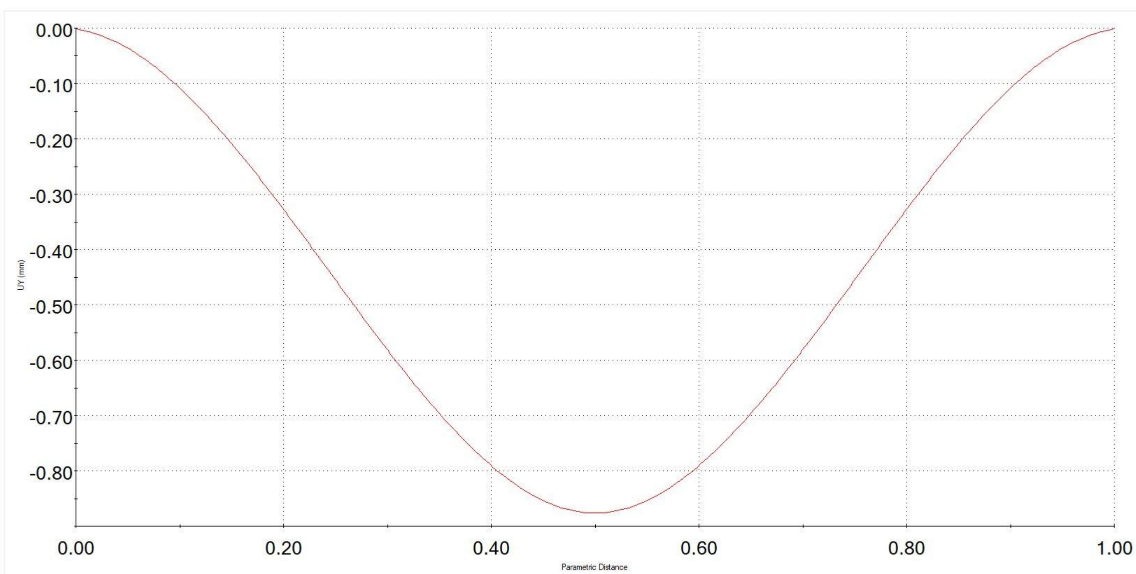
Study name: Full Bar(-Default-)
Plot type: Static nodal stress Stress1



SX (N/mm^2 (MPa))

0.083151, 106.713

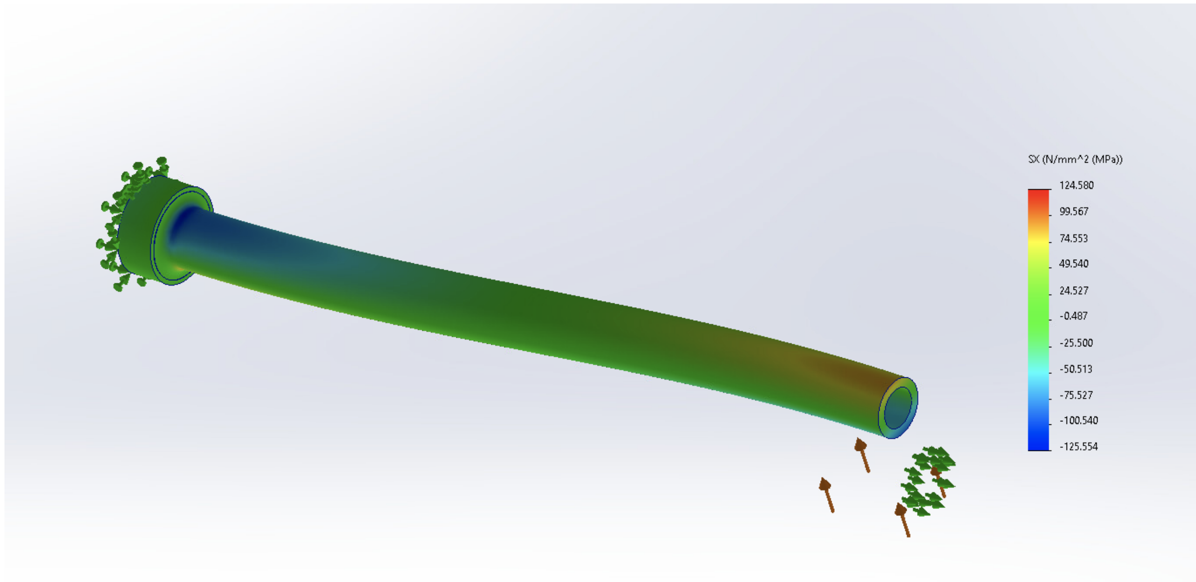
Study name: Full Bar(-Default-)
Plot type: Static displacement Displacement1

