Appendix: Complete transcript

1	Jared:	When I started arguing that the average acceleration—the acceleration would be zero. I didn't think there would be a change. Uh, through this, I've been kind-of thinking about a couple of things. So because the acceleration is the same on both, that means that over time both of them will travel the exact same distance, right? Is that the case?
2	Adolfo:	Mmmmm—
3	Vesal:	Sorry?
4	Jared:	Since the acceleration is going to be the same on both, that means that in the same given time both of them will travel the same distance?
5	Vesal:	Oh, so if you're treating them separately—
6	Jared:	Yeah, let's pretend we're treating them separately for now.
7	Vesal:	[Oh, interesting—
8	Jared:	[So they're gonna travel the same distance?
9	Vesal:	Uh-huh—
10	Jared:	In the same amount of time? Right? [Vesal: Yeah.] Yeah, I just want to make sure that's the case. So if that's the case— [Vesal: Oh, beautiful—] and they're both traveling the same distance— [Vesal: Yeah, yeah] Then there's no way that the acceleration on the center of mass could be zero because it always needs to be closer to the more massive one because that one again is more massive. [Vesal: Ohhhhhf!] Yeah, I was kind-of balancing my pen— [Vesal: Uh-huh, uh-huh—] on my finger and—it isn't very accurate compared to those drawings at all. But if that were moving the same distance that way and that one were moving the same distance that way, ((Jared points to the right)) I would have to somehow move my finger in that direction ((Jared points left)) because if not it would just tip over because there's more mass over here. ((Jared points left)) Is that—
11	Vesal:	Ohh, [shoot—
12	Jared:	[—correct?
13	Adolfo:	[Yes.
14	Vesal:	[—how do we respond to THAT argument?
15	Adolfo:	I was ( )

16	Vesal:	That, that—so, so we have to think about that argument carefully. So, did you guys hear what he said? So like he's saying there's—the acceleration of these guys is the same in opposite directions which means they'll be located the same position—sorry—they'll be located
17	Adolfo:	They'll be—
18	Jared:	same distance away
19	Vesal:	the same distance away from where they started off let's say. Is that what you're saying? [Jared: Yes] And so how—does that cohere with the argument Derrick was making—and I think Mischael also? And someone—a couple of other—and, and Raul. So does—does that cohere? Is that consis—so, if the ce—if the center of mass is not—so, so is the center of mass moving with— [Adolfo: Yeah] You're saying it's not ((Vesal points to Jared))
20	Jared:	Again, I'm saying it that it wouldn't be moving—I mean originally, I was saying it wasn't—
21	Vesal:	Oh, you're saying it would be moving
22	Jared:	but now I'm saying that yes, it would because of that reasoning.
23	Vesal:	Ok, I, I, I thought I was hearing something else.
24	Jared:	Oh, sorry. [My bad
25	Vesal:	[But, but—what, what—um, now it's clear. So if, if they are the same distance away then the center of mass would be in the same—
26	Alejandro:	Yeah, [mov
27	Vesal:	[location, right?
28	Alejandro:	moving
29	Adolfo:	Wait, what?
30	Mischael:	Well, [like ( ) question
31	Vesal:	[Oh, no, sorry, sorry— the—the [opposite.
32	Alejandro:	[It would be moving
33	Jared:	Right.
34	Vesal:	that the center of mass would be moving— ((Vesal points left))
35	Jared:	Yeah, it would have to moving—[that direction ((Jared points left))
36	Vesal:	[—closer to the 2M. [Jared: Mm hm.] ((Jared nods))

