DETECTING GERRYMANDERS:

COMPACTNESS
VS
SAMPLING

BRIEF TOUR OF COMPACTNESS SCORES

ISOPERIMETRY

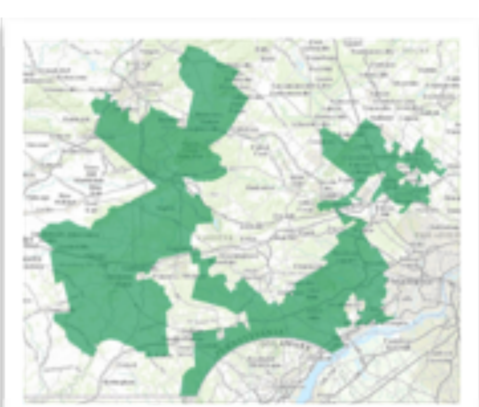
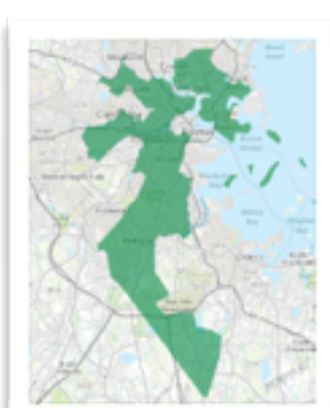
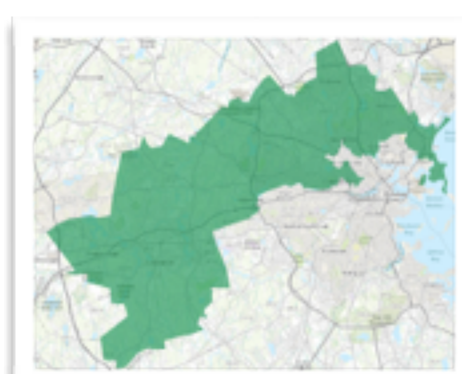
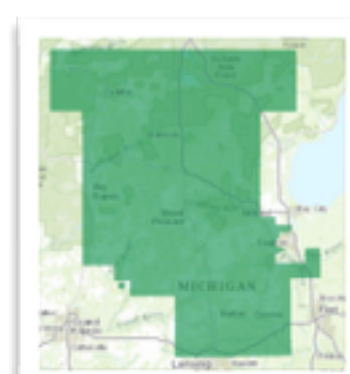
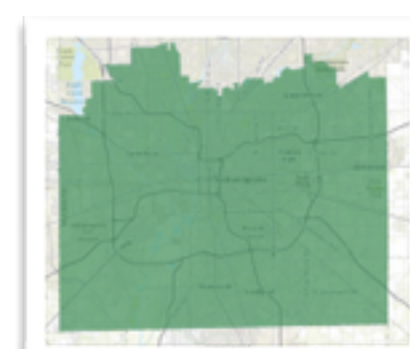
Isoperimetric Theorem: let R be a bounded open subset of the plane whose boundary is a rectifiable curve. Then

$$4\pi A \leq P^2,$$

with equality only for circles. Here area is Lebesgue measure $m(R)$ and perimeter is boundary length $\ell(\partial R)$.

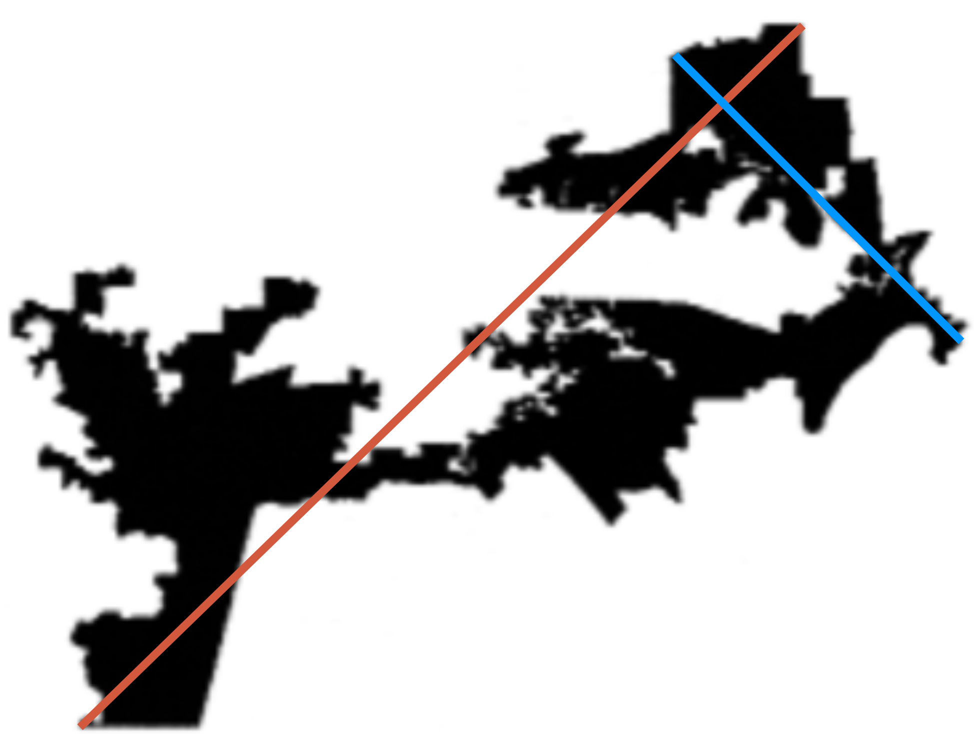
Polsby-Popper score: $\mathrm{PP}(R) = 4\pi A / P^2$.

"Isosquarimetric" version: $\mathrm{PP}'(R) = 16A/P^2$.

SKEW

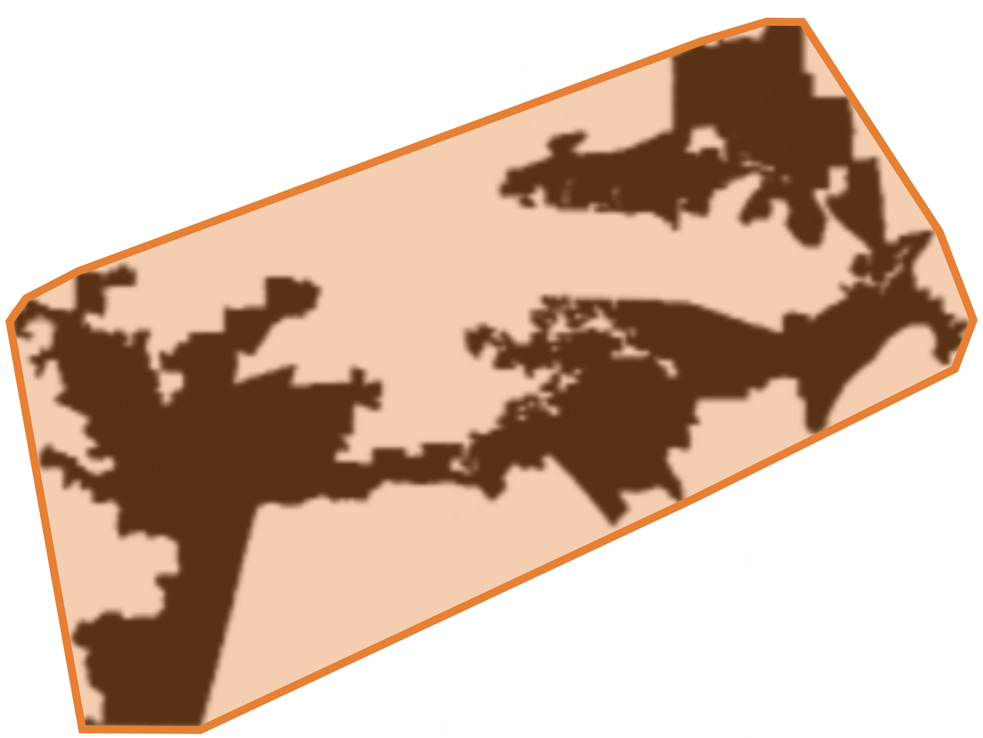
- Measure short axis/long axis, W/L. For instance,
  - L=diameter, W=longest ⊥ cross-section
  - In every direction, bound with rectangle, and choose the one with the most extreme ratio.

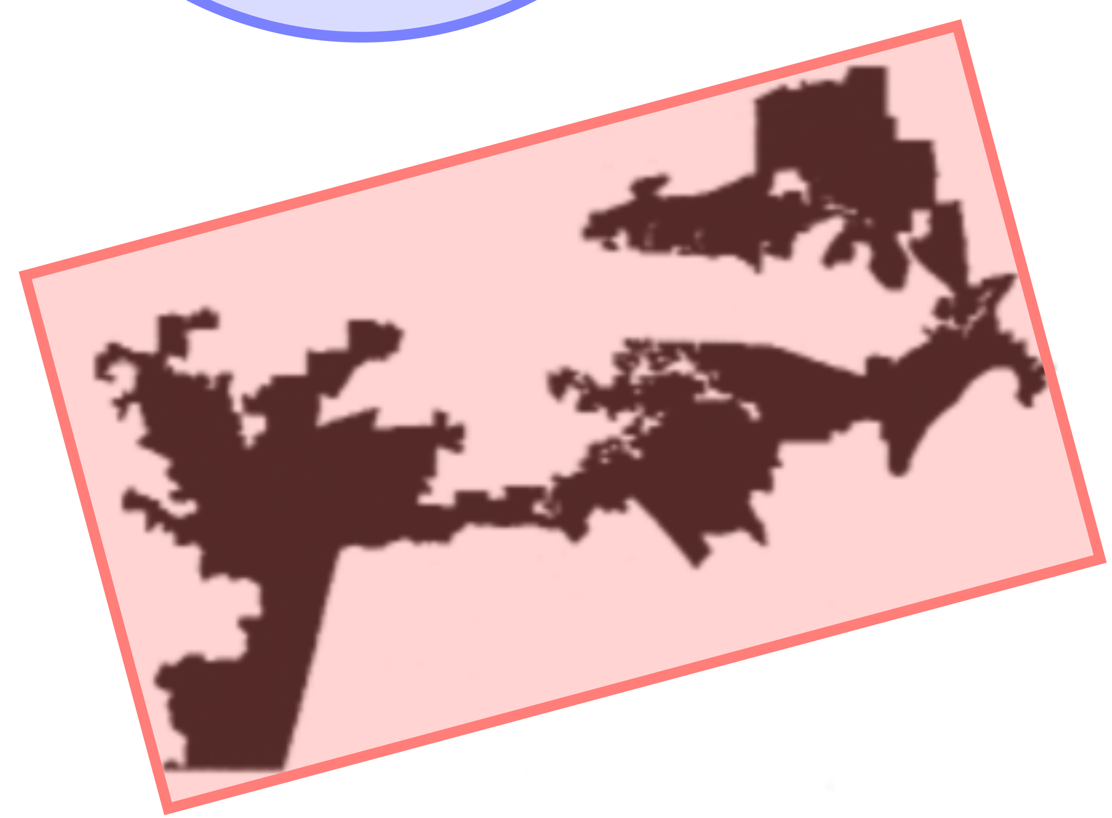


INDENTEDNESS

- Area of the region divided by the area of a comparison figure
    - Convex hull
    - Bounding rectangle
    - Circumscribing circle

*this one is called the **Reock** score*

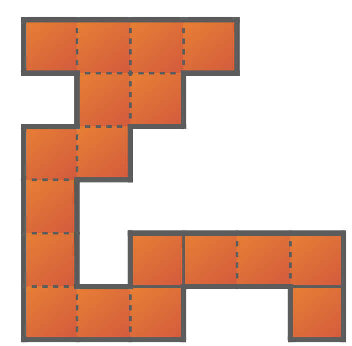




A COMBINATION SCORE

- We might notice: skew and convex hull scores have complementary blind spots.
- **Convex hull** gives a perfect score to arbitrarily long, skinny rectangles.
- **Skew** gives a perfect score to a square that is carved out like a swiss cheese.
- So maybe we can propose a combination **Box Score** that sums the two.