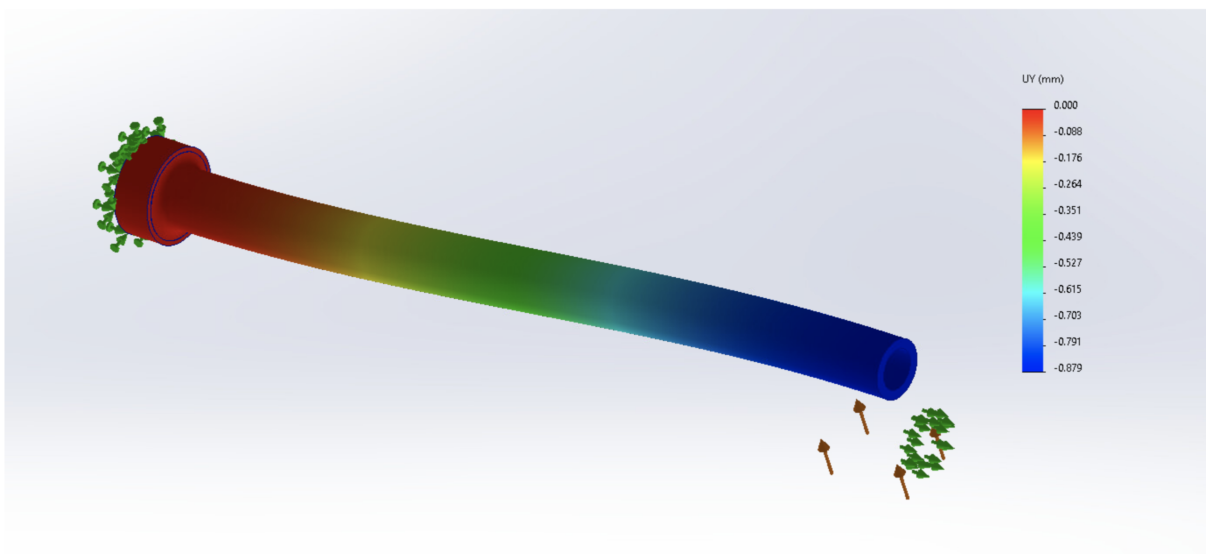


UY (mm)