		Um, otherwise what would happen?
37	Jared:	Otherwise it wouldn't be
		((Jared moves his hands to demonstrate a see-saw))
		the average anymore.
38	Vesal:	Right, it wouldn't be the average anymore. It would be different. [Korri: Mm hm.] Are you sure? Can somebody re—restate it? Just, just so that we—cuz that— that's a really nice way of thinking about, uh—
39	Adolfo:	Well, I think it would be better if we draw it out, cuz—
40	Vesal:	Wanna draw it?
41	Adolfo:	Uh, ok—
42	Jared:	((to Adolfo; Jared smiles))
		Go!
43	Adolfo:	OK.
		((Derrick silently applauds. Adolfo gets up and goes to the board))
44	Vesal:	(Vesal starts to erase the board))
		And then we'll break up into groups—
		((Vesal erases part of the board))
		um, after summarizing—
		((Vesal walks away from the board; Adolfo draws the two bodies on the board: 2M M; Vesal points at Jared))
		Great point.
45	Adolfo:	Now here
		((Adolfo points to the math already up on the board))
		we said accelerations have to be the same because 2F over 2M is F over M, and F over M is F over M. [Vesal: Yeah] So this is going to a, ((Adolfo draws a vector labeled "a" over the Mischael object))
		this is going to a.
		((Adolfo draws a similar vector over the 2M body))
		[Vesal: Yep, yep.]
		Uhhh, this means that they're travel—they're gonna travel—
		((Adolfo draws dotted lines straight down from each body))
		the same distance in the same amount of time because they have the same acceleration. So this one
		((Adolfo starts drawing another instance of the 1M body vertically at the lower end of the dotted line and horizontally to the right of the original instance of 1M))
		is pointing away and this one—

((Adolfo draws another instance of the 2M body horizontally aligned with the new instance of the 1M body and horizontally the same distance away from the original instance of the 2M body as the new 1M body was away from the original 1M body))

46 Mischael: That supports—

47 Adolfo: What?!

48 Mischael: That supports the acceleration being zero.

49 Vesal: Wait, wait—

50 Adolfo: No, no, no—

51 Jared: ((to Mischael))

No, no.

((to Adolfo)

Keep going, keep going.

52 Adolfo: Because here—[that's the center of mass—

((Adolfo points to an asterix he previously drew between the 2M and 1M, closer to the 2M))

53 Vesal: That's where I tripped up—

54 Adolfo: It's impossible for it to be here

((Adolfo draws an asterix representing the center of mass vertically aligned with the bottom ends of the dotted lines and horizontally in the same place as it was originally))

[because—

55 Mischael: Why?

56 Adolfo: —it has to be closer to 2M

57 Mischael: It is closer to 2M, though.

58 Adolfo: Because the 2M is heavier.

59 Mischael: ((Mischael points at the board))

The, the, the rate—the proportionality of the distance away is still the same.

60 Alejandro: Well—

61 Jared: But they travel the s—

62 Mischael: That's what I was thinking. That's what I said.

((others murmur))

63 Alejandro: The thing is that—

64 Mischael: His argument is different. He's saying that they're both accelerating in the *same* way.

65 Alejandro: What?

66 Jared: What?

((others murmur))

67 Mischael: Wasn't that your argument?

68 Jared: Mine's more along these lines again

		((Jared points to the board and then starts to get up))
69	Mischael:	Okay, if I had—if I had, like, a pencil, right, and I'm balancing it on my fingers, if both ends of the pencil continue to grow, the center of mass will still be where I'm balancing on my fingers. So they're growing at the same rate, but the distance away from—the distance is—is still proportional.
70	Adolfo:	I don't know, I don't know if I'm drawing it right.
71	Derrick:	((to Mischael))
		No, but this side—this side of the pencil is double the mass of this side.
72	Mischael:	((to Derrick))
		[I know. That's why the center of mass is closer—
73	Kimmee:	[But the center of mass is closer to the thing that's heavier. So if it's a pencil—but if it's, like, a pencil, then it's like—
		((Adolfo redraws the lower asterix in a new position; see Figure))
		I mean, you're assuming that—like, it's right in the middle. So if you were looking at it from the center of—from the point of the center of mass, it would be growing, like, proportional to each other, from the center of mass. But if you're looking at something, like, heavier
		((Kimmee points to her water bottle))
		than the other thing,
		((Kimmee points to her phone))
		and its growing, well, how could it be, like, how could the center of mass be in the same place? It would have to move. Like—
74	Jared:	[It would have to move *with the* [[*heavier mass.*
75	Mischael:	What do you mean?
76	Kimmee:	[So it would be [[closer. Yeah.
77	Joel:	Um, so I think I know exactly what Jared's saying—where, like—
78	Vesal:	Can you—can you help us with a drawing?
		((Joel gets up and goes to the board))
79	Joel:	Um, so I think I know almost exactly what Jared's saying—where, like—
80	Vesal:	Can you—can you help us with a drawing?
		((Joel goes to the board))
81	Joel:	So if we agree that that's the location of the center of mass, here, right?
		((Joel points to the top asterix in Figure))
		[Vesal: Yep]

