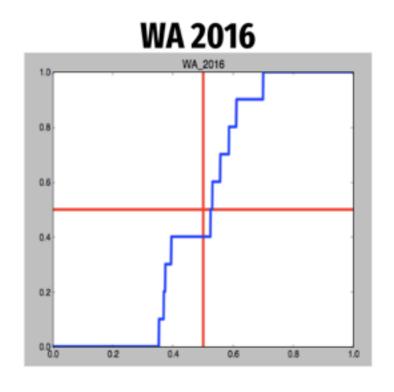
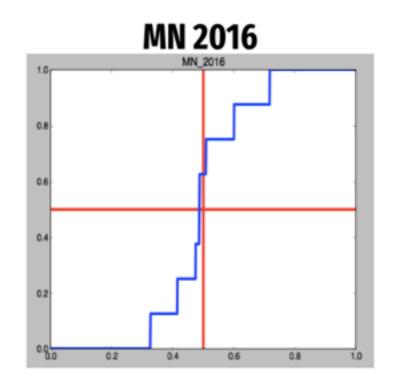
A FORMULA GOES TO COURT

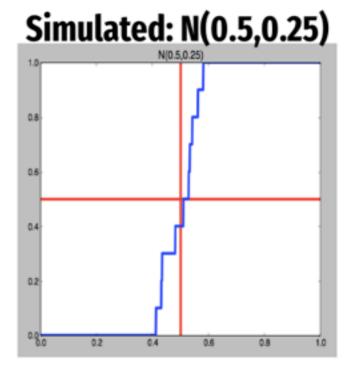
PARTISAN GERRYMANDERING,
THE EFFICIENCY GAP,
AND BEYOND

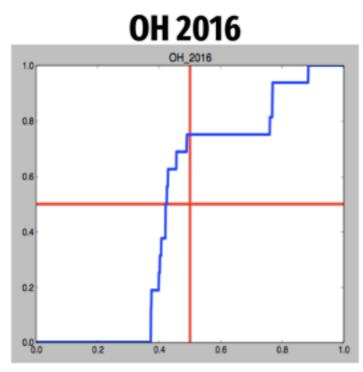
PARTISAN SYMMETRY

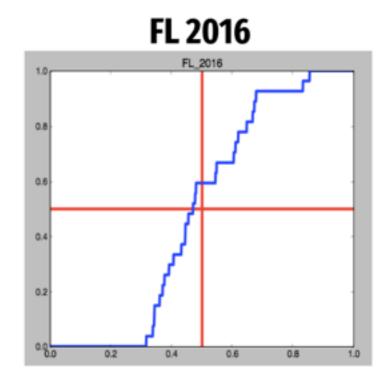
PARTISAN SYMMETRY: USES SEATS-VOTES CURVE

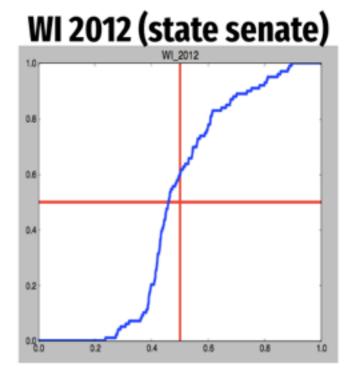




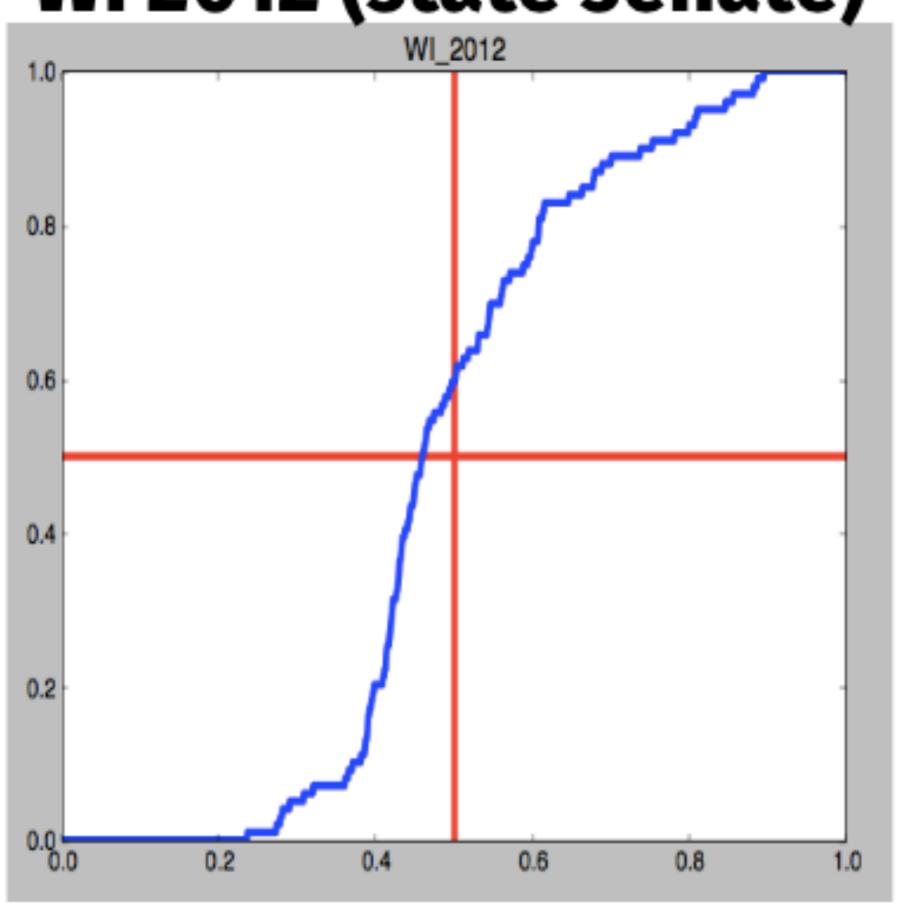




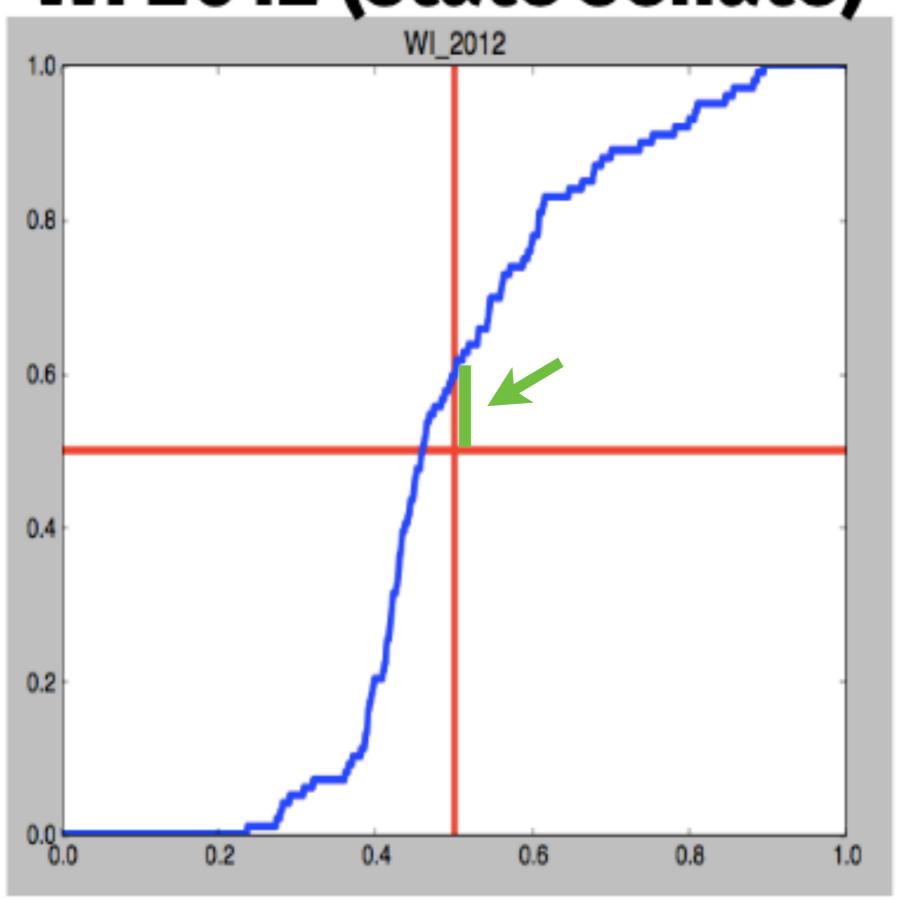




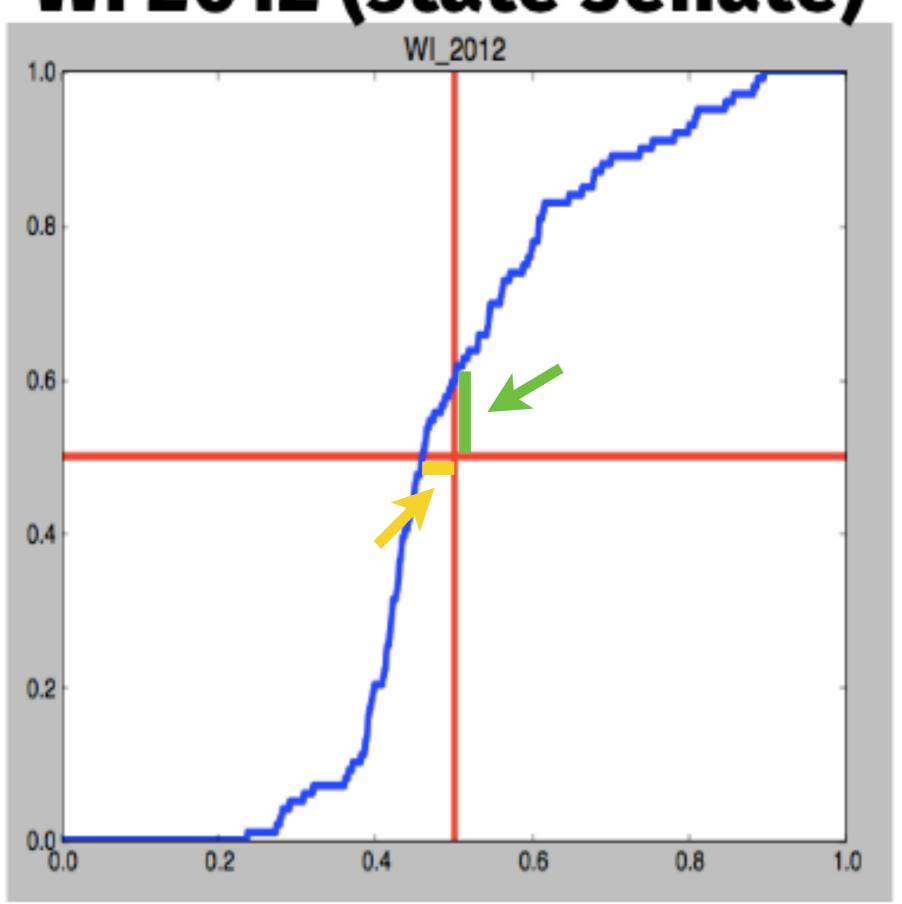
WI 2012 (state senate)



WI 2012 (state senate)



WI 2012 (state senate)



EFFICIENCY GAP

WHAT DOES SCOTUS WANT?

- ➤ Bandemer v Davis (1986): six justices agree partisan gerrymandering is justiciable, but no applicable standard identified.
- ➤ Plurality: "A partisan political gerrymander violates the Equal Protection Clause only on proof of both **intentional** discrimination against an identifiable political group, and an actual discriminatory **effect** on that