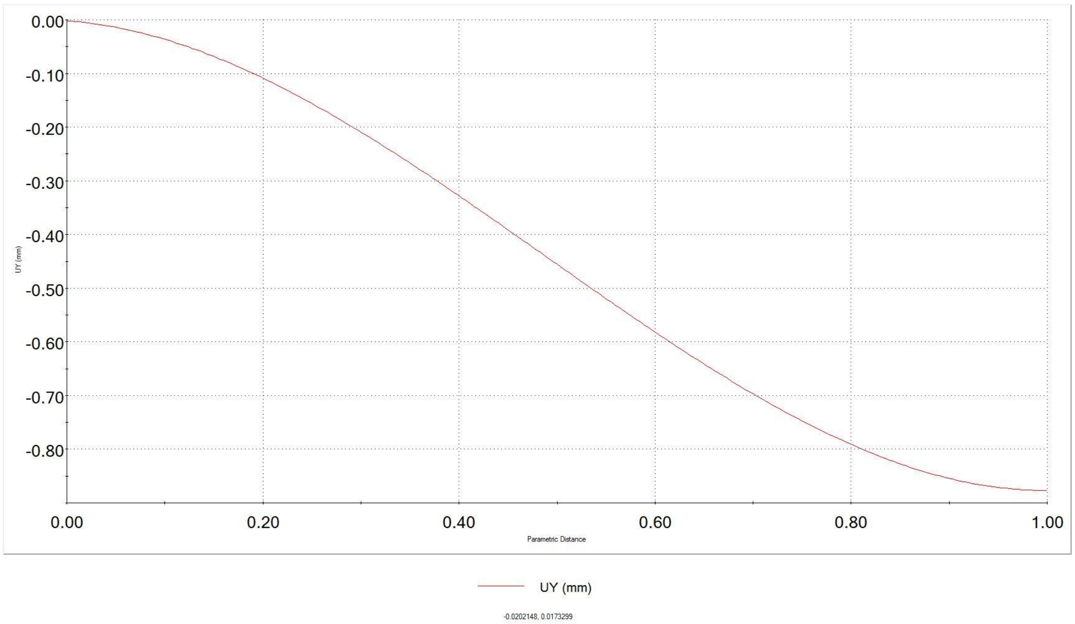
-0.00808081, 0.0303797

- ***Deliverables 3&4, Half Bar:***

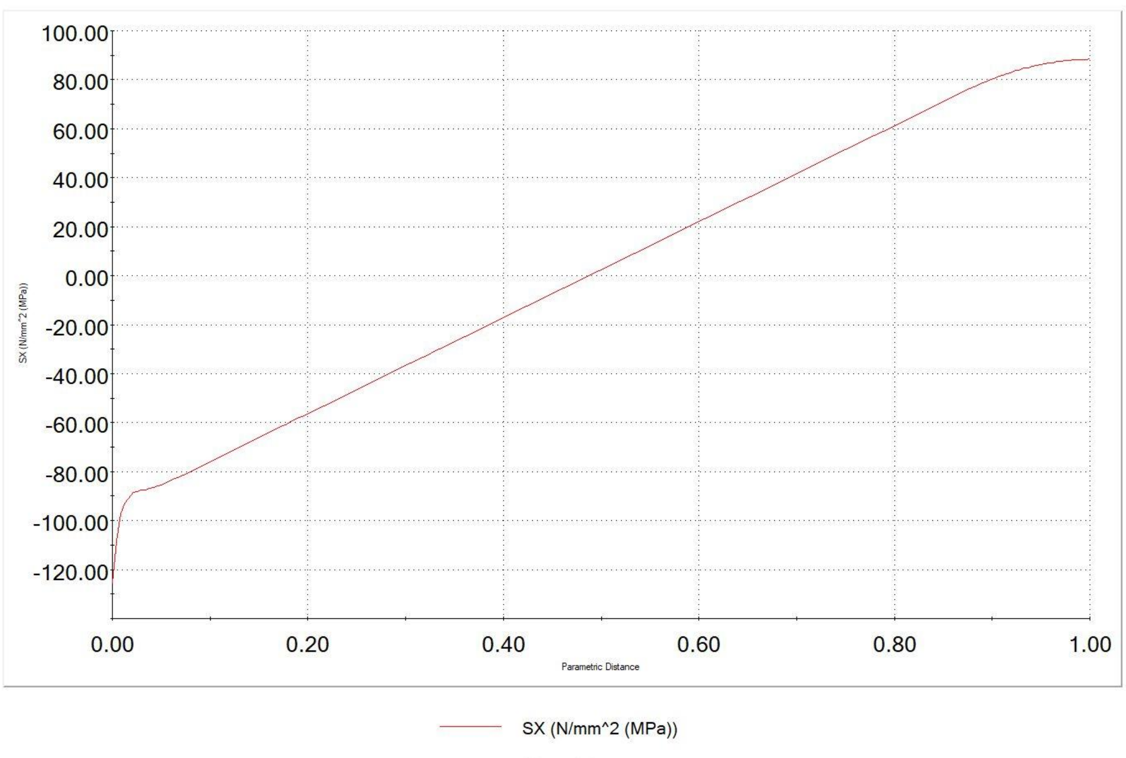