		So, at this point—so it's a lot closer to the, uh, larger mass when you start.
		[Vesal: Yep]
		And as you start separating the distances,
		((Joel moves his hands apart)
		this
		((Joel points to the top asterix again))
		di— this moves—sorry. If they move away at the same rate,
		((Joel moves his hands apart from each other again))
		then THIS distance
		((Joel points to and brackets D in Figure))
		minus this distance
		((Joel points to and brackets L in Figure))
		is going to be the diff—only difference in distance between the two masses, right? To the center of mass? And as they mo—as they grow farther and farther apart, won't that become insignificant?
82	Jared:	I'm not sure what you mean.
		[Derrick: Yeah.]
83	Kimmee:	I don't follow.
84	Mischael:	Um, OK so at the beginning, if you take the distance from the 2M to the center of mass, right, and you put it over the distance from the center of mass to M, that—that proportionality will stay the same if they're moving different—
		((Alejandro raises his hand))
		if they're moving—if they're accelerating at the same rate.
85	Vesal:	So he's
		((referring to Mischael)) saying if they're accelerating at the same
		((Vesal gets up and walks towards the board))
		rate—
86	Joel:	Yeah, and moving the same distance apart.
87	Vesal:	The same distance apart—
		((Vesal walks back away from the board and towards his seat))
88	Joel:	Yeah, so, if this is, like—this ( )
		((Joel points to D in Figure))
		is, like—is one—one whatever unit, from the center of mass.
		((Joel marks D "1"))
		And this
		((Joel points to L and marks it "2"))
		is [two
89	Vesal:	[two units

90	Joel:	then as they keep going farther apart, this could be like—
		((Joel draws a new line delineating D2 in Figure and labeling it “1,000,001”))
91	Vesal:	Three units and six units.
		((others laugh))
		Let’s make it easy.
		(others laugh))
92	Jared:	Why so big?
93	Adolfo:	Yeah, why so, why so big?
94	Jared:	Just 10, just 10.
		((others murmur; Joel erases “1,000,001”))
95	Vesal:	He’s saying, he's saying it would be—
		((to Mischael))
		you would say it’s three and six?
96	Mischael:	Say, say it’s three and six, right? One over two is the same thing as three over six. It’s still proportional.
97	Alejandro:	No, wait—
		((Alejandro motions with his hand and then lets it drop))
98	Vesal:	Would it be proportional?
99	Jared:	Right, and that proportion is still closer to the heaviest one—
100	Joel:	Well I think it’s just adding distances, so is it gonna stay proportional?
		((Alejandro motions with his hand as if to protest, and then silently shakes his hands downwards as if in frustration, before smiling and resting his chin on his arm))
101	Mischael:	Can I, like ( )?
		((gets up to go to the board; others murmur))
102	Alejandro:	Nooo—
		((smiling and shaking his fists))
103	Mischael:	This was—this was one of the original arguments. But like, it’s wrong? I feel like I know—
104	Vesal:	Wait, you’re not going to erase that, are you?
105	Mischael:	No. ((Mischael puts down the eraser, Joel starts to go back to his seat; others laugh))
106	Jared:	Not anymore.
107	Mischael:	Alright, so say this distance
		((Mischael points to the distance between the center of mass and the 2M body before it moved))
		is like, d—
		[Vesal: Yeah]
		and this distance

((Mischael points to the distance between the center of mass and the 1M body before it moved))

is l.