- xy-Hull=1
- xy-Skew=.038
- Box=1.038



- xy-Hull=.5
- xy-Skew=1
- Box=1.5

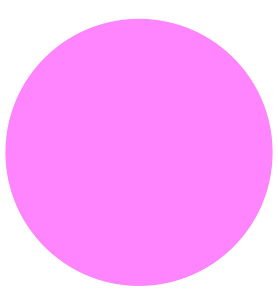
- xy-Hull=1
- xy-Skew=1
- Box=2

DISPERSION

- How spread-out or *sprawling* is your district?
- Average distance between points in a domain or **moment of inertia**.

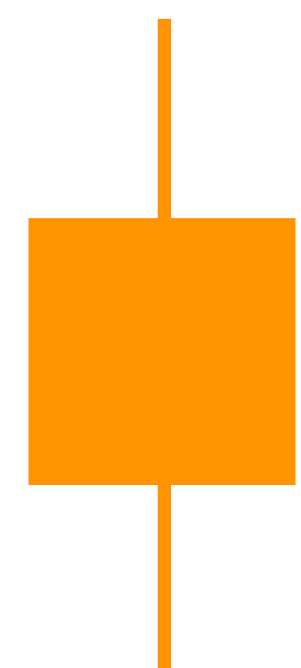
$$\mathbb{E}[\ (x_1-x_2)^2 + (y_1-y_2)^2\ ] = 2\ \mathbb{E}[\ (d(\mathbf{x},\mathbf{x}_0))^2\ ]$$

- Discrete/finite version: Average distance between voters, or from voters to district center
- Could use travel time instead of distance: How long does it take you to go yell at your Rep in a town-hall?
- For all of these, important to normalize for size.



Avg $d(x,y)$ over round disk $\approx .453\ d \approx .511\sqrt{A}$

Avg $d(x,y)$ over square $\approx .369\ d \approx .522\sqrt{A}$



Classical fact: disk has best coefficient of $\sqrt{A}$.

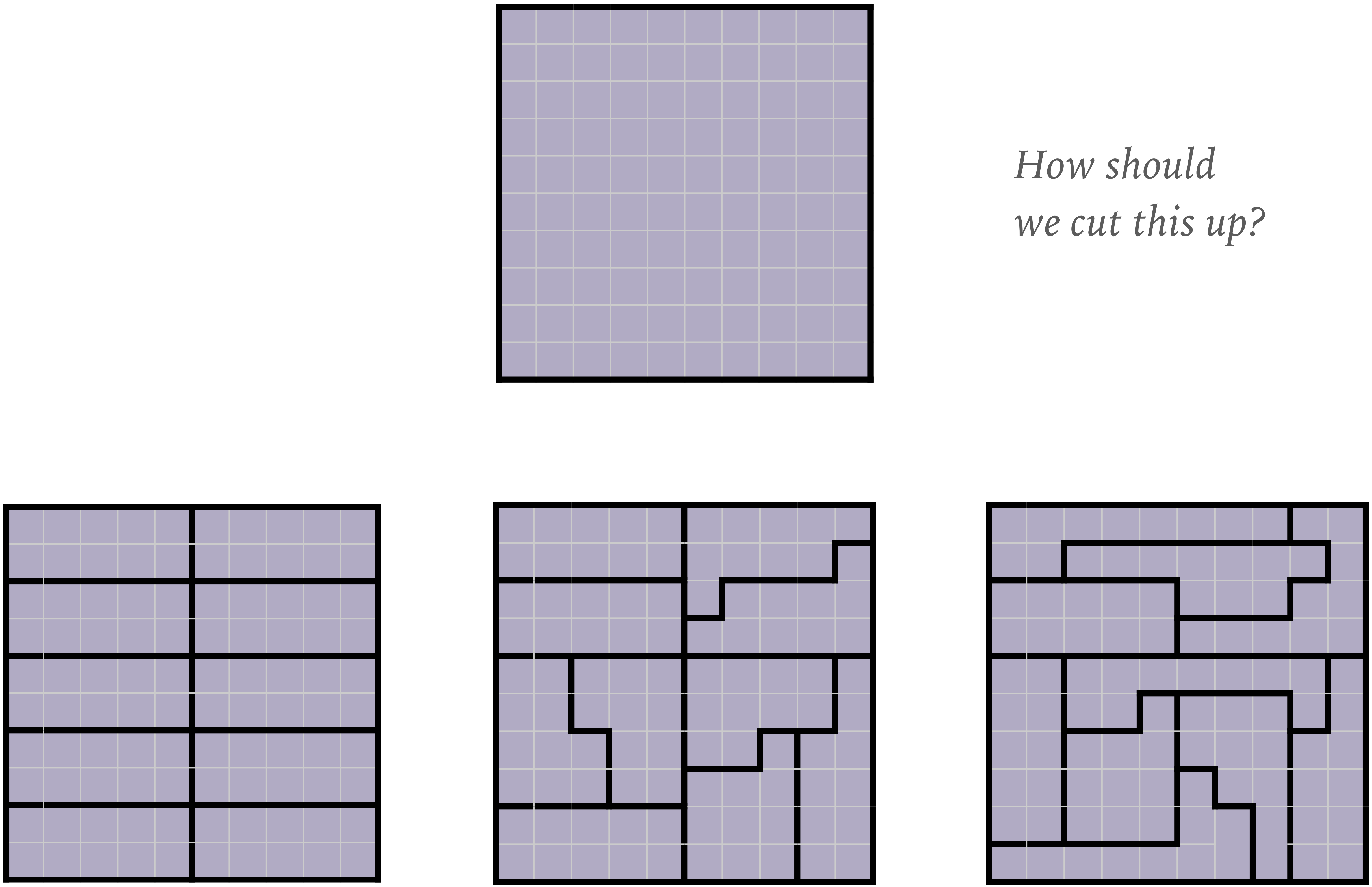
COMPACTNESS: BUT WHY?

WHAT IS COMPACTNESS GOOD FOR?

- How do compactness considerations help promote fair districting?

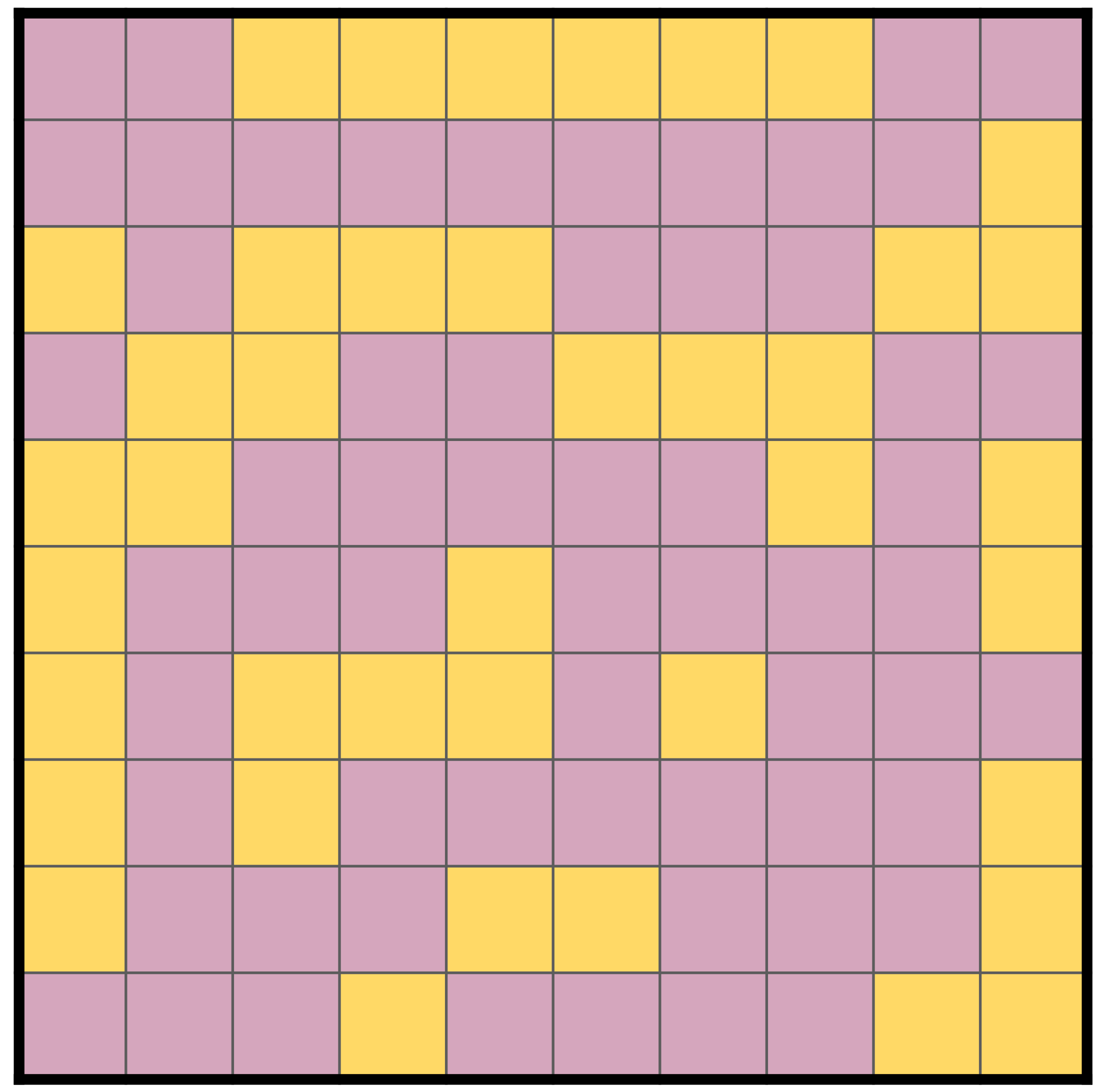
- Two main arguments:
  1. **Any** shape constraints generally limit the power of the map-drawer
  2. Extreme gerrymandering requires eccentrically shaped districts