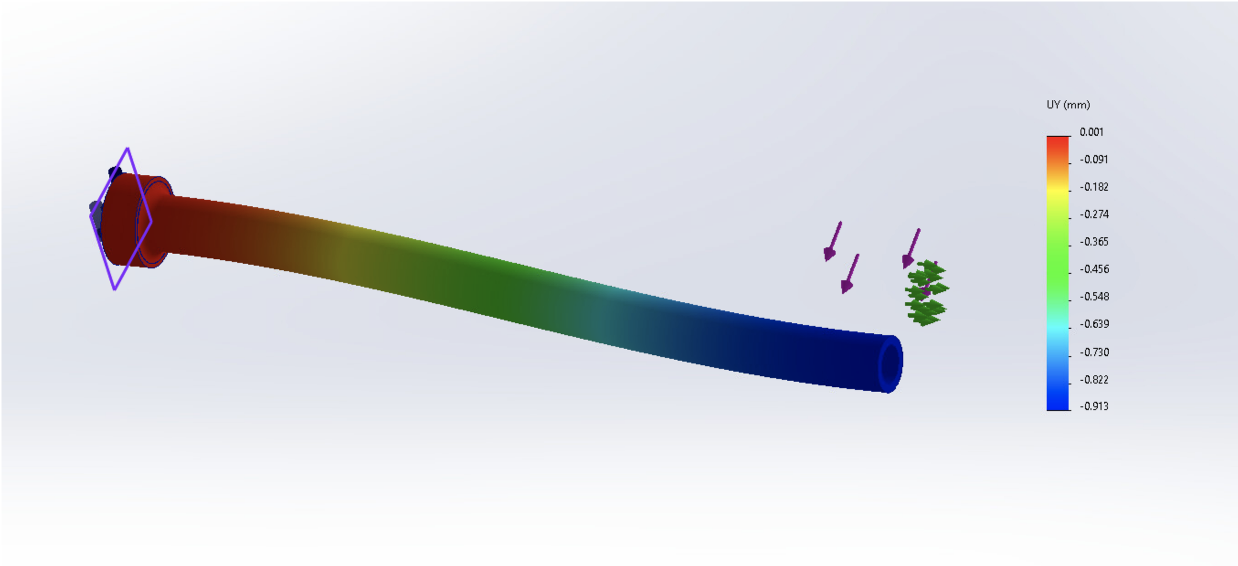
Study name: Half Bar(-Default-)
Plot type: Static displacement Displacement1

