Editorial

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Research biography of a distinguished expert in the field of inverse problems: Professor Eric Todd Quinto

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Abstract: This article gives a brief overview of the research in microlocal analysis, tomography, and integral geometry of Professor Eric Todd Quinto, Robinson Professor of Mathematics at Tufts University, along with the collaborators and colleagues who influenced his work.

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The research of Eric Todd Quinto, Robinson Professor of Mathematics at Tufts University, spans Radon transforms, microlocal analysis, and their application to a broad range of problems in tomography. His career would not be as rich without the support of important mentors and friends over the years, and this article outlines his work through his collaborations.

Professor Quinto was lucky to meet Nobel Laureate and Tufts Physics Professor Allan Cormack, a pioneer in tomography, when he started at Tufts in 1977. Cormack introduced him to the field of tomography, and they wrote several articles together. In 1980, he arranged for Professor Quinto to attend the first Oberwolfach tomography workshop, where Quinto met founders of the field, including Frank Natterer and Alfred Louis. They discussed important tomography problems and an applied way of thinking and became lifelong friends. Many of Louis' students, including Bernadette Hahn, Peter Maaß, Andreas Rieder, Gaël Rigaud, and Thomas Schuster became mathematical collaborators, as well as good friends, sharing mathematics and life.

Professor Quinto has been lucky to work with talented and creative younger researchers, including Gaik Ambartsoumian, Raluca Felea, Jürgen Frikel, Christine Grathwohl, Jakob Jørgensen, Esther Klann, Venky Krishnan, Peer Kunstmann, Cliff Nolan, Ronny Ramlau, James Webber, and others, as well as Tufts colleagues Christoph Börgers and Fulton Gonzalez.

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Sharing mathematics through teaching and mentoring his students has always been satisfying to Professor Quinto, and he has enjoyed doing research with many talented undergraduates and his Ph.D. students Yiying Zhou, Aleksei Beltukov, Anuj Abhishek, and Alejandro Coyoli.

Tomography

The transform that models X-ray tomography is a special case of the generalized Radon transform that Professor Quinto studied in graduate school. When he received his Ph.D. from MIT in 1978, the field of tomography was just beginning. Mathematicians such as Larry Shepp and Kennan Smith introduced the field to mathematicians by presenting the theory and practical applications in mathematical language. It was a perfect time to start in the field. Intrigued by Allan Cormack's work in exterior tomography, Quinto developed several reconstruction algorithms, which he tested on industrial data [14, 16].

Later, Jan Boman introduced Professor Quinto to his former student Ozan Öktem who was working on single particle electron microscopy. Öktem, Quinto, and biologist Ulf Skoglund developed simple and effective algorithms for this highly ill-posed limited data problem and tested them on real data. Professor Quinto enjoyed learning about the biological and practical challenges of this application [17, 18].

Some of Professor Quinto's most important work involves the melding of his expertise in tomography with his expertise in microlocal analysis. His first article joining these two themes, [15], was among the articles that introduced microlocal analysis to applied tomographers. In it, he described visible and invisible singularities in limited-data tomography. Scientists found the article readable and useful for their work, and Quinto has written other introductory articles, such as [11]. He continued to use these ideas to explain visible and invisible features as well as artifacts in a range of tomographic problems (e.g., [5, 6, 20]. See also [10, 12]. Now, microlocal analysis has permeated tomography.

To describe the results, we introduce the wavefront set. If f is a distribution, its wavefront set, $\mathrm{WF}(f)$ describes points at which f is not smooth *and* directions in which it is not smooth at those points. For example, if f is the characteristic function of a set A with smooth boundary, then $\mathrm{WF}(f)$ is the normal bundle to $\mathrm{bd}(A)$. That is, f is not smooth at all points on $\mathrm{bd}(A)$ and at each point $\mathbf{x} \in \mathrm{bd}(A)$, the wavefront directions are (co)normal to $\mathrm{bd}(A)$ at $\mathbf{x}$. When a Radon transform is a Fourier integral operator, it manipulates wavefront set in precise ways. See [11] for an elementary introduction to microlocal analysis with tomographic applications.