TEST ON A SIMPLE EXAMPLE



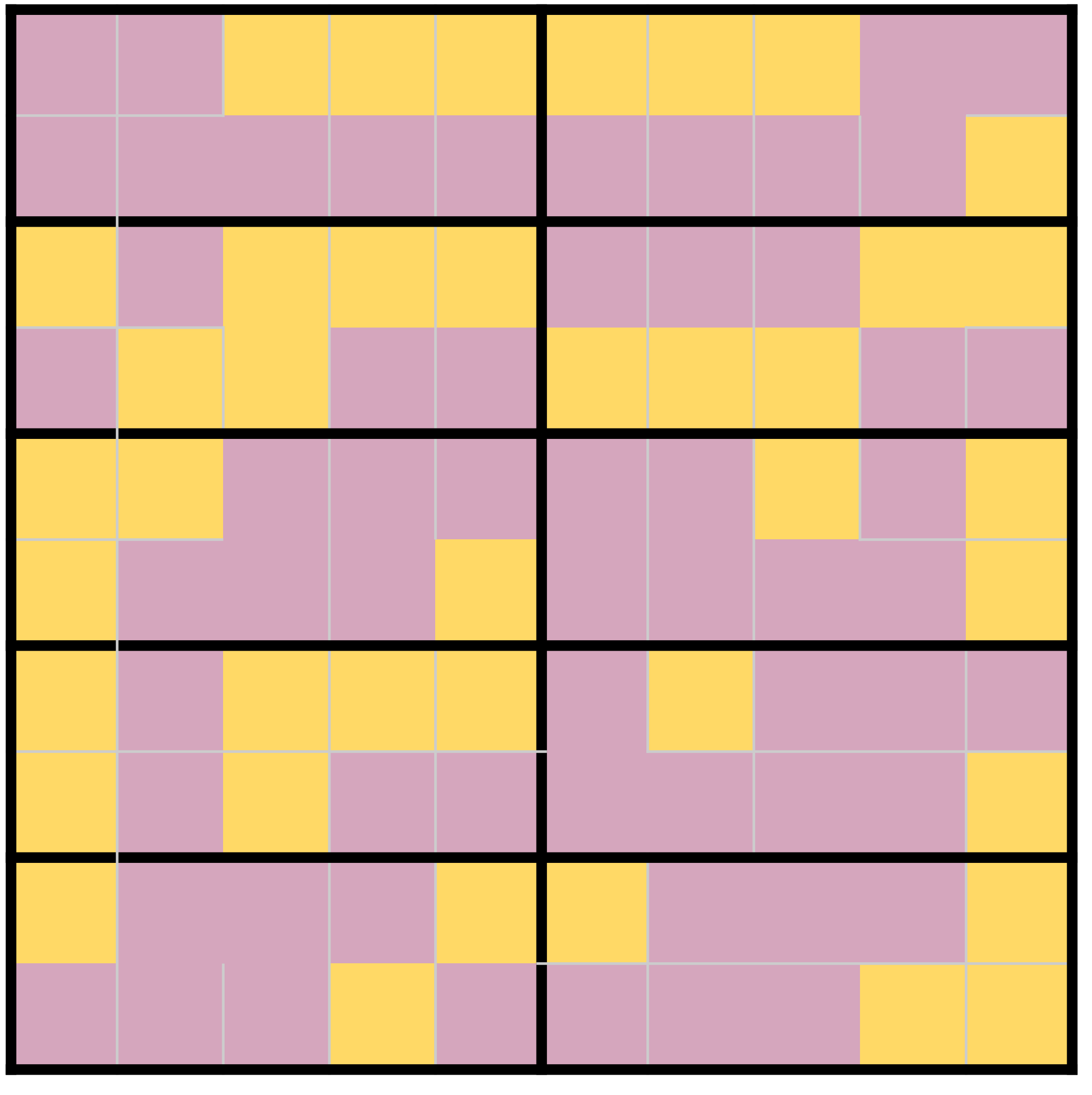
How should we cut this up?

10×10 grid, 40% orange voters

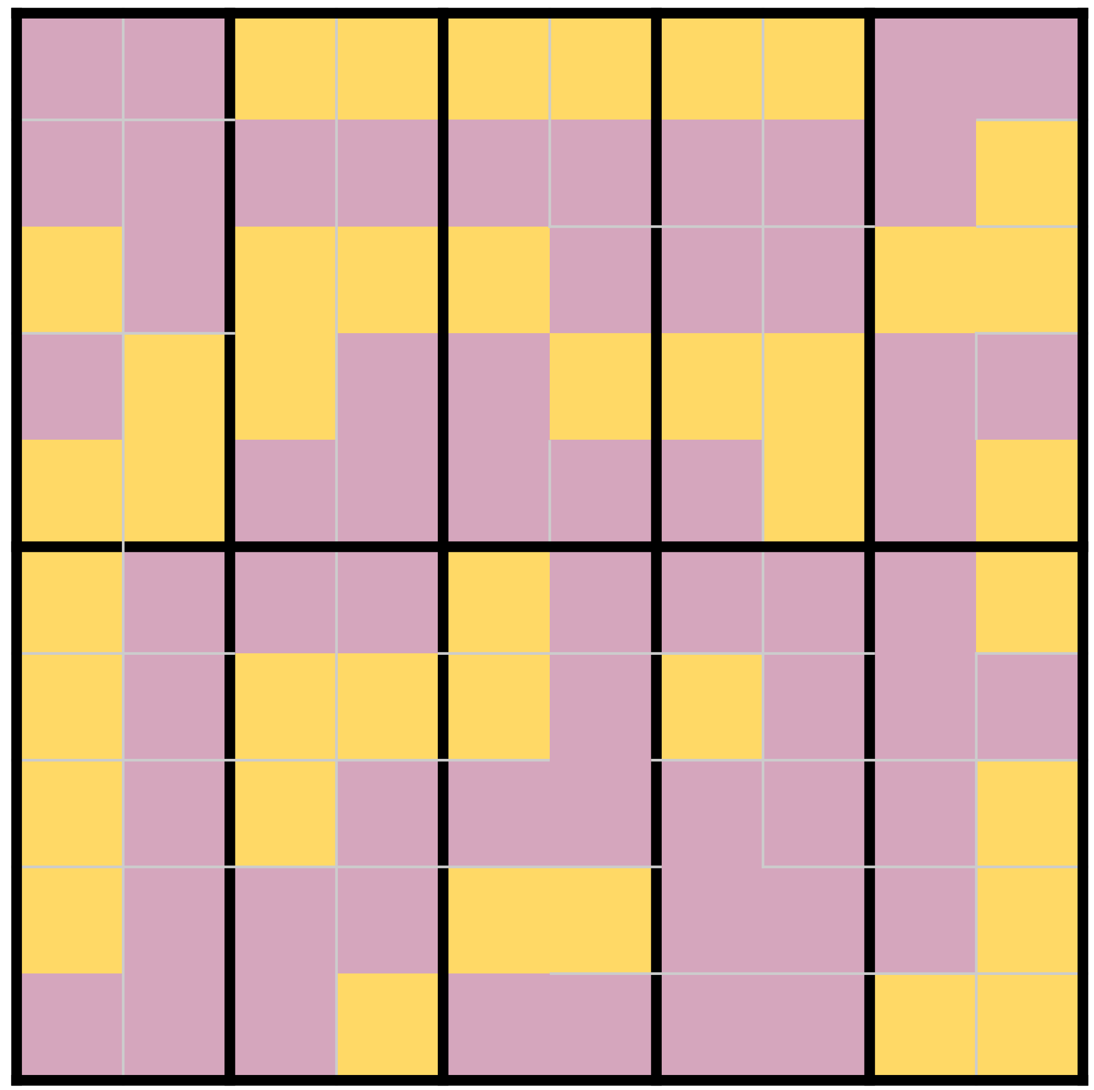


if we make 10 districts, orange “should” win 4 seats, right?