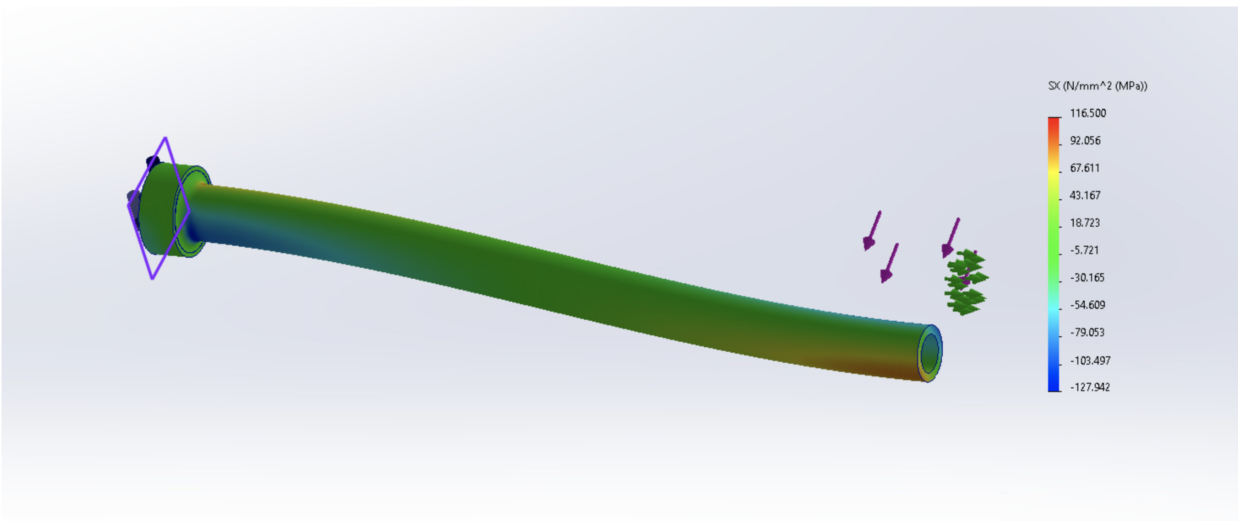


Study name: Half Bar(-Default-)
Plot type: Static nodal stress Stress1

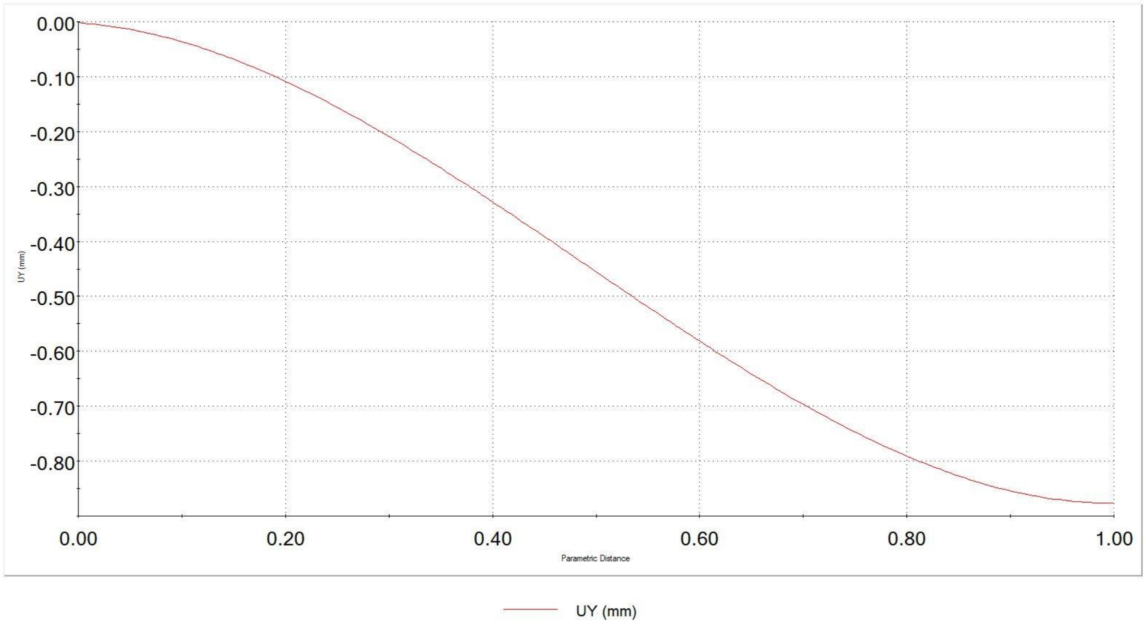