For $(\omega, p) \in S^1 \times \mathbb{R}$ let $\ell(\omega, p)$ be the line perpendicular to ω and containing the point $p\omega$. Then the Radon line transform of a compactly supported function f is

$$Rf(\omega, p) = \int\limits_{\mathbf{x}\in\ell(\omega,p)} f(\mathbf{x})\,\mathrm{d}x_\ell,$$

the transform R_1 defined in (1) below for the line transform with $\mu = 1$. The dual transform is defined for $g \in L^2_{\mathrm{loc}}(S^1 \times \mathbb{R})$ and $\mathbf{x} \in \mathbb{R}^2$ as

$$R^* g(\mathbf{x}) = \int\limits_{\omega\in S^1} g(\omega, \mathbf{x}\cdot\omega)\,\mathrm{d}\omega$$

the integral of g over parameters for all lines containing $\mathbf{x}$.

Much of Professor Quinto's research focuses on limited data X-ray CT, in particular which singularities of a function f are visible and which are invisible in tomographic reconstructions. In addition, he analyzes when there can be artifacts or singularities added to the reconstruction that are not in the original object. If limited data are given over an open set $A \subset S^1 \times \mathbb{R}$, we define the *limited data Radon transform (for A)* to be $R_A f = \mathbb{1}_A Rf$, where $\mathbb{1}_A$ is the characteristic function of A. Thus, $R_A f(\omega, p)$ is the given data of Rf for $(\omega, p) \in A$ and is zero otherwise. Sometimes $\mathbb{1}_A$ is replaced by a smooth approximation supported in A, suppressing both some visible singularities and artifacts.

Theorem 1 ([5, 15]). *Let D be an elliptic pseudodifferential operator in the p-variable, and let f be an integrable function or a distribution of compact support. Let A be an open symmetric subset of $S^1 \times \mathbb{R}$, let $(\omega, p) \in A$ and let $\mathbf{x} \in \ell(\omega, p)$. Then*

$$(\mathbf{x}, \omega) \in \mathrm{WF}(f) \iff (\mathbf{x}, \omega) \in \mathrm{WF}(R^* D R_A f).$$

If $(\omega, p) \in \mathrm{bd}(A)$, *then artifacts can occur on the line* $\ell(\omega, p)$, *i.e., there can be points* $\mathbf{y} \in \ell(\omega, p)$ *such that* $(\mathbf{y}, \omega) \in \mathrm{WF}(R^* DRf_A f)$ *but* $(\mathbf{y}, \omega) \notin \mathrm{WF}(f)$.

For complete data, all singularities of f are visible: $\mathrm{WF}(R^* DRf) = \mathrm{WF}(f)$.

Therefore, if $(\omega, p) \in A$, $\mathbf{x} \in \ell(\omega, p)$ and $(\mathbf{x}, \omega) \in \mathrm{WF}(f)$, then $(\mathbf{x}, \omega)$ is a visible singularity of f under R_A, and these singularities are the only singularities that will be visible at points on $\ell(\omega, p)$ and in direction ω. Note that the line $\ell(\omega, p) = \ell(-\omega, -p)$ so the statements in the theorem are valid if ω is replaced by $-\omega$.

A precise analysis of these added artifacts including geometric descriptions is in [5]; sometimes the artifacts can be entirely along lines $\ell(\omega, p)$ for $(\omega, p) \in \mathrm{bd}(A)$ and other times they can be on curves generated by points on boundary lines. Quinto and coauthors have generalized these ideas to a broad range of limited data problems such as photoacoustic tomography [6], seismic imaging, and Compton tomography (e.g., [19, 20]), and these are an important part of his current research.

Integral geometry

Professor Quinto was fortunate to have mentors including his Ph.D. advisor, Victor Guillemin, and informal advisor Sigurdur Helgason. Guillemin developed seminal ideas in microlocal analysis, and he proved that Radon transforms are Fourier integral operators in the highly influential book with Sternberg [8]. Professor Quinto's first article [13] uses their framework to analyze the microlocal properties of generalized Radon transforms satisfying the Bolker condition, and this was also used in his later work in both tomography and integral geometry.

Sigurdur Helgason proved fundamental properties of Radon transforms and taught Quinto their properties, including elegant support theorems, which Quinto generalized. To describe the first of these support theorems, we consider the hyperplane transform. Let $\mu : \mathbb{R}^n \times S^{n-1} \to \mathbb{R}$ be a smooth nowhere zero function. For $(\omega, p) \in S^{n-1} \times \mathbb{R}$, define the *generalized Radon hyperplane transform*

$$R_\mu f(\omega, p) = \int_{\mathbf{x} \in H(\omega,p)} f(\mathbf{x}) \mu(\mathbf{x}, \omega)\, \mathrm{d}x_H, \tag{1}$$

where $\mathrm{d}x_H$ is Lebesgue measure on the hyperplane $H(\omega, p) = \{\mathbf{x} \in \mathbb{R}^n : \mathbf{x} \cdot \omega = p\}$.

