

New reference for Theorem 2.2 and Corollary 2.5 in WAVELET METHODS FOR A WEIGHTED SPARSITY PENALTY FOR REGION OF INTEREST TOMOGRAPHY

I thank Alexander Katsevich for letting me know of a beautiful recent theorem of his in [3] that subsumes Theorem 2.2 and Corollary 2.5 of our article. I will quote both theorems and explain why theirs is stronger than ours. We let R be the Radon line transform in the plane.

Theorem 2.2' [4] Let f be a distribution of compact support and let Ω be an open connected set. Let U be an open disk contained in Ω .

- (i) Assume f is constant on U .
- (ii) Assume Rf is zero for all lines intersecting Ω .

Then f is the zero function.

In our article [4], the theorem was stated for functions, since that was what was needed, however, as noted at the start of its proof it is valid for distributions of compact support (a standard convolution argument with an approximate identity can be used to prove it for distributions, as stated here (using $R(f * g) = Rf *_p Rg$)).

The theorem in [3] is as follows.

Theorem 8.1 [3] Let f be a distribution of compact support and let Ω be an open connected set. Let U be an open disk contained in Ω .

- (i) Let S be the connected component of $\text{supp}(f)$ containing Ω . Assume there is an open neighborhood V of S and an function F that is analytic on V and such that $F = f$ on U .
- (ii) Assume Rf is zero for all lines intersecting Ω .

Then f is the zero function.

The difference between their theorem and ours is in condition (i). Their condition (i) hypothesis subsumes ours because the analytic function F in their condition (i) can be constant: under the hypotheses of our theorem, one can extend the constant value of f in U to a constant function, F , in a neighborhood of $\text{supp}(f)$. Then one applies his theorem with our hypotheses to show $f = 0$.

This conclusion that $f = 0$ knowing only interior data is remarkable for several reasons. The interior Radon transform is not injective in general, even in the ROI.

In fact, most of the earlier theorems, such as the ones in [2, 5] inferred only that $f = 0$ in the ROI, Ω . However, the two theorems above conclude $f = 0$ everywhere (see also the theorem in [6]).

Many of the previous theorems can be proven using microlocal analysis in [1]; it is straightforward to use microlocal arguments to show these theorems in [2, 5] (inferring that f is zero in Ω given that the line transform of f is zero on lines meeting Ω and f is zero on an open subset U of Ω). Our Theorem 2.2' and Theorem 8.1 in [3] seem more difficult to prove using microlocal analysis, because it is generally not easy to use microlocal arguments to get outside the ROI; one knows the entire analytic wavefront set of f in the ROI from its Radon data but the Radon data does not determine all wavefront directions of f outside the ROI.

References

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