- ***Deliverables 5&6, Half Bar:***



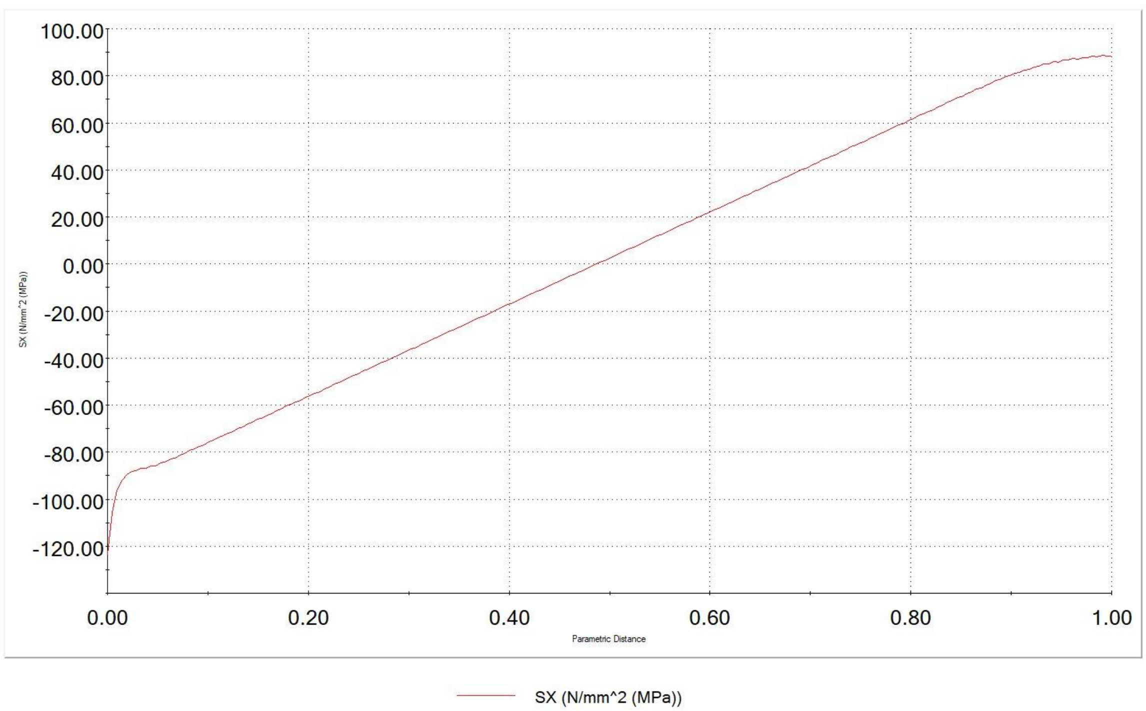


Study name: Half Bar (Bolts)(-Default-)
Plot type: Static displacement Displacement1