Professor Quinto's work with Jan Boman combined their perspectives and expertise and was the start of a rich collaboration. Jan Boman introduced Quinto to a powerful microlocal analytic regularity theorem of Kawai, Kashiwara, and Hörmander [9, Theorem 8.5.6] that allows one to infer support restrictions on a function at the boundary of its support if a normal direction is not in its analytic wavefront set. Many Radon transforms are real analytic Fourier integral operators, and they used this theorem and the analytic microlocal regularity of these FIO to eat away at supp(f). This is the idea behind their first support theorem.

Theorem 2 ([3, Theorem 2.2]). *Let* $\mu : \mathbb{R}^n \times S^{n-1} \to \mathbb{R}$ *be a strictly positive real-analytic function that is even in* ω. *Let* $f \in \mathcal{E}'(\mathbb{R}^n)$ *and let W be a connected symmetric open set in* $S^{n-1} \times \mathbb{R}$. *Assume for some* $(\omega_0, p_0) \in W$ *that f is zero in a neighborhood of the hyperplane* $H(\omega_0, p_0)$. *If* $R_\mu f(\omega, p) = 0$ *for all* $(\omega, p) \in W$, *then the support of f is disjoint from the union of hyperplanes* $\bigcup_{(\omega,p)\in W} H(\omega, p)$.

To our knowledge, this was the first result to use analytic microlocal analysis to prove support theorems for generalized Radon transforms. Important results for analytic Radon transforms on hyperfunctions and distributions have subsequently been proven using other microlocal techniques.

The proof of Theorem 2 is simple enough to describe. One starts with a hyperplane $H(\omega_0, p_0)$ in W that is disjoint from supp(f) and moves it through the connected set W to a hyperplane $H_1 = H(\omega_1, p_1)$ that meets supp(f) at some point $\mathbf{x}_1$ *and* supp(f) is on one side of H_1. By [9, Theorem 8.5.6], $(\mathbf{x}_1, \omega_1) \in \mathrm{WF_A}(f)$. However, because $R^* R_\mu$ is an analytic elliptic pseudodifferential operator [3, Lemma 3.4] and $R_\mu f = 0$ in a neighborhood of (ω_1, p_1), $(\mathbf{x}_1, \xi_1)$ is not in the analytic wavefront set of f. This contradiction shows that $\mathbf{x}_1 \notin \mathrm{supp}(f)$ and no hyperplane in W meets supp(f).

Inspired by seminal work of Gelfand on admissible complexes, and a fundamental article by Greenleaf and Uhlmann [7], Boman and Quinto proved support theorems for Radon transforms on line complexes in $\mathbb{R}^3$

(see [4]). Subsequently, Quinto and coauthors, including Eric Grinberg, used analytic microlocal analysis to prove support theorems for a range of Radon transforms on other sets.

Professor Quinto's work in the 1990s and 2000s with Mark Agranovsky on stationary sets for the wave equation was rewarding because Professor Agranovsky's deep knowledge of harmonic analysis complemented his expertise in microlocal analysis. They had fun bouncing ideas off of each other by e-mail and creating new mathematics.

Let $S(\mathbf{x}, r)$ be the sphere centered at $\mathbf{x} \in \mathbb{R}^n$ and of radius $r > 0$. Then the spherical transform of f is defined as

$$Rf(\mathbf{x}, r) = \int_{S(\mathbf{x},r)} f(\mathbf{x})\, d\mathbf{x}_S,$$

where $d\mathbf{x}_S$ is the surface measure on the sphere $S(\mathbf{x}, r)$. Since f is trivially recoverable from data over all $(\mathbf{x}, r)$, they consider a more restrictive domain for Rf. Let $S \subset \mathbb{R}^n$ and consider the spherical transform restricted to $(\mathbf{x}, r) \in S \times \mathbb{R}_+$.