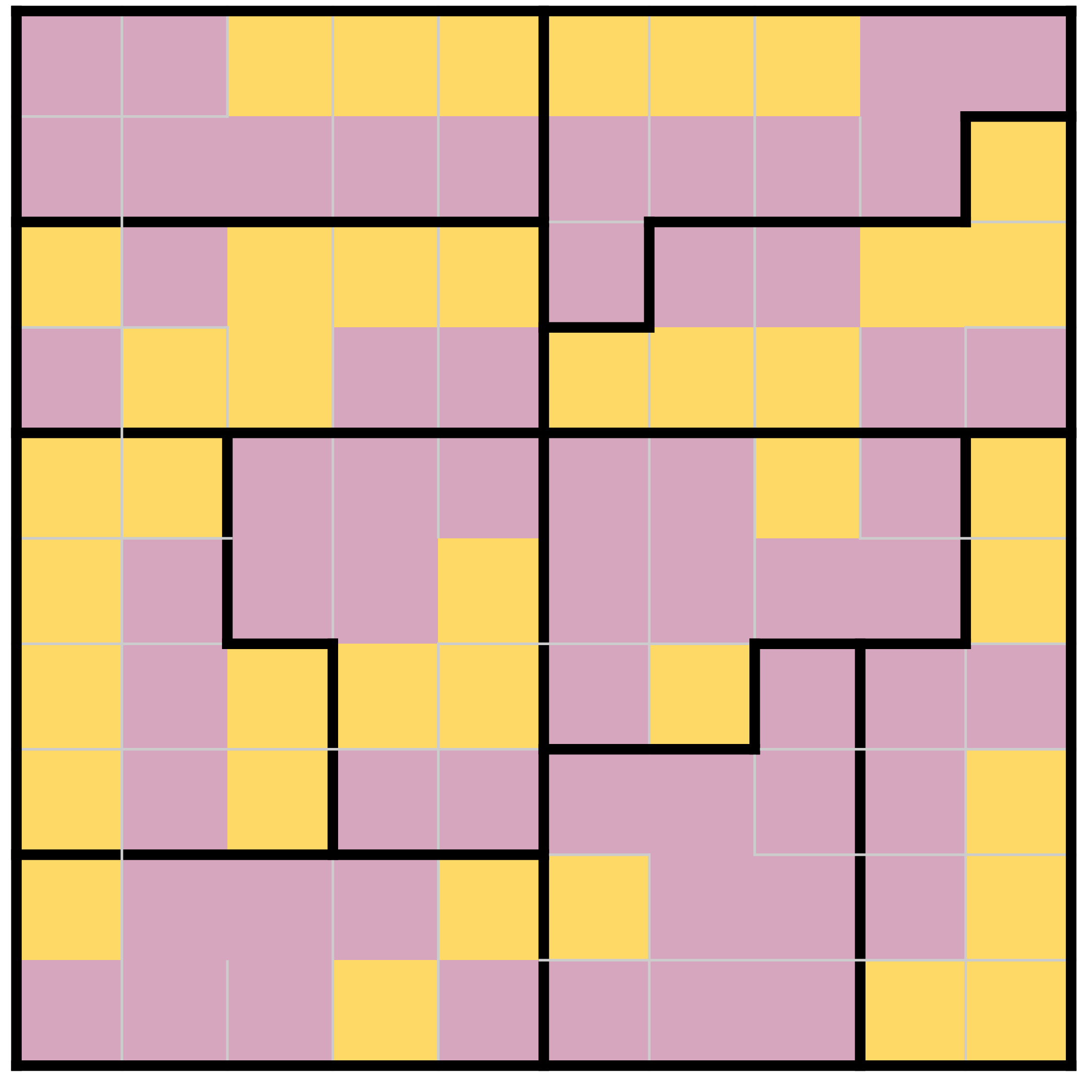
A



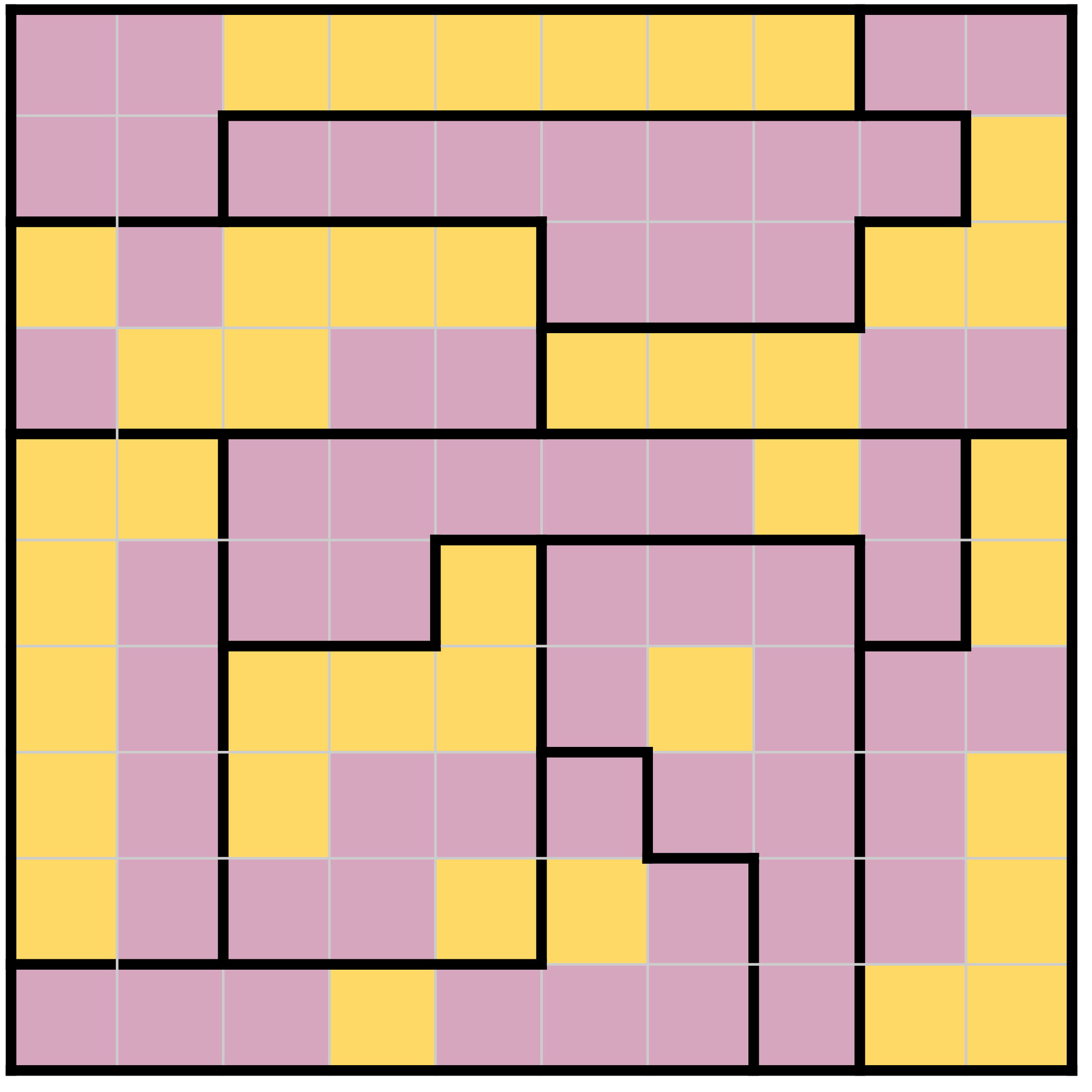
B



c

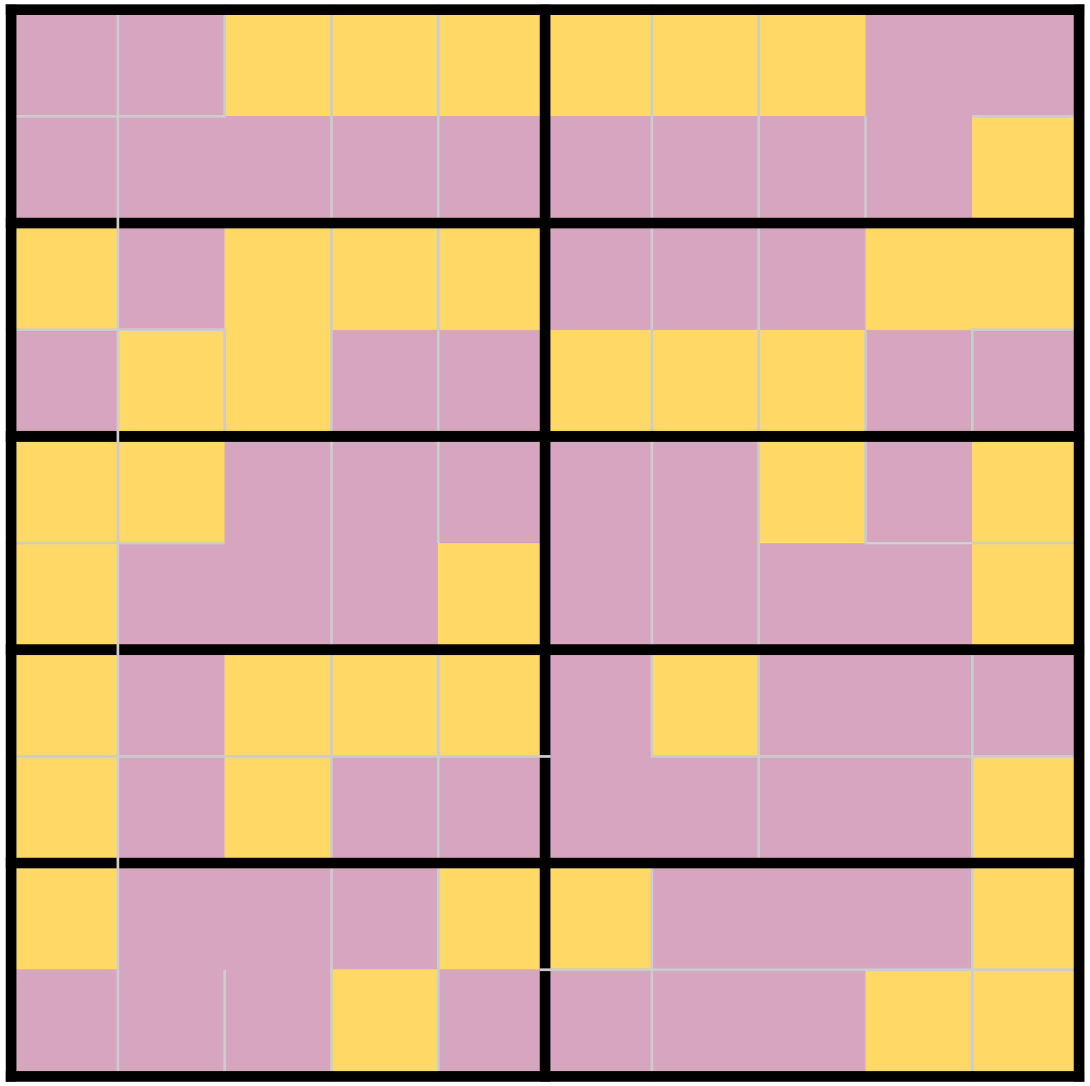


D



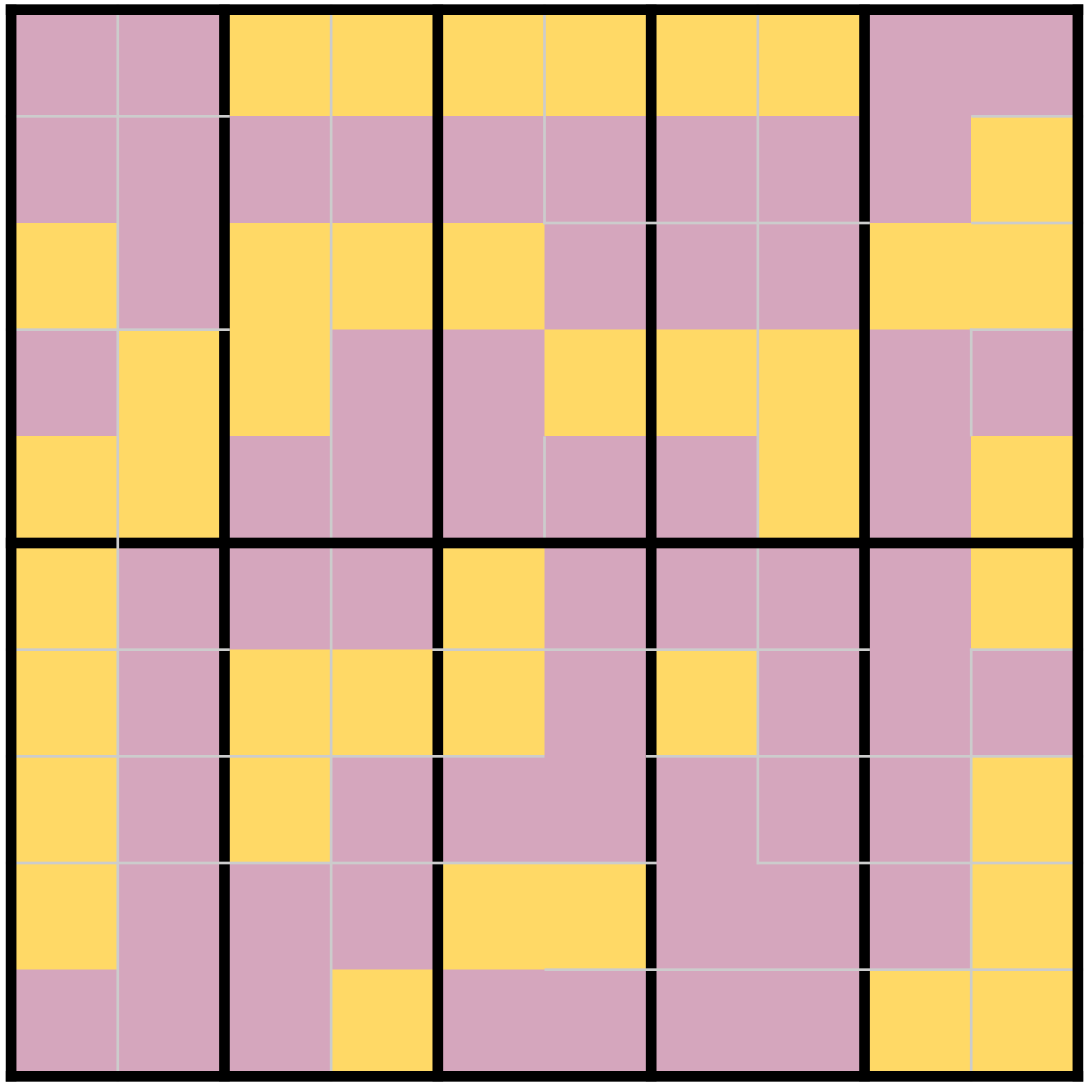
2 safe orange seats + 1 toss-up

A



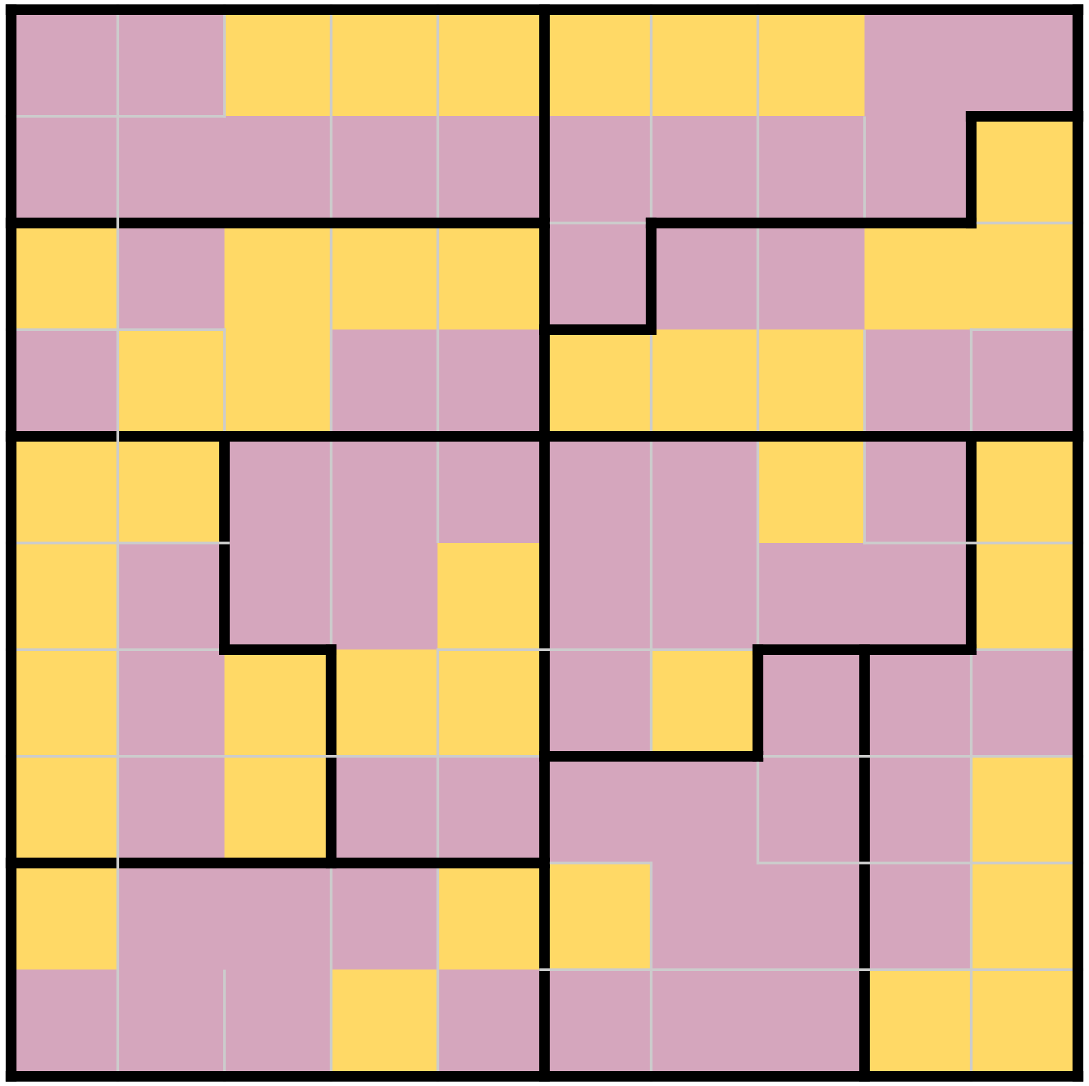