group... [U]nconstitutional discrimination occurs only when the electoral system... will **consistently** degrade... a group of voters' influence on the political process as a whole"
- ➤ Vieth v Jubelier (2004): explicitly rejects all then-proposed standards.
- ➤ LULAC v Perry (2006): still seeking manageable standard. Symmetry tests are deemed somewhat attractive but potentially troubling because of speculation/counterfactuals.

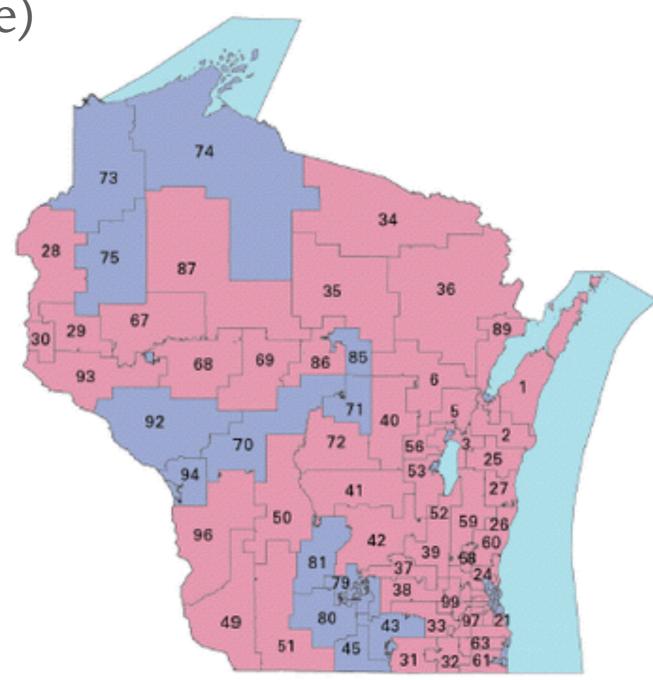
ENTER EFFICIENCY GAP

Whitford v. Gill (pending case)

"Partisan Gerrymandering and the Efficiency Gap"

Nicholas Stephanopoulos & Eric McGhee

University of Chicago Law Review, 2015



ENTER EFFICIENCY GAP

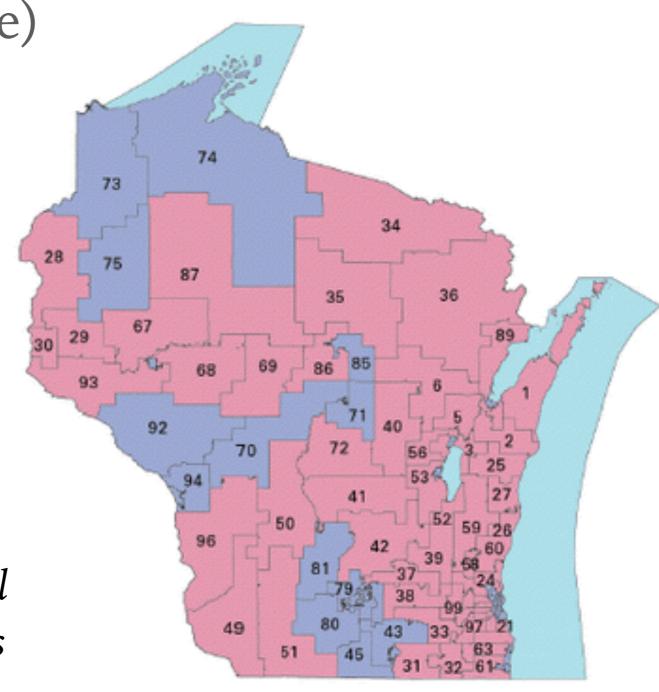
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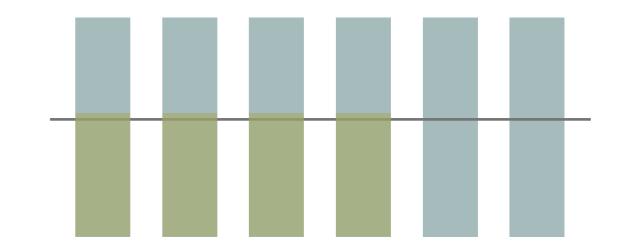
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new score called EG "captures, in a single tidy number, all of the packing and cracking decisions that go into a district plan."