Definition 3. The set $S \subset \mathbb{R}^n$ is a *set of injectivity* for R on compactly supported functions if, whenever $f \in C_c(\mathbb{R}^n)$ and $Rf(\mathbf{x}, r) = 0$ for all $r \in \mathbb{R}_+$ and all $\mathbf{x} \in S$, then $f = 0$.

Agranovsky and Professor Quinto developed theorems to characterize sets of injectivity for the spherical transform in a range of papers starting for the plane [1]. Let Σ_N be the set of lines centered at the origin in the plane and going through the $2N$-th roots of unity. We call any rigid motion of a Σ_N a *Coxeter system* of lines.

Theorem 4 ([1]). *The set $S \subset \mathbb{R}^2$ is a set of injectivity for the circle transform R on compactly supported functions if and only if S is not a subset of the union of a Coxeter system and a finite set of points.*

Therefore, if S consists of curves, any one of which is not a line, then S is a set of injectivity. In fact, S can be any infinite number of points that is not contained in any Coxeter system.

The proof outline is as follows. Assume f is a nontrivial compactly supported function in the plane and $Rf(\mathbf{x}, r) = 0$ for all $(\mathbf{x}, r) \in S \times \mathbb{R}_+$. This means that the polynomial functions

$$Q_k(\mathbf{x}) = \int_{r=0}^{\infty} r^{2k} Rf(\mathbf{x}, r) = \int_{\mathbf{y}\in\mathbb{R}^n} \|\mathbf{y} - \mathbf{x}\|^{2k} f(\mathbf{y})\, d\mathbf{y}$$

are all zero for $\mathbf{x} \in S$. If k_0 is the smallest value of k such that Q_k is nontrivial, then Q_{k_0} is a harmonic polynomial. For $\mathbf{x} \in S$, $Q_{k_0}(\mathbf{x})$ is zero, therefore S is contained in the zero set of a harmonic polynomial. The authors then use subtle properties of harmonic polynomials in the plane to show that the zero sets of such polynomials are either in a Coxeter system or have at least two connected components that are smooth curves that are bounded away from each other. Then, they use analytic microlocal analysis to show that if $Rf(\mathbf{x}, r) = 0$ for all $\mathbf{x} \in S$ and $r > 0$, then f must be zero. This final argument is easiest to see when the two curves are parallel lines, ℓ_1 and ℓ_2. The null space of R with centers restricted to ℓ_1 consists of odd functions about ℓ_1 and same for ℓ_2. If the compactly supported function f integrates to zero over all circles centered on both ℓ_1 and ℓ_2, then it is odd about both ℓ_1 and ℓ_2, but this implies $f = 0$ since any nonzero function odd about two parallel lines has unbounded support. The results have implications for density of translates of radial functions and solutions to the wave equation and other PDEs (see [1]). Their proofs start from ideas of Lin and Pinkus.

Agranovsky and Quinto also partially characterized stationary sets for the spherical transform in $\mathbb{R}^n$ as well as for the spherical transform functions defined in crystallographic domains in $\mathbb{R}^n$ (see [2]) along with work with their friend Peter Kuchment.

For more information about Professor Quinto's research, see https://sites.tufts.edu/tquinto/. Articles in the bibliography provide more complete references to work in the field.

Professor Quinto is not only an outstanding organizer of various international conferences, including numerous SIAM conferences and the Inverse Problems: Modeling and Simulation (IPMS) conference series, thus playing a leading role in the inverse problems community, but also above all a good-natured and friendly person.

Apart from his notable contributions to the field of inverse problems, Professor Quinto is also known for his warm personality, soft-spoken way and spirit of camaraderie and of course, for his booming laughter, that brings instant cheer to everyone around. Professor Quinto is a special human being loved by everyone in the inverse problems community and not only! For decades, Professor Quinto has been a friend to hundreds of young patients as a volunteer at Boston Children's Hospital, bringing smiles and good cheer, rocking the babies, talking and playing with the kids and even helping them with Maths homework. In 2003 in recognition of his leadership as a volunteer, Professor Quinto received the Bob Groden Distinguished Service Award, a Children's Hospital humanitarian honor. Professor Quinto's compassion and earnestness obviously came also through to his students who have been praising him for being caring and excited about teaching [21]. A truly brilliant mathematician with a big heart!

On behalf of his colleagues, students, and friends from all over the world, we would like to wish Todd every success in his scientific work and much happiness with his wife, Judy Larsen, and daughter Laura Quinto.

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