0 safe orange seats + 3 toss-up

B



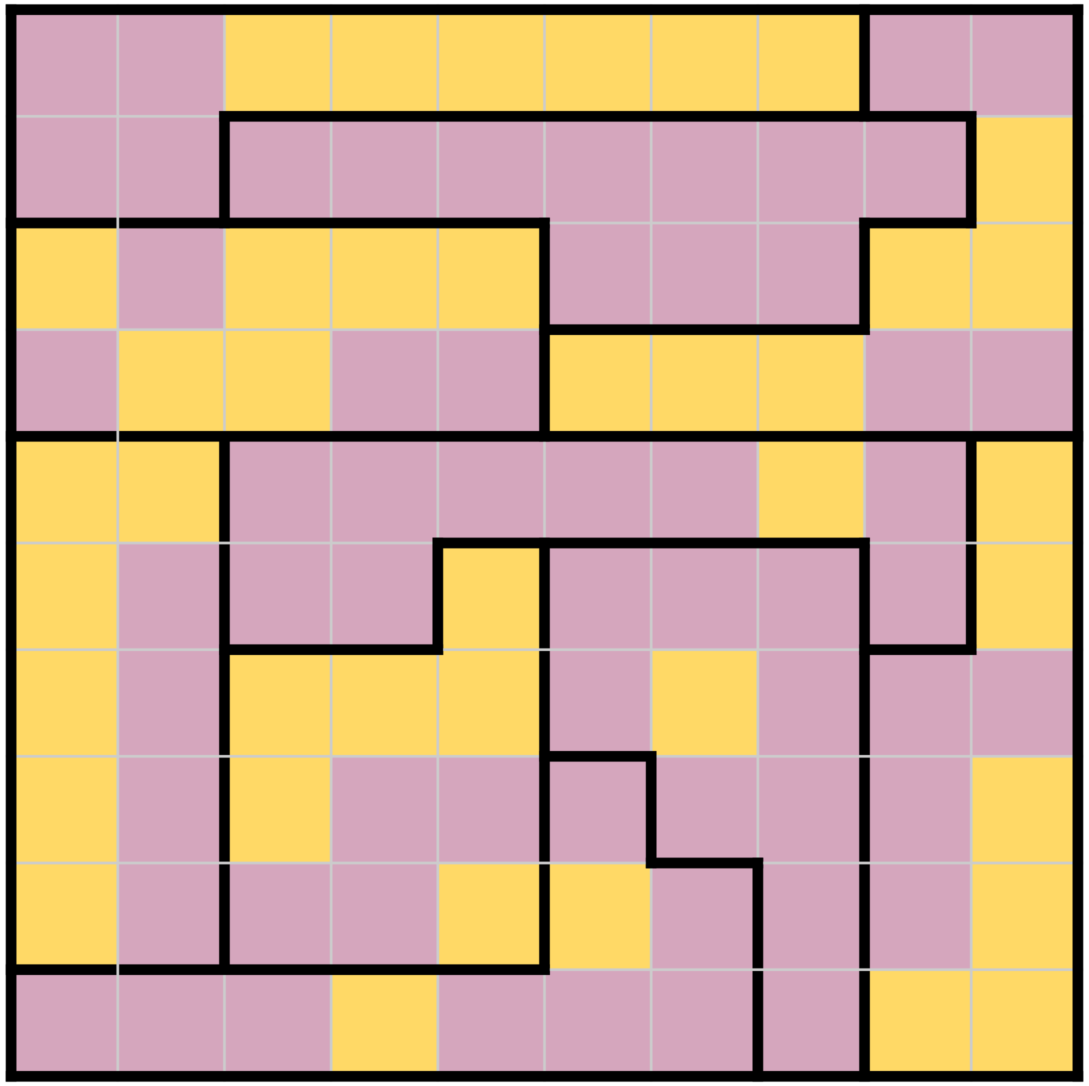
4 safe orange seats

c



6 safe orange seats

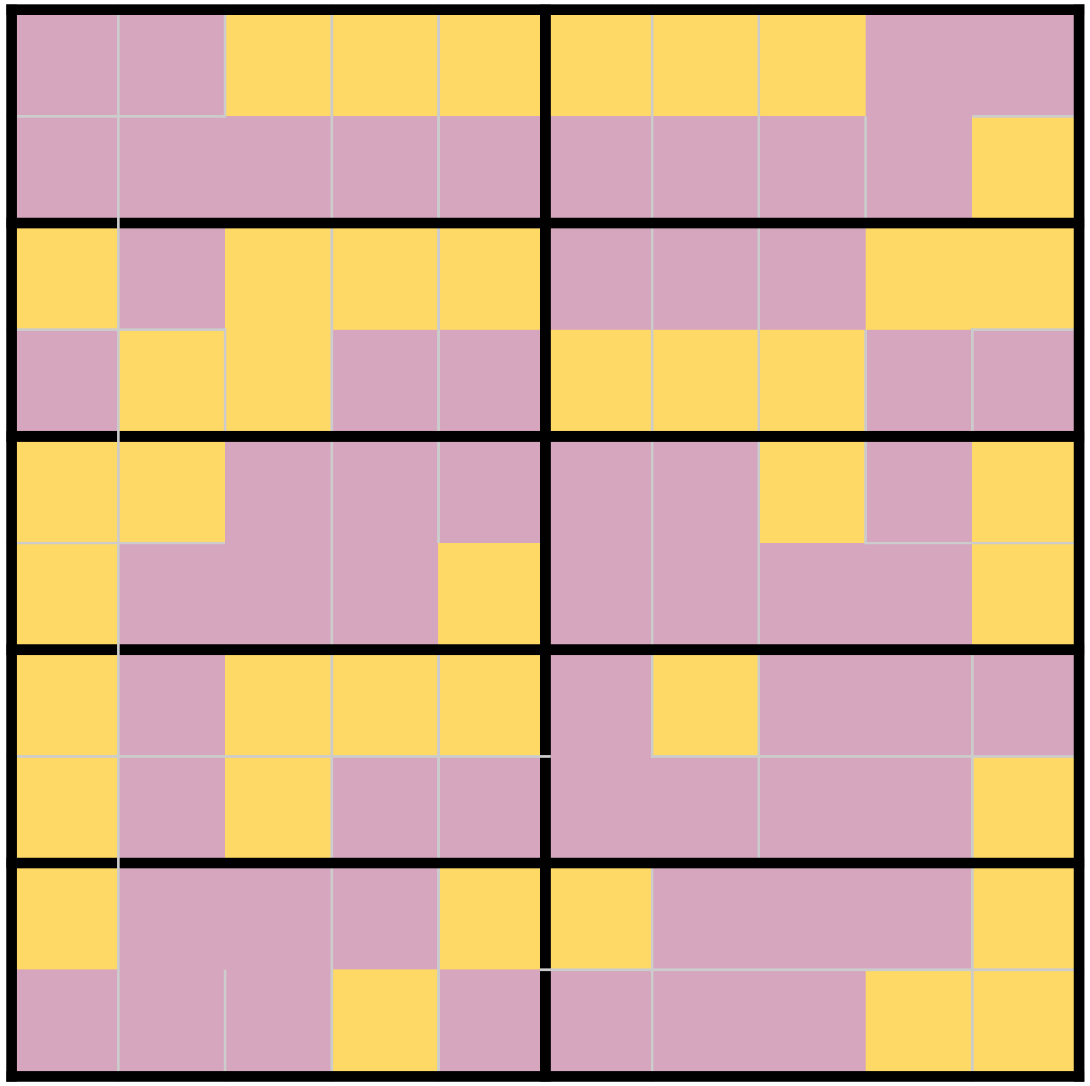
D



*This very successful gerrymander is brought to you by “**packing**” and “**cracking**”*