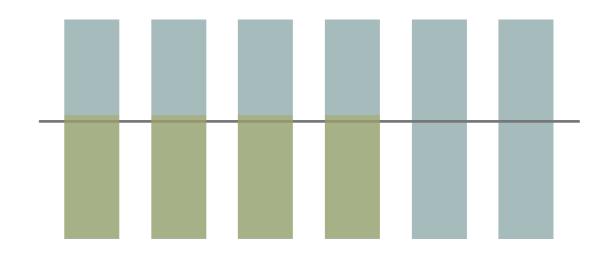


$$EG = \sum_{i=1}^{S} \frac{W_i^A - W_i^B}{T}$$



$$EG = \frac{W^A - W^B}{T}$$

Let's say you "waste" (a) all votes in a district you lose, and (b) excess votes in a district you win.

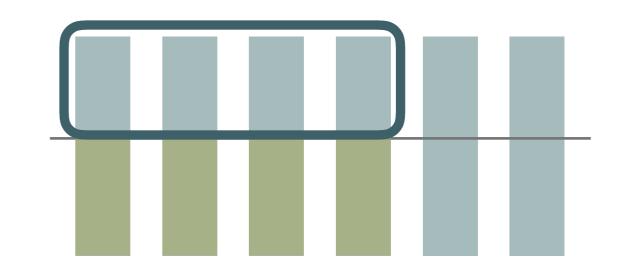


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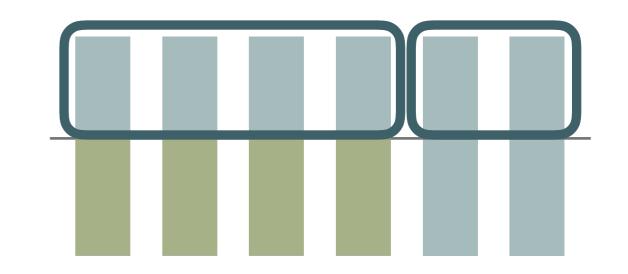


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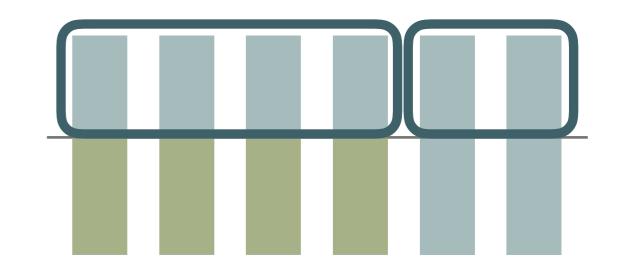


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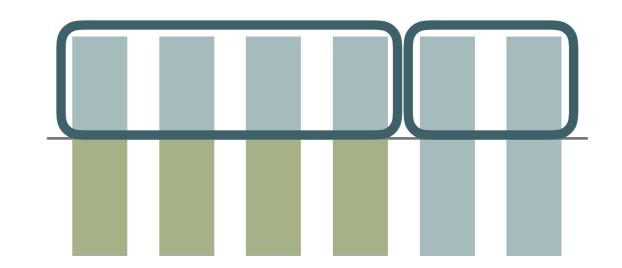


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- ➤ You can just look at the wasted vote differentials in each district as a proportion of the vote that turned out.
- ➤ If all districts have the same turnout, then we get significant simplification.



$$EG = \sum_{i=1}^{S} \frac{W_i^A - W_i^B}{T}$$



$$EG = \frac{W^A - W^B}{T}$$

➤ Extremely simple idea: add up wasted votes.

i

1

2

3

4

5

All

i	T_i^A	
1	95	
2	40	
3	75	
4	45	
5	45	
All	300	

i	T_i^A	$T_i{}^B$
1	95	5
2	40	60
3	75	25
4	45	55
5	45	55
All	300	200

i	T_i^A	T_i^B	Winner
1	95	5	A
2	40	60	В
3	75	25	A
4	45	55	В
5	45	55	В
All	300	200	2A:3B

i	T_i^A	T_i^B	Winner	W_i^A
1	95	5	A	45
2	40	60	В	40
3	75	25	A	25
4	45	55	В	45
5	45	55	В	45
All	300	200	2A:3B	200

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2	40	60	В	40	10
3	75	25	A	25	25
4	45	55	В	45	5
5	45	55	В	45	5
All	300	200	2A:3B	200	50

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In our example: EG = 150/500 = 0.3

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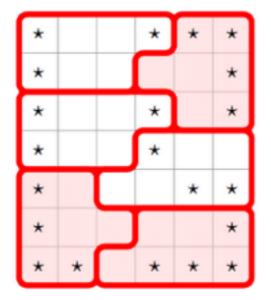
Efficiency gap:

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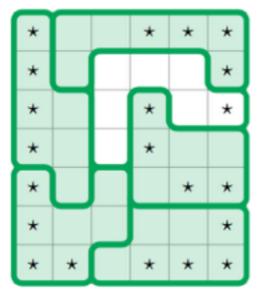
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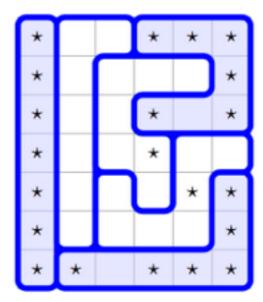
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Plan I: EG = 0



Plan II: EG = -1/3



Plan III: EG = 0

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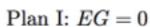
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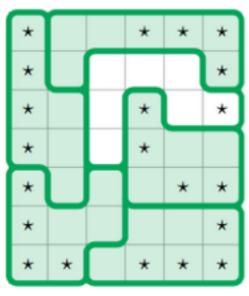
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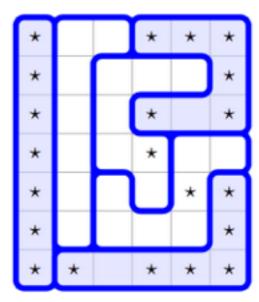
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Plan II: EG = -1/3



Plan III: EG = 0

➤ Suppose $T^A = \frac{1}{2} + t$ and $S^A = \frac{1}{2} + s$.

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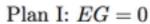
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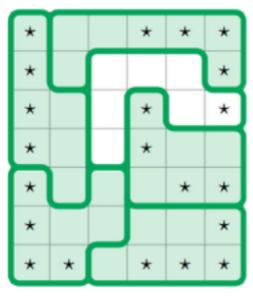
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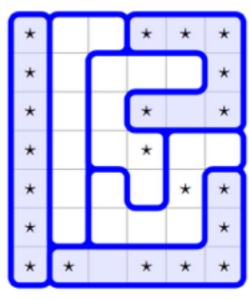
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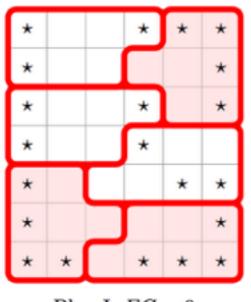
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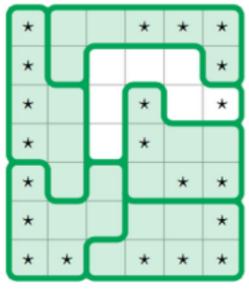


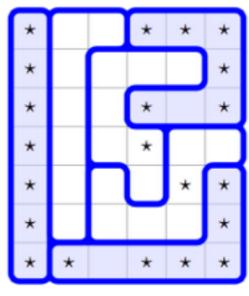
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➤ Suppose $T^A = \frac{1}{2} + t$ and $S^A = \frac{1}{2} + s$.

(Vote margin; seat margin.)