[Vesal: Yeah—]

So the proportion—the proportionality of d over l

((Mischael writes "d/l" on the board above Adolfo's representation of the system before—the bodies moved))

and if this moves—if they're both accelerating

((Mischael points at the representation of the bodies before they moved and motions that they move away from each other some set amount))

away at the same—rate, d over l

((Mischael points to that expression on the board))

will always stay the same.

[Alejandro: No.]

Because they're moving away at the same, heh, rate.

108 Alejandro: No, no.

((Alejandro points at the board and Korri raises her hand. Alejandro then looks at the rest of the class.))

Heh—

109 Jared: And in order for them—

110 Mischael: Even if, even if this

((Mischael points to the distance between the new position of the 2M body and the original center of mass))

is longer, this

((Mischael points to the new position of the 1M body and the original center of mass))

will be even longer.

[Jared: Yeah]

So d over l

((Mischael points to that expression on the board))

will stay the same. If this

((Mischael points to and labels d in Adolfo's original drawing))

is one, this

((Mischael points to and labels l in Adolfo's original drawing))

is two, right, and then it

((Mischael points at the 2M body first represented before it moved and then represented after it moved))

accelerates so this becomes three,

((Mischael labels the distance between the new position of 2M and the original center of mass "3"))

this

		((Mischael points at the 1M body first represented before it moved and then represented after it moved))
		will accelerate to be six.
		((Mischael labels the distance between the new position of 1M and the original center of mass “6”))
111	Jared:	And doesn't that mean that for the proportion to stay the there—
112	Mischael:	So one over two
		((Mischael points to the “1” and “2” on the board))
		is still—is still equal to three over six.
		((Mischael points to the “3” and “6” on the board))
113	Jared:	For the proportions to stay the same
114	Mischael:	[The proportions will stay the same.
115	Jared:	[that means the x will always have to be closer to the heavier mass, right?
		((Jared motions with hands representing the positions of the bodies and the distance between the 2M body and the center of mass as small relative to the distance between the center of mass and the 1M body.))
116	Mischael:	It IS closer—it’s THREE
		((Mischael points to the distance marked “3”))
		compared to SIX.
		((Mischael points to the distance marked “6”))
117	Jared:	Right, it IS closer, but that’s exactly what we’re saying—
118	Mischael:	[So it’s NOT MOVING.
119	Jared:	[It IS closer. ((others murmur)) But HOW COULD IT DO THAT IF IT WASN’T MOVING?
120	Vesal:	Wait, wait, sorry, sorry. Ok, great, great.
		((Vesal smiles and points to Mischael; Talisa is also smiling here))
		Uh, I just wanna—you?
		((Vesal points to Korri))
121	Korri:	Ok, yeah.
122	Vesal:	Korri?
123	Korri:	Um, so, just cuz it stays proportional and, like, I’m combining what both of them are saying, but it seems like they’re disagreeing for a weird reason. Like, I think that—like, okay—Mischael is suggesting that the proportion is gonna stay the same—it’s not gonna, like, percentage-wise, like, go more towards one or the other objects—like, three to six, three here—like, whatever. It’s the same distance. ( ) Whatever, so—but he’s
		((Korri seems to be referring to Jared))