2 safe orange seats + 1 toss-up

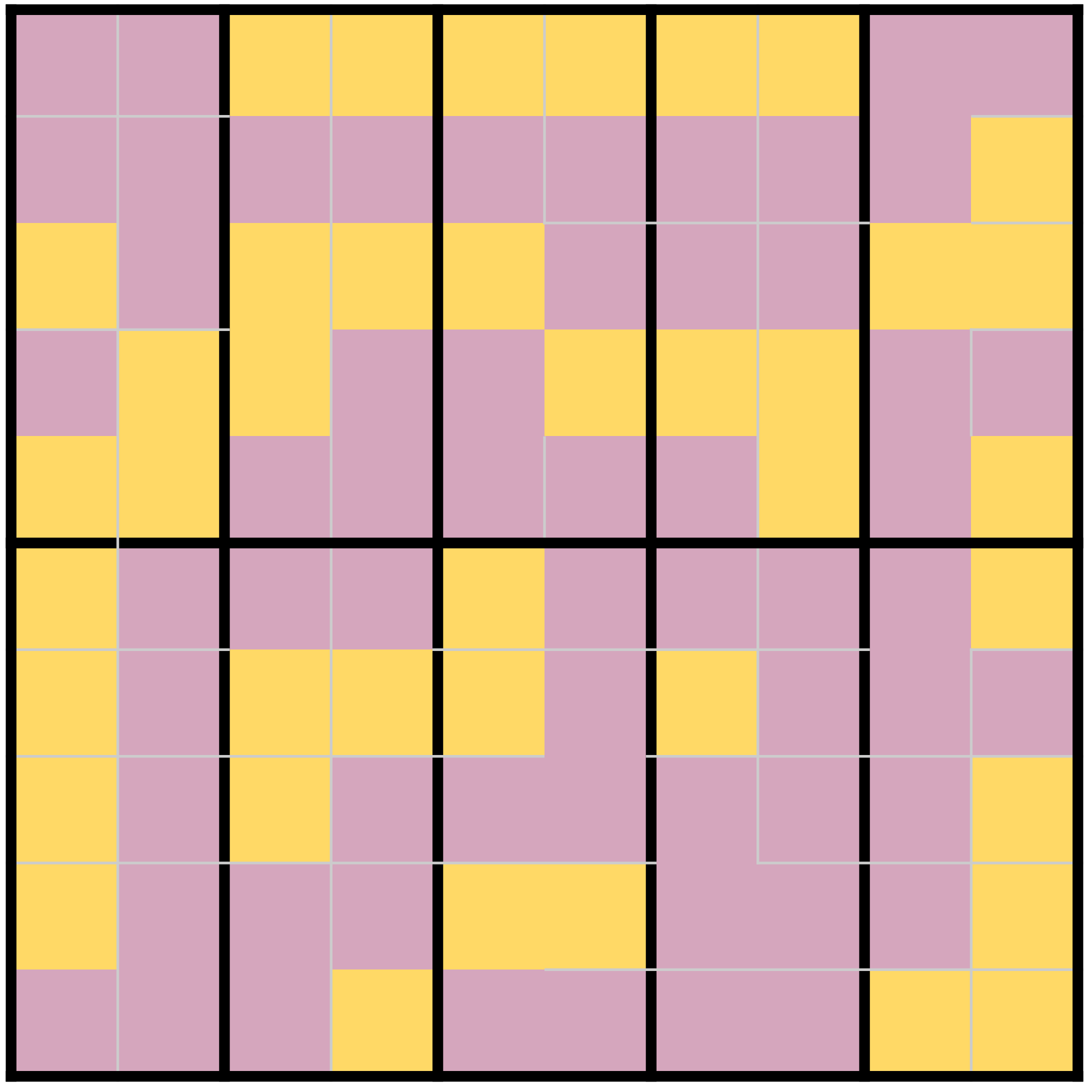
A



total P	16A/P^2	Box score	CHull	AMOI
140	0.816	1.4	1	24.16

0 safe orange seats + 3 toss-up

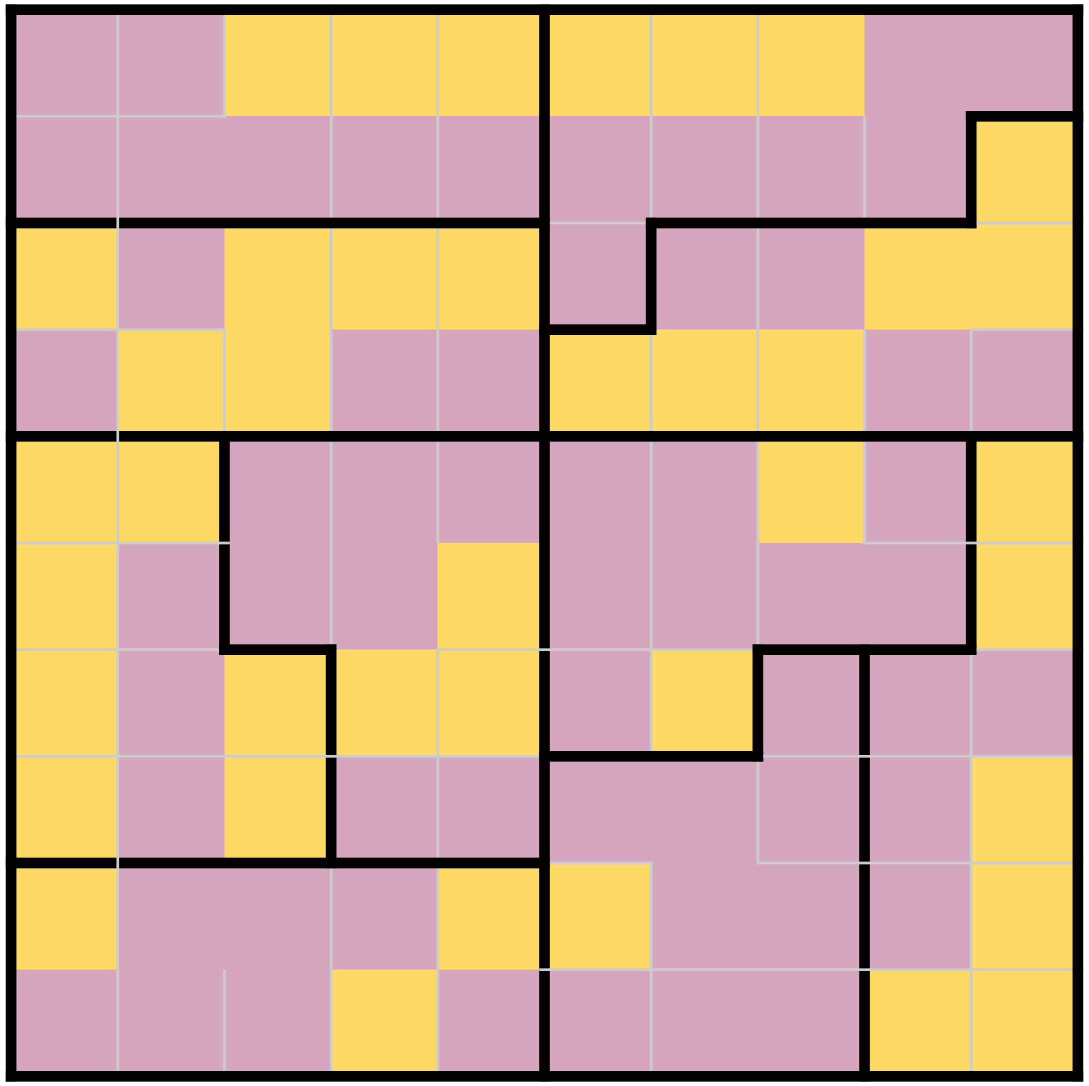
B



total P	16A/P²	Box score	CHull	AMOI
140	0.816	1.4	1	24.16

4 safe orange seats

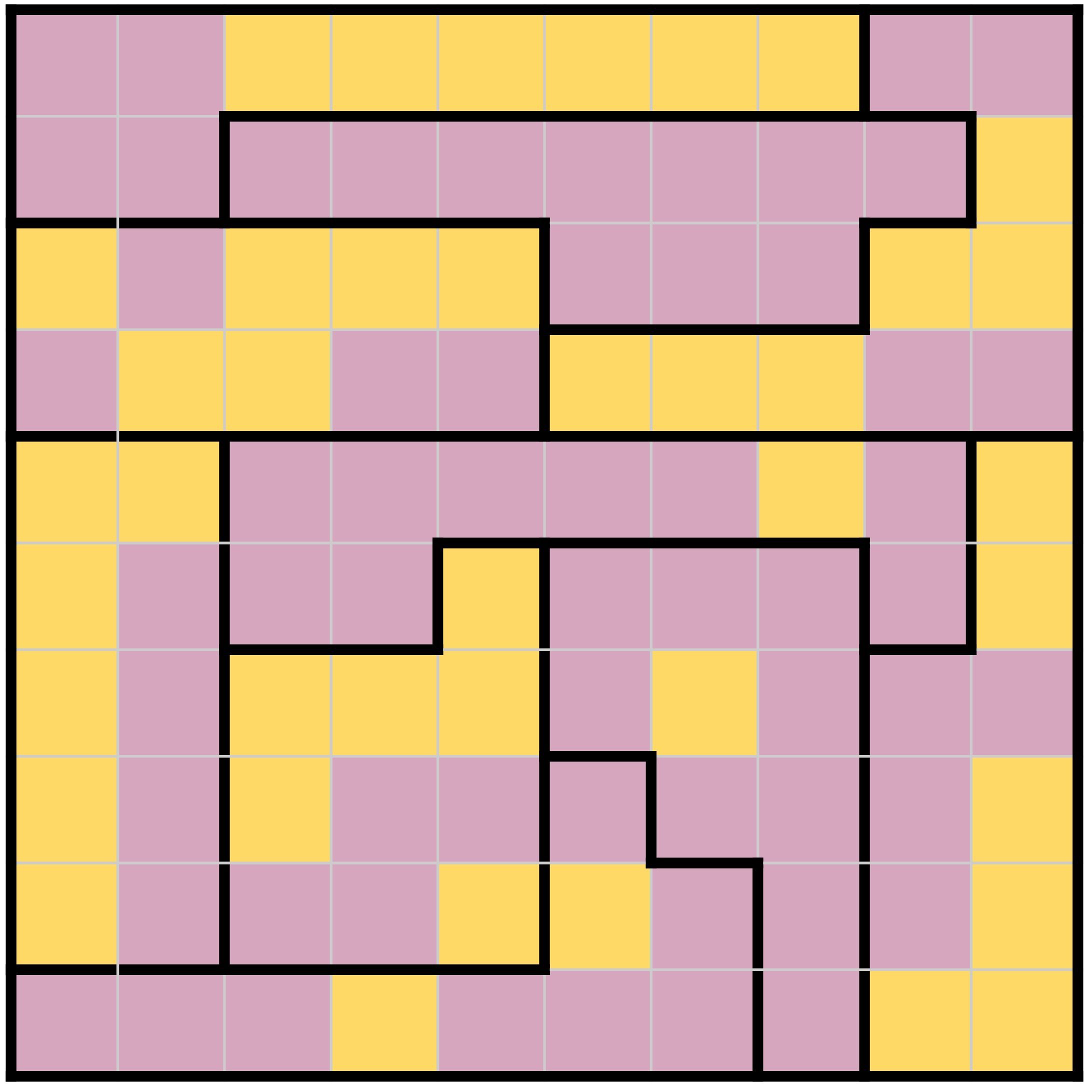
c



total P	$16A/P^2$	Box score	CHull	AMOI
144	0.778	1.423	0.921	22.66

6 safe orange seats

D



total P	16A/P²	Box score	CHull	AMOI
186	0.579	1.189	0.921	34.84

SO, DOES COMPACTNESS DETECT GERRYMANDERING?

ON THIS SIMPLE EXAMPLE IT KIND OF WORKS

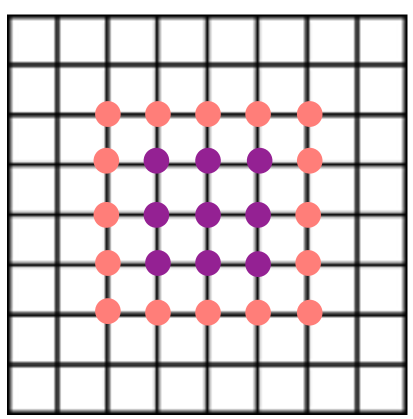
	total P	$16A/P^2$	xy skew	xy hull	Box score	CHull	AMOI
A/B	140	0.816	0.4	1	1.4	1	24.166
C	144	0.778	0.573	0.85	1.423	0.921	22.666
D	186	0.579	0.453	0.736	1.189	0.826	34.846