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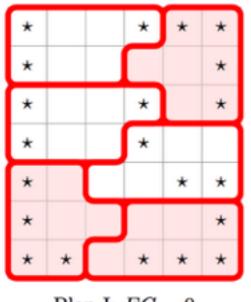
Plan II: EG = -1/3

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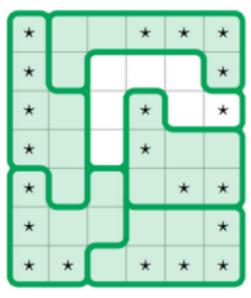
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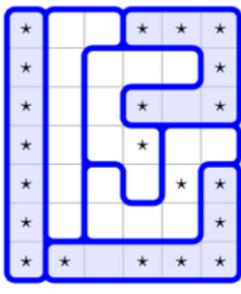
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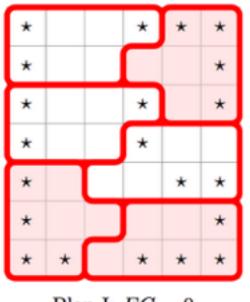
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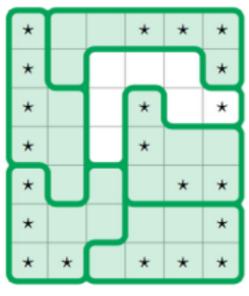
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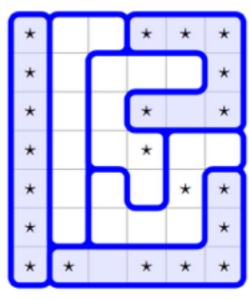
$$EG = 2t - s$$







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DISTRICT CONTRIBUTIONS CANCEL OUT AND DISAPPEAR

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1	95	5	A	45	5	40
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All	300	200	2A:3B	200	50	150

➤ Here, A got 60% of the votes (vote margin t=.1) and 40% of the seats (seat margin s=-.1).

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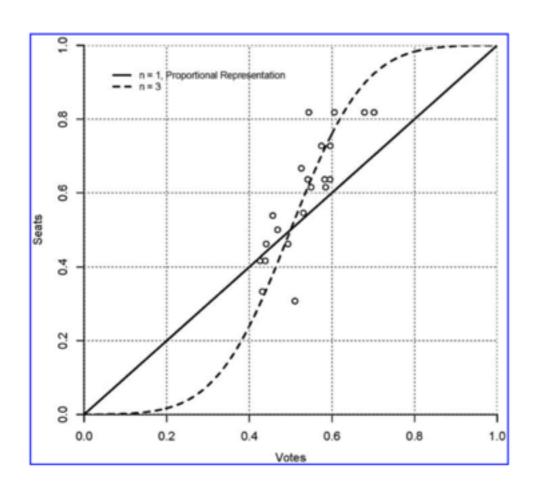
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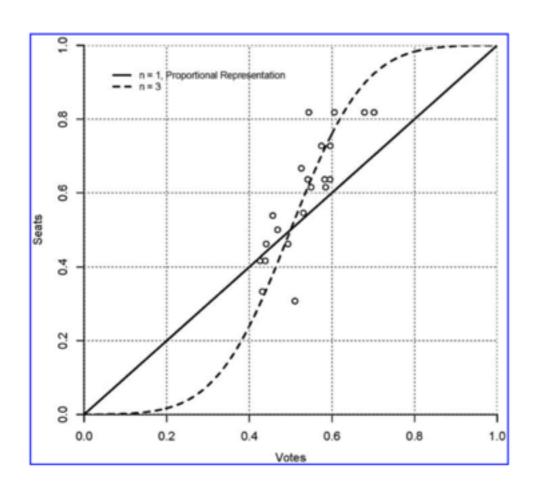
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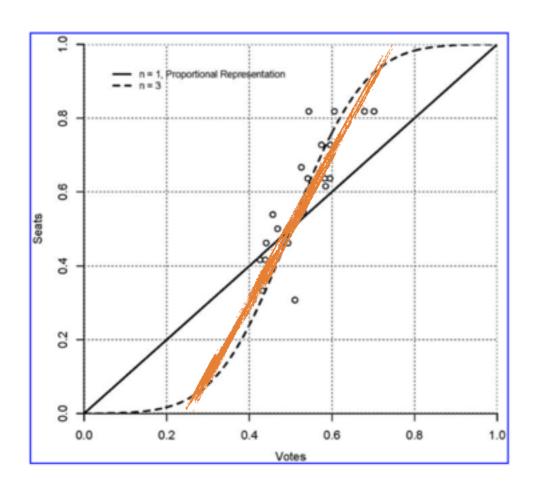
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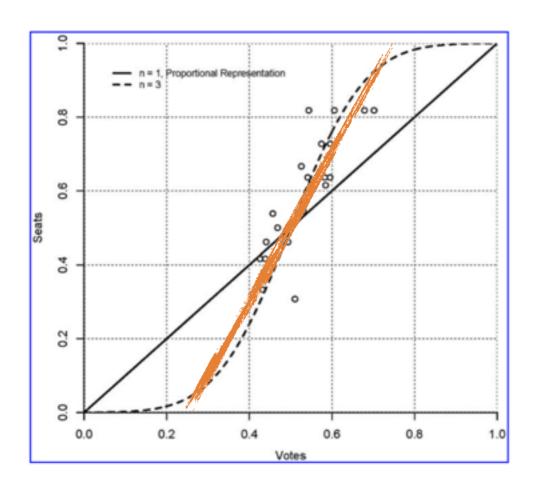
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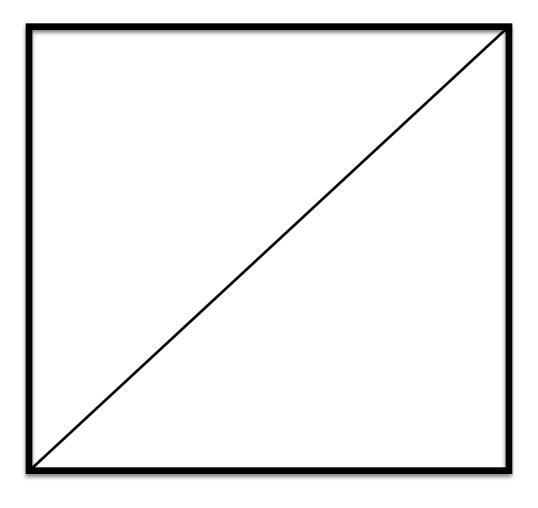
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- ➤ In other words, *EG* adoption commits you to the view that with 60% of the vote, a party *should* get 70% of the seats. Authors claim this as a feature, not a bug!