		saying that it has to be accelerating, though, because of, um, you would have to move faster to maintain that proportion. So I that think the—the reason zero is there is it's supposed to kinda trick you into thinking, like, "Oh if the proportions stayed the same then there is no acceleration." But it HAS to be accelerating to maintain that proportion—so if it was going at constant velocity the proportion would be all messed up ( ) the distance won't be increasing the same in time.
124	Vesal:	OK
125	Korri:	Does that make sense?
		((Korri looks at Vesal))
126	Vesal:	Yeah, Joel? Joel?
		((Vesal points to Joel; Alejandro pumps his arms downwards as if antsy to contribute))
127	Adolfo:	Mischael, Mischael!
		((Adolfo points his arm towards the board strongly, nearly raising from his seat, then lowers his arm; others laugh))
128	Mischael:	I think that this whole system is wrong.
129	Adolfo:	NO, WAIT!
		((Adolfo moves his hands in an "X" as if to say "time out"))
130	Mischael:	And that they're both—they're not, they're not both
		((Mischael motions at Adolfo's original representations of the bodies, before movement of the masses))
		accelerating in these different places. They're both accelerating the same
		((Mischael motions left))
		way.
131	Adolfo:	NOooo.
		((Adolfo waves his arms; others laugh))
		WHAT??
132	Mischael:	The original—
		((Mischael points at Adolfo))
		the original argument, right, was that it's one system.
		((Mischael points to the original representation of the system at the top of the board which is boxed in))
		If it's one system, how are they both accelerating different
		((Mischael separates his hands horizontally away from each other))
		directions? They're both accelerating the same way.
		((Mischael motions to the left from Adolfo's original drawing))
		So the center of mass

		((points to the original position of the center of mass in Adolfo's lower drawing and indicates that it should move leftward))
		has to be accelerating the same way.
		((laughter from others; Mischael then points at parts of Adolfo's original drawing again))
133	Vesal:	Joel? Joel? OK—
134	Student:	What?
135	Alejandro:	Wait, no, no, Let me explain—
136	Jared:	The "system" doesn't mean ( )…as a whole
		((Jared motions with arms to encompass an entire system))
137	Adolfo:	No, no
138	Alejandro:	One minute.
		((Alejandro gets up and goes the board))
		I will do—give me a minute. Like count me a minute. Just give me a minute.
139	Jared:	58 seconds
		((Jared taps his watch; others murmur))
140	Alejandro:	So when I think of proportionality, I think of "times two". OK? So, let's say—let's—let's go back to the astronaut, no?
		[Student: OK.; Student: What?]
		So let's say the wrench is six meters and the astronaut is two meters, no?
		((Alejandro writes "6/2" on the board))
		[Mischael: Yeah] And then let's say "proportionality" is "multiply", no?
		((Alejandro writes some other things on the board, maybe "x 2"))
		So then the astronaut will be four meters from the center
		((Alejandro writes "4" in the denominator))
		of the mass and here will be 12,
		((writes "12 in the numerator))
		no? So this
		((Alejandro points to the 12/4))
		is changed, no? Significantly, no? By six
		((points to the denominator))
		and this
		((Alejandro points to the numerator))
		by two. While here,
		((Alejandro points to Adolfo's original model of the 2M, Mischael example))
		the distance changed the same.

		((Adolfo points excitedly towards the board))
		So we would say this
		((Alejandro labels d “2”))
		is two and this
		((Alejandro labels l “6”))
		is six. And this will be
		((Jared nods))
		plus five, plus five—
		((Alejandro writes “+5” next to the “2” and the “6”))
141	Students:	Yeah!
		((several voice their agreement))
142	Jared:	There you go, there you go.
		((Jared indicating to Mischael))
143	Alejandro:	Six over two is the same as 11 over seven.
		((Alejandro writes “6/2 = 11/7” on the board))
		Ooh, that’s not—
		((Alejandro erases something; others laugh))
144	Alejandro:	Is this
		((Alejandro boxes the 6/2 = 11/7))
		equal? No, that’s not—
145	Adolfo:	They’re moving the same—
146	Korri:	Wait, where you’d get plus—why are you adding the distances that way? Like it’s—
147	Alejandro:	Because I’m—okay, so I’m—since this acceleration’s the same
		((Alejandro uses hands to indicate equal distances away from the original positions of the bodies))
148	Mischael:	[OK, but in one—in one case—
		((Mischael points at the board))
149	Yan:	—the distances
		[will be the same
150	Vesal:	[He's saying because the acceleration is the same, they cover the same amount of ground—the same amount of ground.
151	Student:	Yeah.
		((others murmur))
152	Alejandro:	((Alejandro crosses arms, maybe to indicate movement))
		So let’s say the amount of ground is five, so six plus five is 11, then divided by six—no, shit, now I’m confused.
		((others laugh))
153	Adolfo:	No, that’s right!
154	Derrick:	No, you were right. You were right.
155	Jared:	You’re good, you’re good.