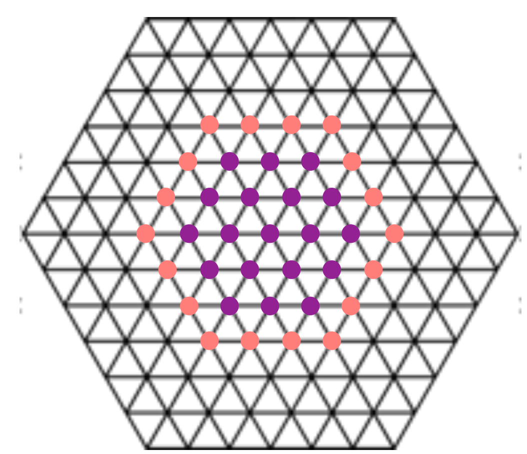
The six-seats outcome (D) is picked out as unreasonable on the basis of shape, but this method gives no guidance about the 2.5 seats (A) vs 1.5 seats (B) vs 4 seats (C).

DISCRETE COMPACTNESS?

USING THE GRAPH

- Many possible interventions in compactness. One simple idea (current project with Bridget Tenner): DISCRETIZE your geometry
- Use discrete/coarse definitions of area and perimeter, counting **area** of a district as the total number of nodes and **perimeter** as the number of boundary nodes





$A = n^2$, $P = 4n - 4$

$A/P^2 \to 1/16$

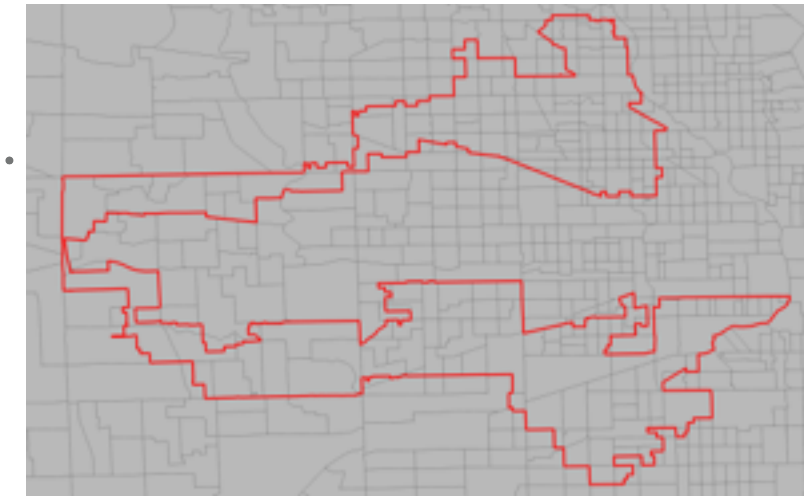
$A = 3n^2 - 3n + 1$, $P = 6n - 6$

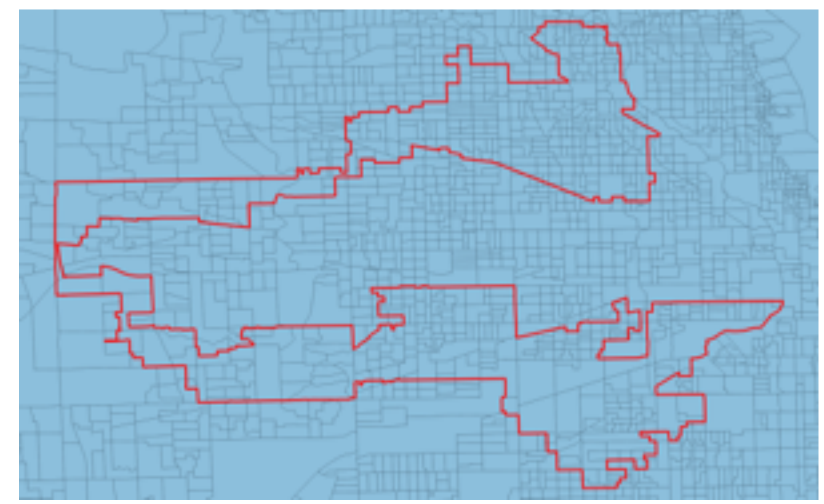
$A/P^2 \to 1/12$

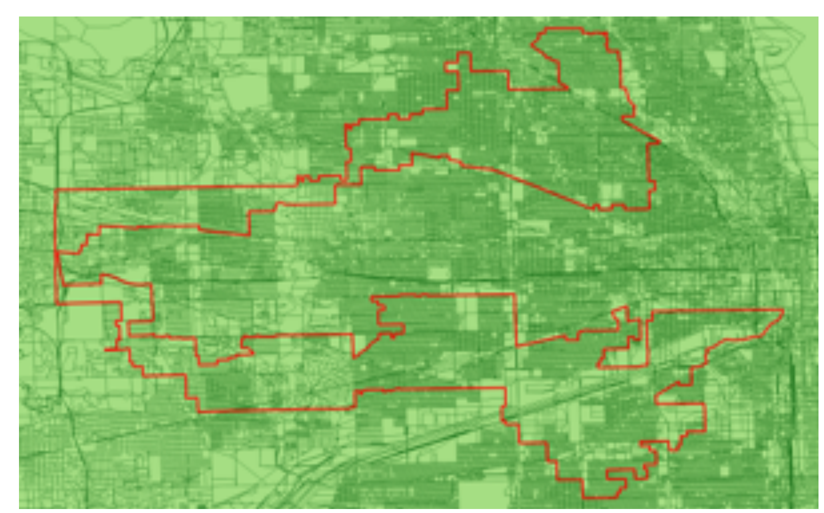
- Behaves well under refinement if the pattern is stable