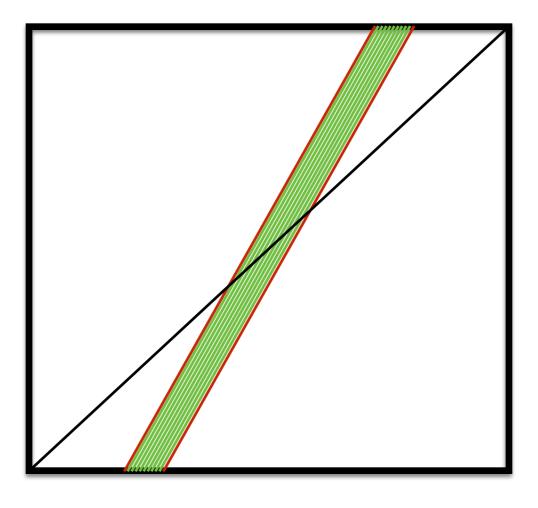


Seats

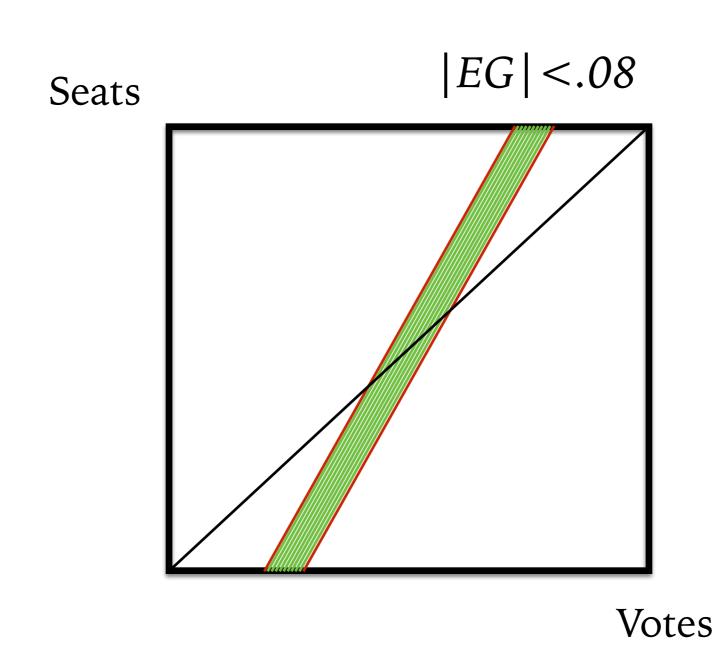


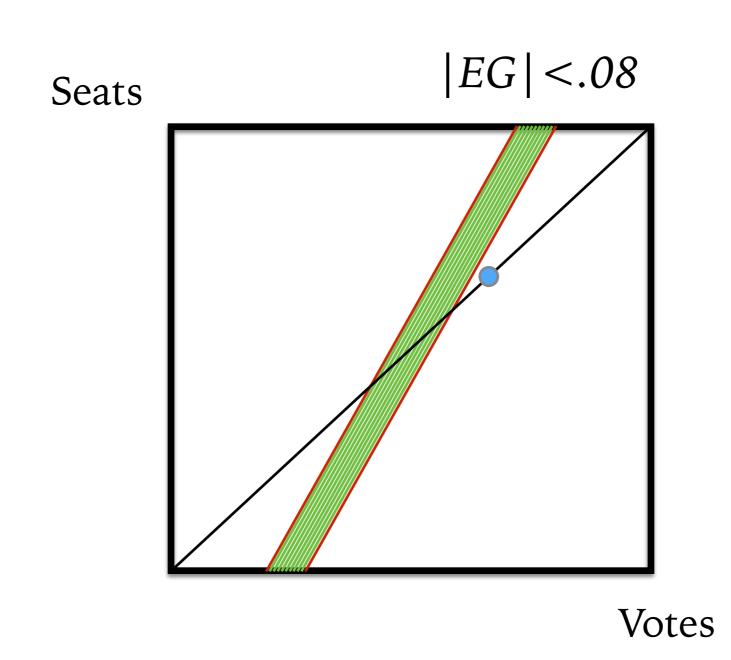
Votes

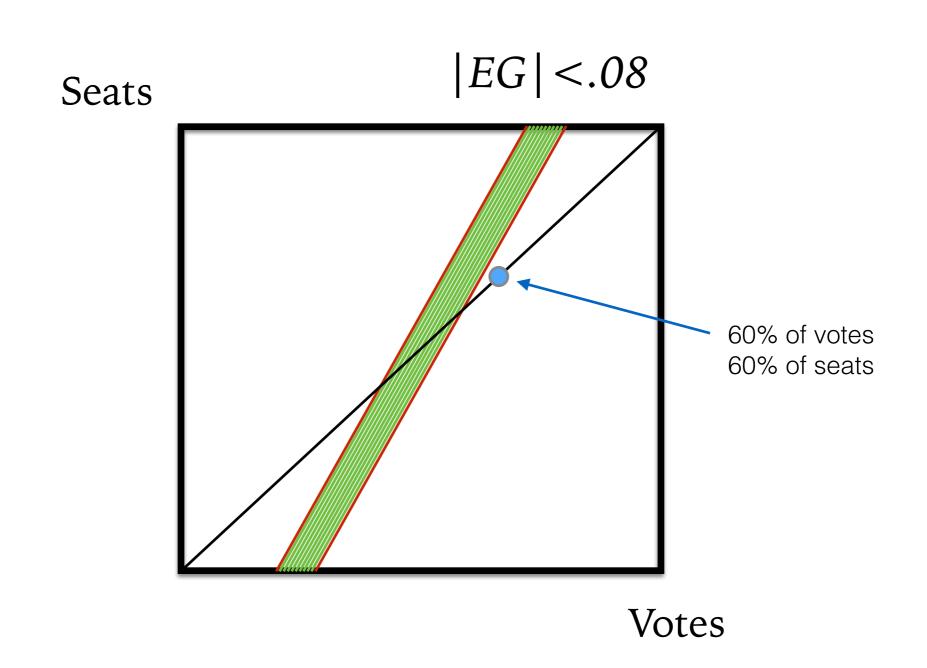
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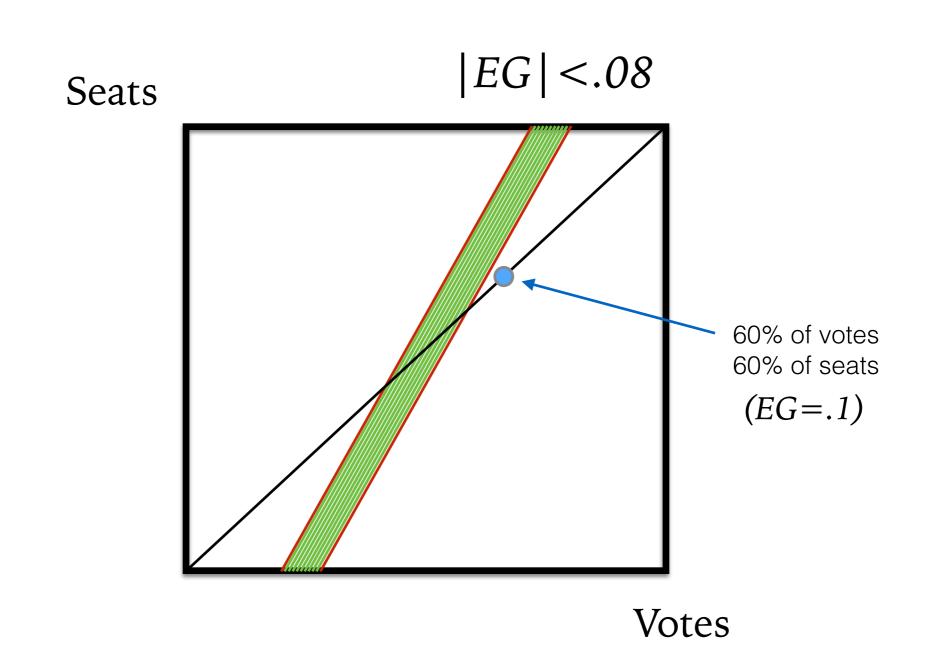


Votes









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 - demonstrate intent
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- ➤ How is it faring? Whitford backed off *EG* more at each level of appeal. Meanwhile appearing in press, studies, numerous cases.
- ➤ Interestingly, current NC case is split in two approaches: LWV v Rucho based on *EG*, Common Cause v Rucho based on *sampling and outliers*....

EG: TURNOUT ISSUES

RECENT PAPER OF ELLEN VEOMETT STUDIES TURNOUT EFFECTS

District	1	2	3	4	5	6	7
Turnout	260,409	278,236	316,467	246,220	192,875	273,296	255,533
District	8	10	11	12	13	14	17
Turnout	236,379	312,600	225,548	283,115	221,242	259,685	245,728
District	19	21	22	23	24	25	26
Turnout	203,475	356,031	305,543	228,965	275,635	310,196	319,080
District	27	31	32	36			
Turnout	230,580	284,588	229,171	218,565			

Table 2: Republican-won Districts in the 2016 Texas congressional election.

District	9	15	16	18	20	28	29
Turnout	188,523	177,479	175,229	204,308	187,669	184,442	131,982
District	30	33	34	35			
Turnout	218,826	126,369	166,961	197,576			

Table 3: Democrat-won Districts in the 2016 Texas congressional election.