156	Alejandro:	Yeah, I'm good.
157	Adolfo:	That is not ( )
158	Alejandro:	Initial is 6, initial is two, and then [plus five, plus five—
159	Derrick:	[Yeah
160	Alejandro:	—makes 11 and seven, and six plus two is not the same as
161	Adolfo:	[Yeah, but the—
		((Adolfo waves his hand diagonally))
		but the—yeah, they—
162	Alejandro:	[11 over seven.
		While here,
		((Alejandro points to the top of the board)
		since, since—
163	Mischael:	Well at the top you're multiplying by two and at the bottom [you're adding
164	Alejandro:	But I'm multiplying by two. Yeah, but this is in the case of the ASTRONAUT—
		((Alejandro writes something on the board near the top))
165	Adolfo:	With not—with INTERNAL forces.
		((to Mischael))
166	Jared:	Yeah, internal. These are the EXTERNAL forces.
167	Korri:	( ) ((others murmur))
168	Alejandro:	The astronaut. You know—you know when he pushed—like, you know when he pushes the wrench, no?
		((Alejandro pantomimes the astronaut throwing the wrench))
		[Student: Yeah.]
		And the force affects the both is the same. Same
		[FORCE
169	Derrick:	[is the same.
170	Adolfo:	[YESSS.
171	Alejandro:	But the—but the astronaut is more massive,
		[you know,
172	Student:	[so—the acceleration—
173	Alejandro:	—even though it's the same force, its more massive, so he's gonna go back SLOWER,
		((Alejandro pantomimes moving back slowly))
		while the wrench is going, like wooo,
		((Alejandro backs up quickly; others laugh))
		...you know? Pretty fast.
		((others laugh))
		So it's gonna be like this, something like this one's going to be faster, this one's going to be slower.

		((Alejandro motions to indicate one object moving quickly to the left and another object moving slowly to the right.))
		But the proportionality would be—
174	Derrick:	[is the same
175	Alejandro:	[the same.
		((Alejandro points to "d/l" on the board))
		[Kimmee: Yeah.]
		But here the distance between those two is the same.
		((moves his hands apart from a fixed point at equal rates.))
		So here's adding
		((Alejandro points to the current example))
		and here's multiplying,
		((Alejandro points to the astronaut example))
		you know, like—
		((Alejandro bows; Derrick claps))
176	Vesal:	But you're saying—But what I'm seeing here is that you're ADDING
		((Vesal gets up and walks towards the board and points at the current example—Adolfo's— drawing of the original system))
		The proportionality is not main— is not
		[maintained
177	Student:	not maintained
178	Alejandro:	Yeah, it's NOT maintained.
179	Vesal:	So therefore, what?
		((Vesal directs this question at Alejandro and steps away from the board, returning to his seat))
180	Alejandro:	Therefore, I change my mind and I go with Jared
		((points at Jared))
		that it's not zero.
		((Alejandro motions at Vesal))
		Acceleration is not zero since—so, so the x
		((Alejandro circles the x representing the original center of mass that Adolfo drew))
		has to be MOVING with the mass—with THIS MASS
		((Alejandro points to the 2M mass, motioning to the left))
		While for the astronaut,
		((Alejandro points towards the top of the board))
		since it's proportional, you can say you can keep the mass—the center of the mass is
		zero. In the astronaut case, the center of the mass is zero
		((Alejandro writes something on the board))
		because it's proportional. But here,

((Alejandro points at the parts of Adolfo's original drawing))

since it's not proportional, x

((Alejandro circles x again in Adolfo's original drawing and makes a small leftward arrow))

has to be moving with, with um, with 2M.

((Alejandro points to the 2M body in Adolfo's original drawing))

And the way that I thought about it is, um, as Jared said, if you keep it right in the middle—

((Alejandro picks up a chalkboard eraser to demonstrate its balance point))

uh, this is bad, this is equal—

((others laugh))

but, OK, like, what I thought about it is…