DISCRETIZED POLSBY-POPPER

- D-Tenner: compare discrete A/P^2 to classical
- PP is subject to coastline paradox, empty space effects, even sensitive to map projection—dPP corrects
- Relative rankings seem quite stable as you change the units
- Sees urban density
- BUT… seems to allow crazy-looking districts if they cut through low-population areas



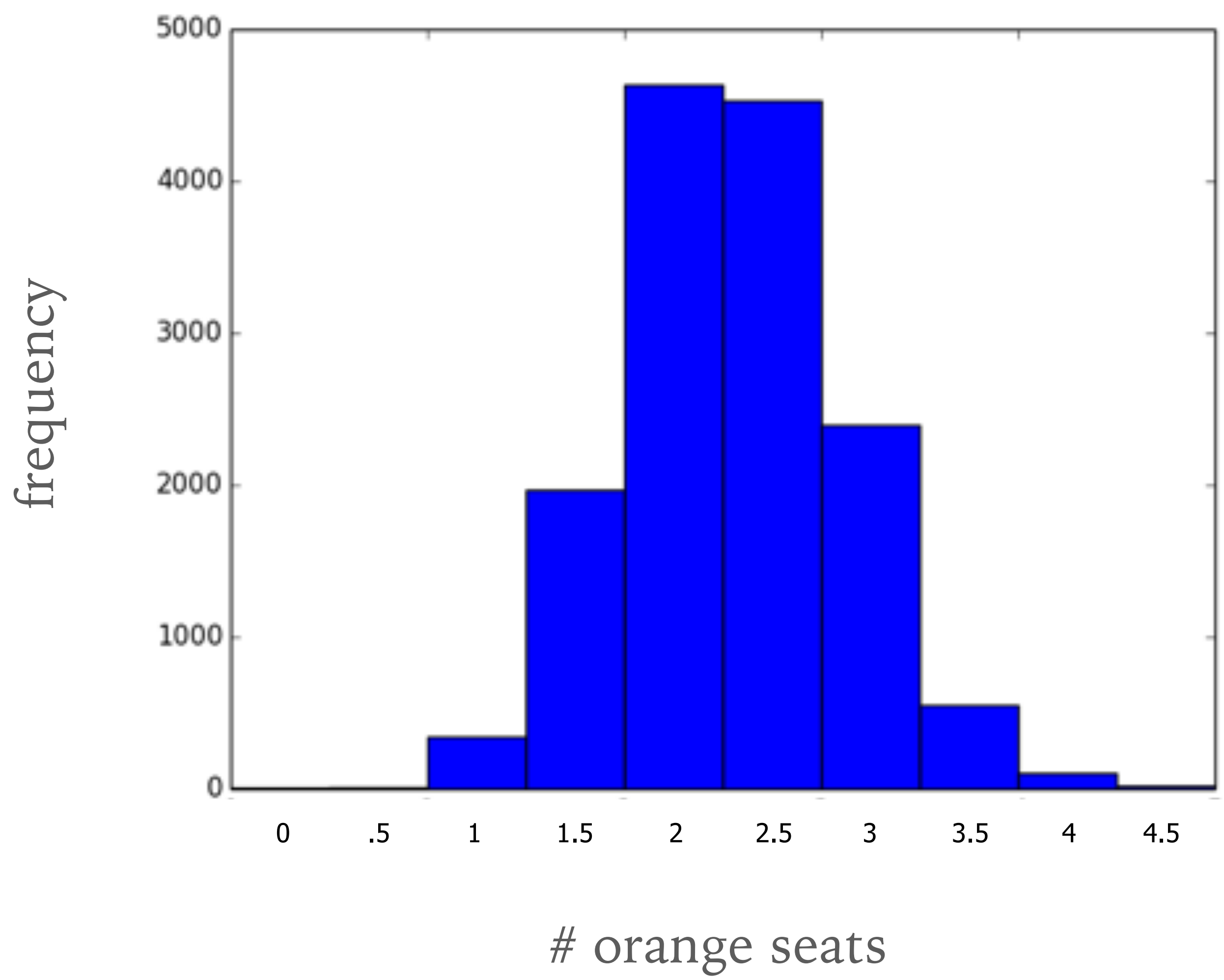




NOW THROW OUT COMPACTNESS AND USE AN ENSEMBLE INSTEAD

HOW EXTREME IS YOUR GERRYMANDER?

I can tell you first-hand that the extreme gerrymander was harder to find. Let's model that with a computer.



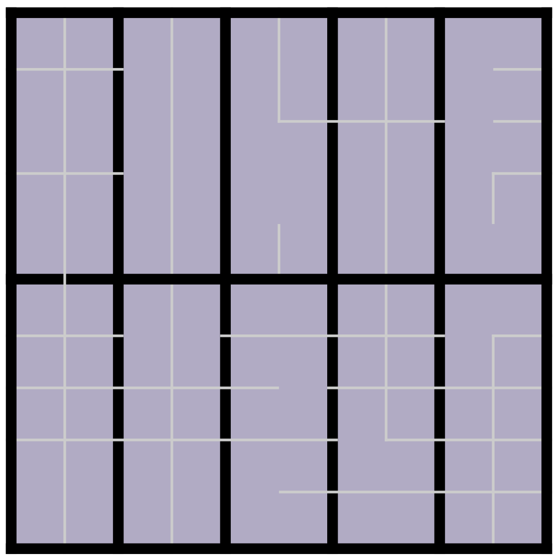
We set up python runs that start with a districting plan and make *pair swaps*, checking contiguity of the proposal.

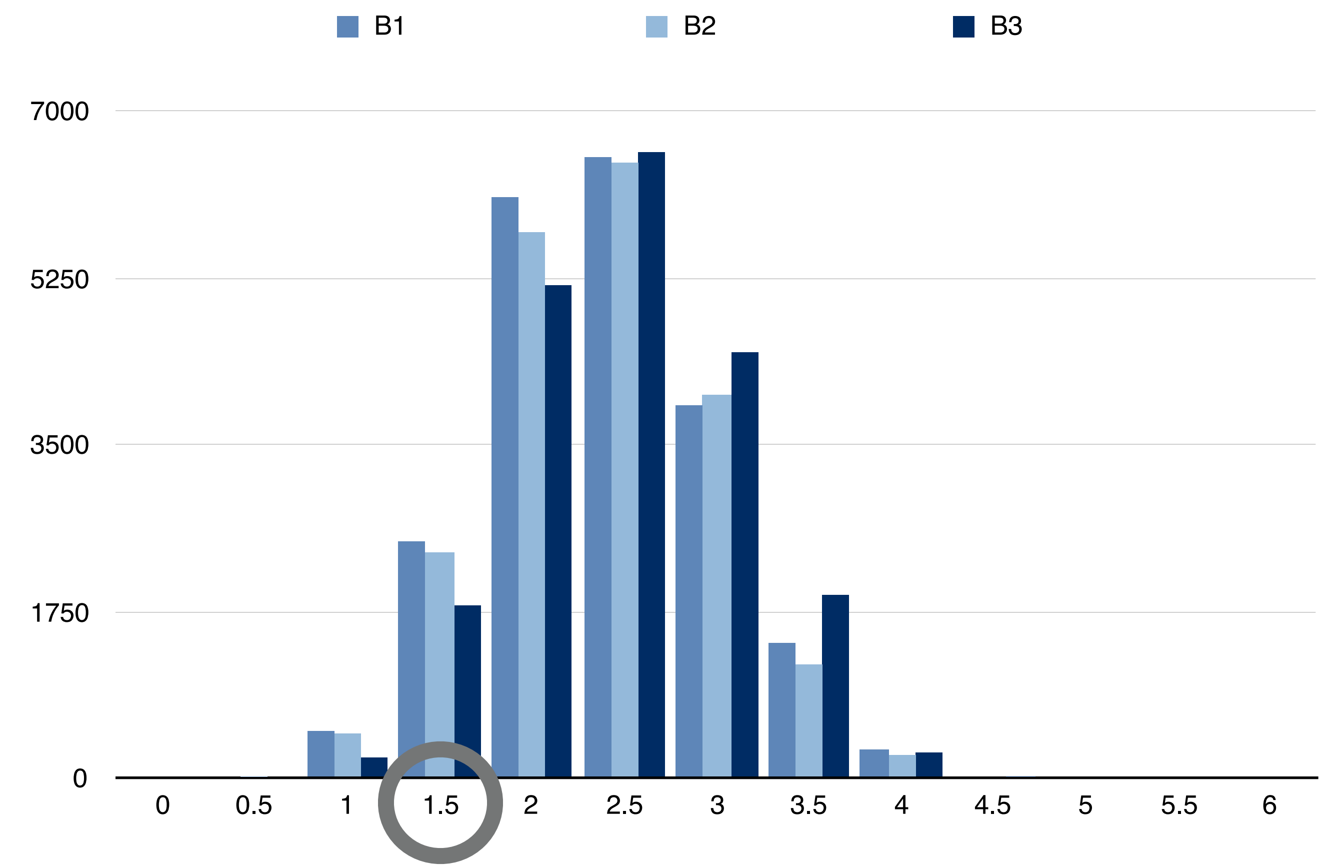
(So population equality and contiguity are maintained, but compactness is ignored.)

This run began with 10 vertical columns (3 Orange seats).

Orange had 40% of votes, but got ≥40% of seats in only 0.8% of the 14,564 maps produced by taking 100,000 random steps

Start with plan B (1.5 orange seats)





Start with plan D (6 orange seats)