State	AZ	CA	FL	GA	IL	IN	MD	MA	MI	MN	MO
n	9	53	27	14	18	9	8	9	14	8	8
ρ	1.42	1.11	1.07	0.99	1.14	1.18	1.08		1.11	1.08	1.10
M/m	2.15	4.41	1.62	1.55	2.06	1.42	1.18	1.34	1.47	1.19	1.34
State	NJ	NY	NC	OH	PA	TN	TX	VA	WA	WI	
n	12	27	13	16	18	9	36	11	10	8	
ρ	1.26	1.11	0.94	1.14	1.00	1.10	1.48	1.09	0.91	1.16	
M/m	1.96	1.83	1.27	1.34	1.56	1.31	2.82	1.42	1.65	1.53	

Table 4: Turnout ratios in all states with at least 8 congressional districts

Notes: her S is seat share, V is vote share;

 $S^* = S-1/2$, $V^* = V-1/2$ are margins

State	AZ	CA	FL	GA	IL	IN	MD	MA	MI	MN	MO
n	9	53	27	14	18	9	8	9	14	8	8
ρ	1.42	1.11	1.07	0.99	1.14	1.18	1.08		1.11	1.08	1.10
M/m	2.15	4.41	1.62	1.55	2.06	1.42	1.18	1.34	1.47	1.19	1.34
State	NJ	NY	NC	OH	PA	TN	TX	VA	WA	WI	
n	12	27	13	16	18	9	36	11	10	8	
ρ	1.26	1.11	0.94	1.14	1.00	1.10	1.48	1.09	0.91	1.16	
M/m	1.96	1.83	1.27	1.34	1.56	1.31	2.82	1.42	1.65	1.53	

$$EG = S^* - 2V^* + \frac{S(1-S)(1-\rho)}{S(1-\rho) + \rho}$$

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n	9	53	27	14	18	9	8	9	14	8	8
ρ	1.42	1.11	1.07	0.99	1.14	1.18	1.08		1.11	1.08	1.10
M/m	2.15	4.41	1.62	1.55	2.06	1.42	1.18	1.34	1.47	1.19	1.34
State	NJ	NY	NC	OH	PA	TN	TX	VA	WA	WI	
n	12	27	13	16	18	9	36	11	10	8	
ρ	1.26	1.11	0.94	1.14	1.00	1.10	1.48	1.09	0.91	1.16	
M/m	1.06	1.83	1 27	1.3/	1 56	1 31	2.82	1.49	1.65	1.53	

$$EG = S^* - 2V^* + \frac{S(1-S)(1-\rho)}{S(1-\rho) + \rho}$$

Table 4: Turnout ratios in all states with at least 8 congressional districts

Notes: her S is seat share, V is vote share; $S^*=S-1/2$, $V^*=V-1/2$ are margins EG-preferred seat share is

$$S = \frac{\rho(4V - 1)}{\rho(4V - 1) + 3 - 4V}$$

State	AZ	CA	FL	GA	IL	IN	MD	MA	MI	MN	MO
n	9	53	27	14	18	9	8	9	14	8	8
ρ	1.42	1.11	1.07	0.99	1.14	1.18	1.08		1.11	1.08	1.10
M/m	2.15	4.41	1.62	1.55	2.06	1.42	1.18	1.34	1.47	1.19	1.34
State	NJ	NY	NC	OH	PA	TN	TX	VA	WA	WI	
n	12	27	13	16	18	9	36	11	10	8	
ρ	1.26	1.11	0.94	1.14	1.00	1.10	1.48	1.09	0.91	1.16	
M/m	1.96	1.83	1.27	1.34	1.56	1.31	2.82	1.42	1.65	1.53	1

$$EG = S^* - 2V^* + \frac{S(1-S)(1-\rho)}{S(1-\rho) + \rho}$$

Table 4: Turnout ratios in all states with at least 8 congressional districts

Notes: her S is seat share, V is vote share; $S^*=S-1/2$, $V^*=V-1/2$ are margins EG-preferred seat share is

$$S = \frac{\rho(4V - 1)}{\rho(4V - 1) + 3 - 4V}$$

Theorem (V, 2018)

Fix rational numbers 1/4 < V < 3/4 and 0 < S < 1. Consider an election with vote share V, seat share S, and EG=0. Then

$$\rho = \frac{S(3-4V)}{(1-S)(4V-1)}$$

VEOMETT'S THEOREM FOR EG=0

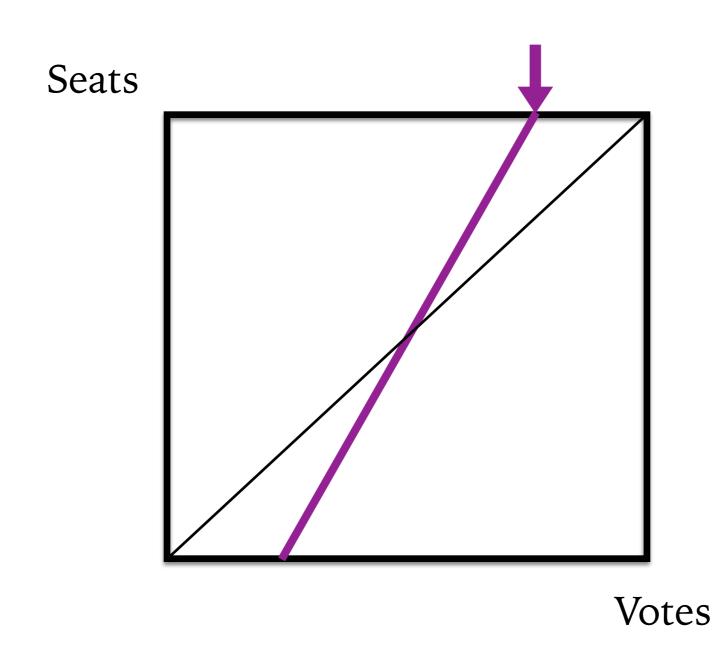
Theorem (V, 2018)

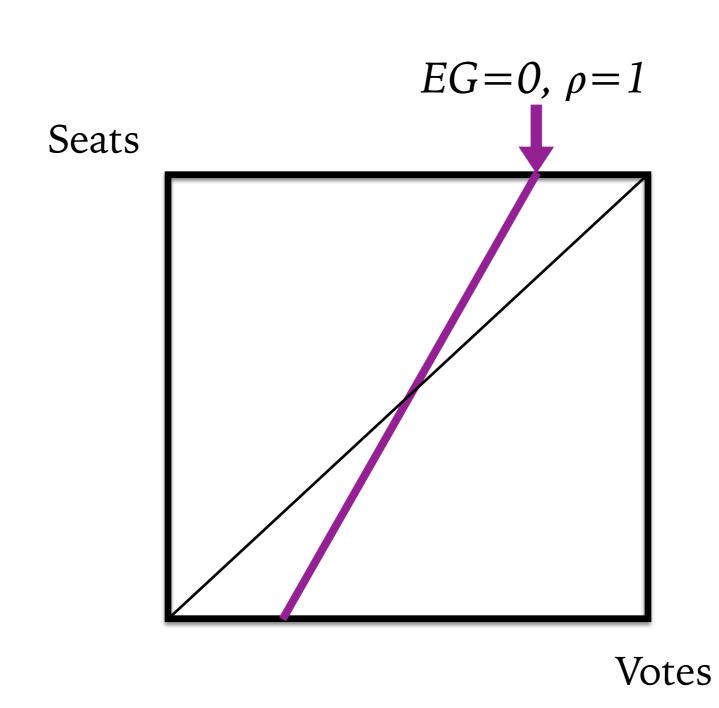
For any rational numbers 1/4 < V < 3/4 and 0 < S < 1, there exists election data with vote share V, seat share S, and EG = 0.