0.0440868, 0.0316252

Study name: Half Bar (Bolts)(-Default-)
Plot type: Static nodal stress Stress1



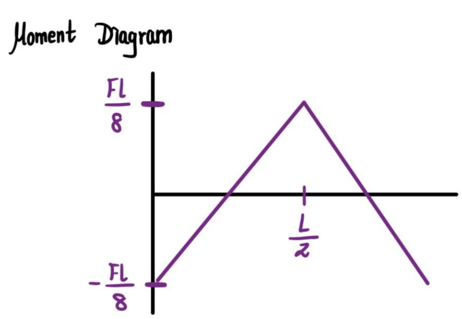
0.0108611, 93.361

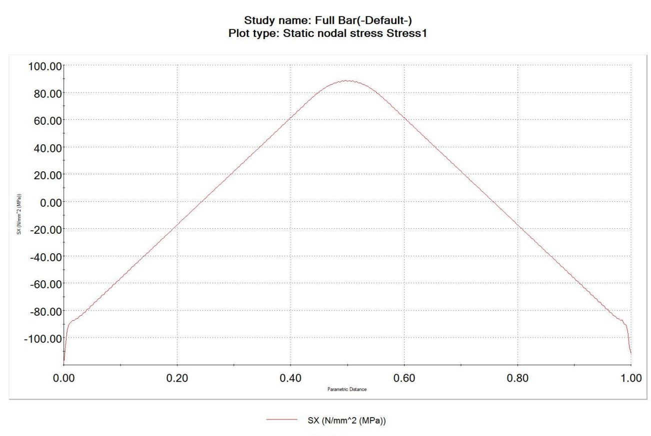
	From steps h&i	Deliverables 1&2	Deliverables 3&4	Deliverables 5&6
Maximum normal stress (MPa)	103.752	88.581	88.627	88.492
Maximum deflection (mm)	-0.976	-0.878	-0.879	-0.913

- Comparing the values found beforehand and commenting on the sources of differences:

The stress found using the theoretical calculations (103.752 MPa) was greater than that found using the FEA (with the FEA results being relatively close to each other). In addition, the deflection found using the theoretical calculations (-0.976) was also different from the deflection value found in the FEA. These differences could be due to the fixed conditions and mesh density used when creating the FEA. For example, using a more refined mesh might have given more accurate results. Furthermore, the theoretical calculations can be thought of as approximations, so differences in values could be due to any assumptions made in order to simplify the calculations.

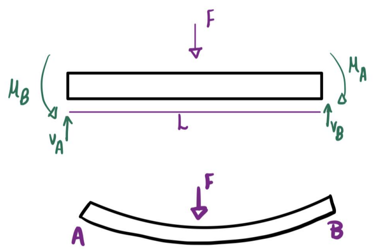
- Comparison between the moment diagram and the stress plot from the FEA:

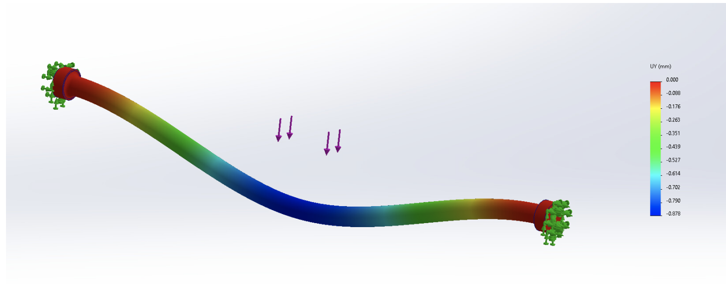




As observed in both the moment diagram and the FEA plot, each displays a curved shape, affirming the accuracy in approximation of our calculations.

- Vertical deflection along the beam, compared to the deflection curve from the FEA:





As observed, the schematic representing the vertical deflection of the beam closely mirrors the FEA representation. We determined how the beam behaves at the endpoints and identified the points along the beam where the deflection is expected to be at its maximum.

Conclusion:

The goal of this project was to theoretically determine the normal stress and deflection of a pull-up bar subjected to a concentrated load in the middle and compare this to the results of a Finite Element Analysis of the same conditions. The theoretical calculations were performed by drawing free body diagrams and finding reaction forces and moments at the supports in order to calculate the normal stress and deflection developed in the bar. The normal stress found using theoretical calculations was greater than the stress found using the FEA, while the displacement was found to be relatively accurate. This difference may be due to the fixed conditions and mesh density used to create the FEA.

In this project, we learned to find the normal stress and deflection of the pull-up bar by first finding the reaction forces and moments at the supports of the bar. We learned to apply the flexure formula and use the moment function to determine the stress and deflection. This project also taught us to use SolidWorks to visualize and model how a fixed bar behaves under deformation. We learned to represent the simulation using various types of fixtures, including symmetry and wall fixtures. Our final outcome included results from three studies: using a full bar and two fixed wall connectors, a half bar with a single wall connector using a symmetry fixture, and a half bar with a single wall connector using a wall fixture. Seeing the results of these studies showed us the different ways to perform a Finite Element Analysis while getting the same results.