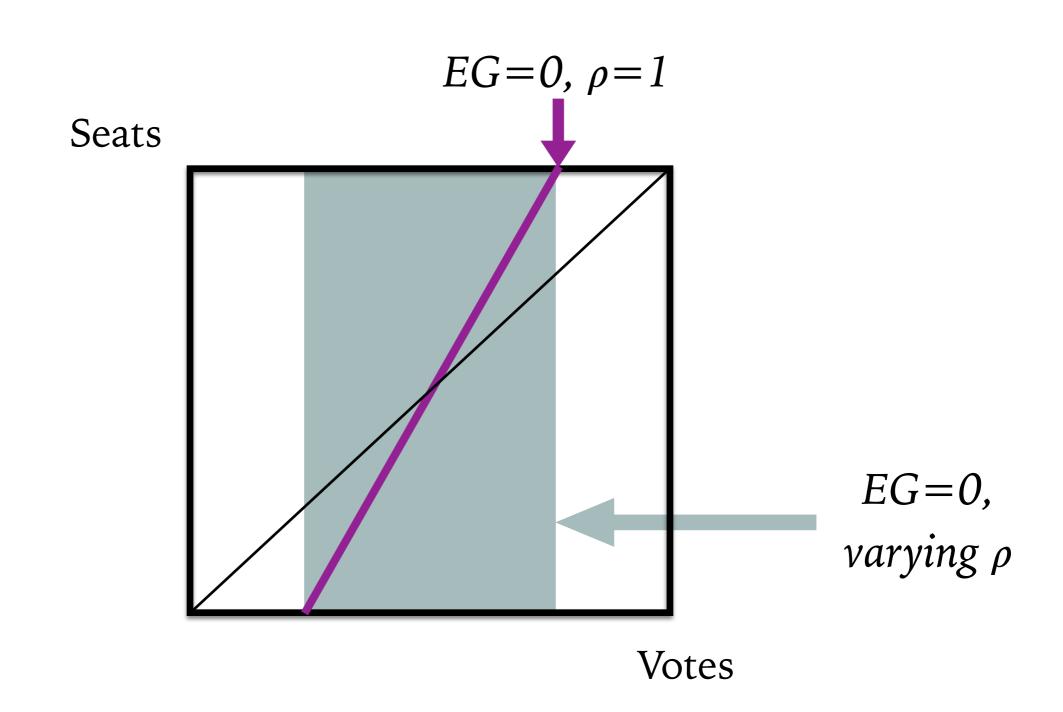
Proof 2:

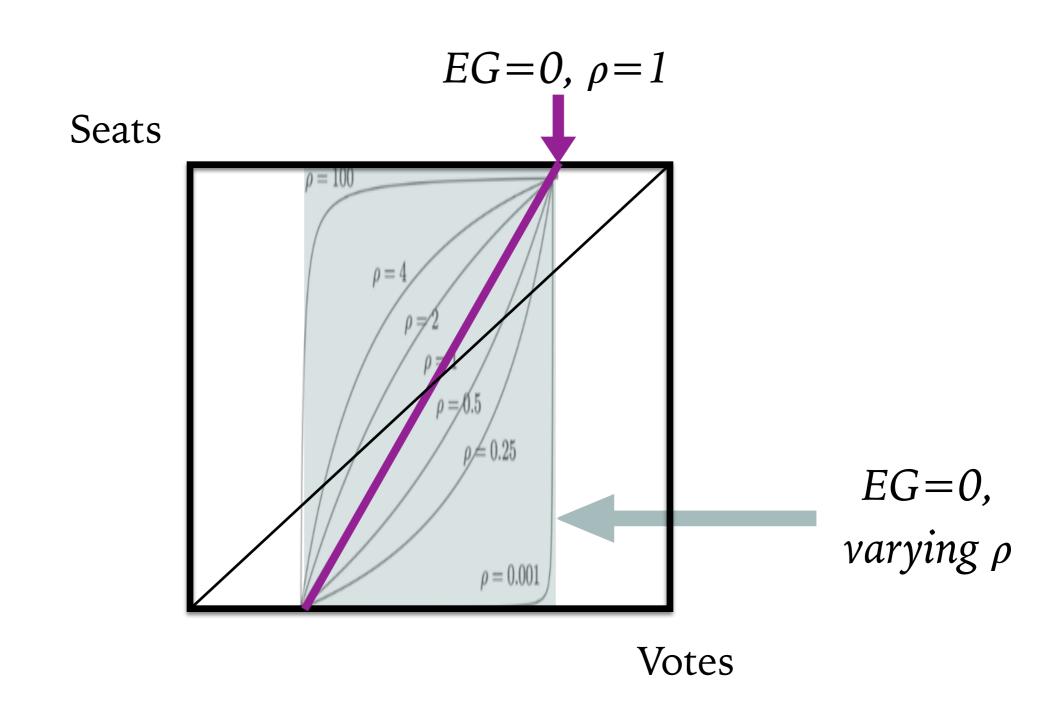
Any election with EG=0 can be constructed as follows:

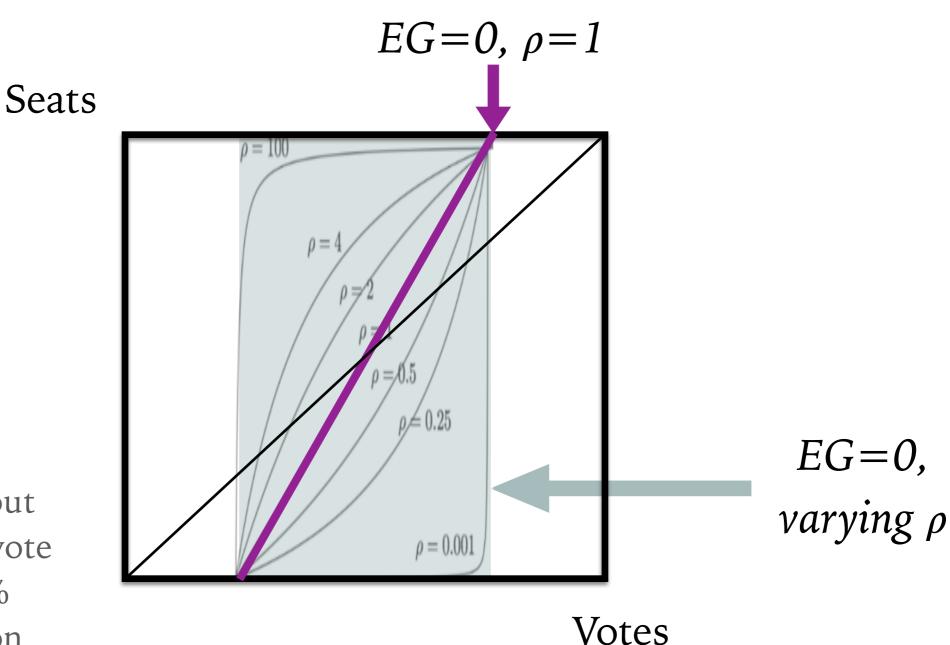
- Start with "empty election" (no votes), fixed districts won by each party.
 - Note that EG=0.
- Add votes to keep EG=0, and maintain each district's winner:
 - Three winning votes to A, one losing to B.
 - Three winning votes to B, one losing to A.
 - One winning vote to A, one winning vote to B.
 - One vote to each party in district A won, One vote to each party in district B won.











Punchline: in TX, with current turnout patterns, a 50-50 vote would require 60% Dem representation to have EG=0

EG: MEASUREMENT ISSUES

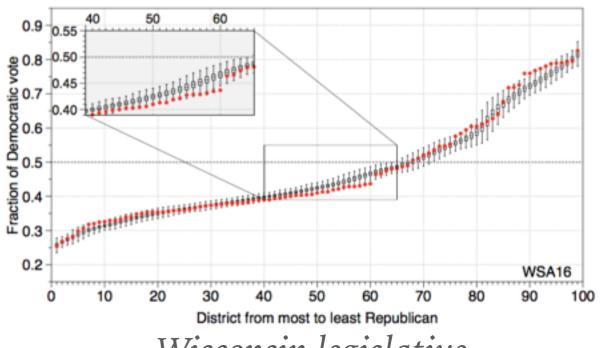
➤ It's not obvious which vote totals to assign to precincts.

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 - ➤ Imputation issues (uncontested races, incumbency effects)

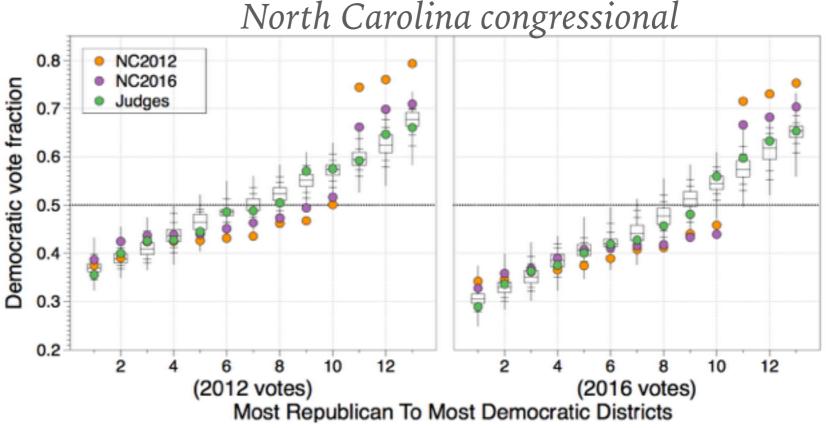
- ➤ It's not obvious which vote totals to assign to precincts.
 - ➤ Imputation issues (uncontested races, incumbency effects)
- ► It's not obvious which Δ to use. (i.e., where to place the precincts)

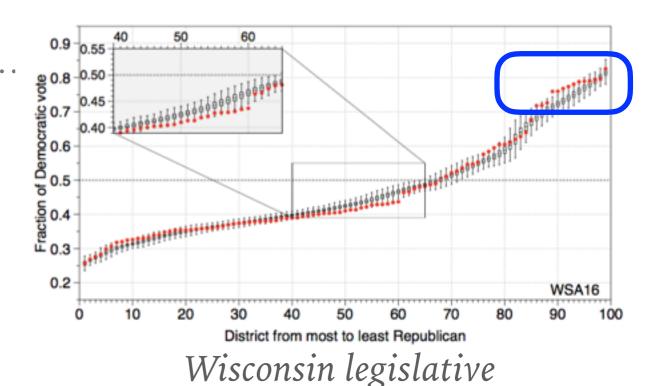
- ➤ It's not obvious which vote totals to assign to precincts.
 - ➤ Imputation issues (uncontested races, incumbency effects)
- ➤ It's not obvious which Δ to use. (i.e., where to place the precincts)
 - > cf. Ohio, Pennsylvania

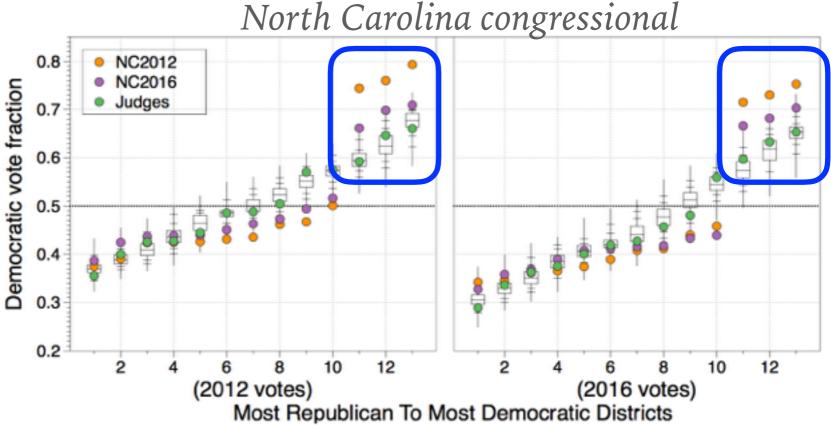
OTHER IDEAS

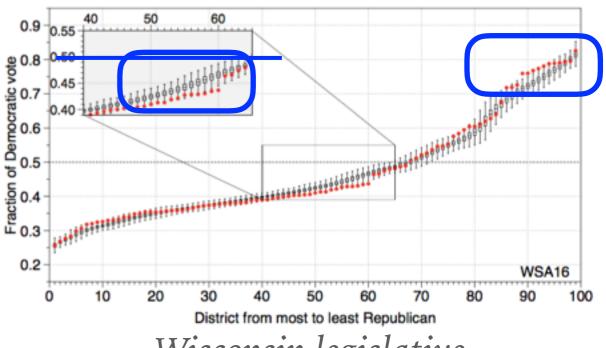


Wisconsin legislative

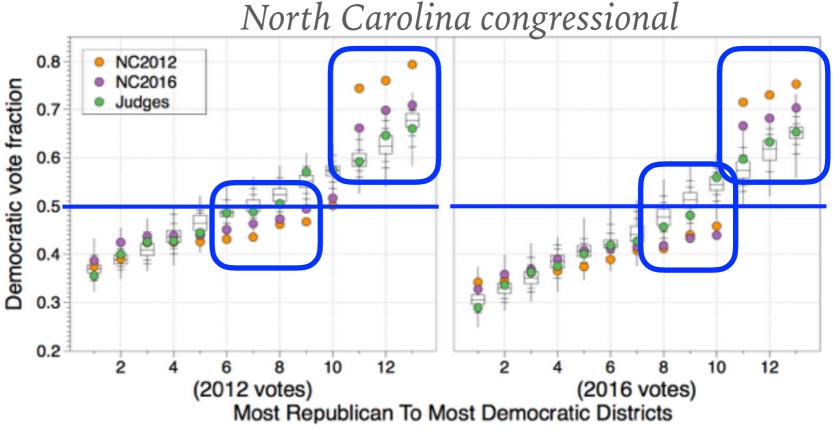


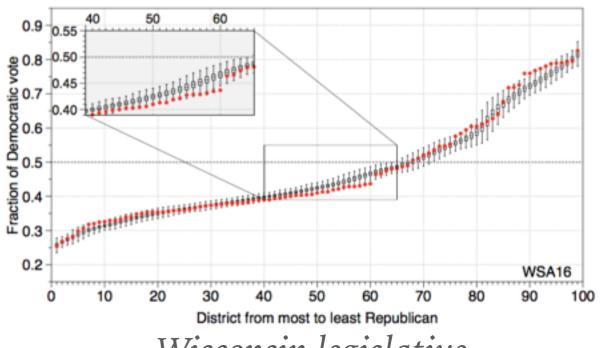




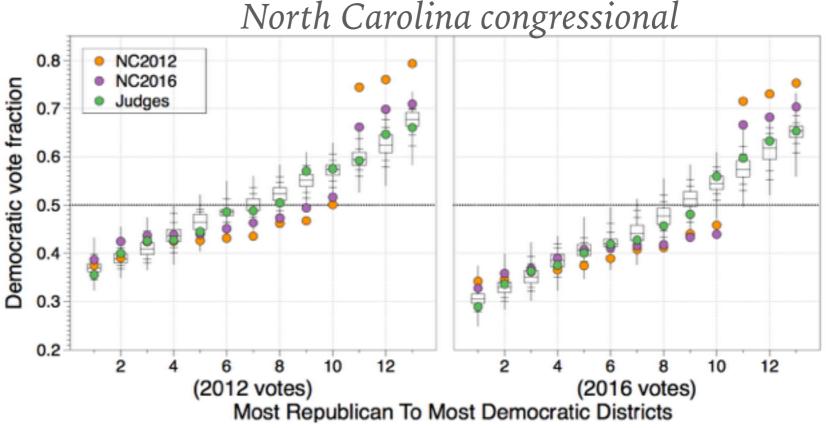


Wisconsin legislative



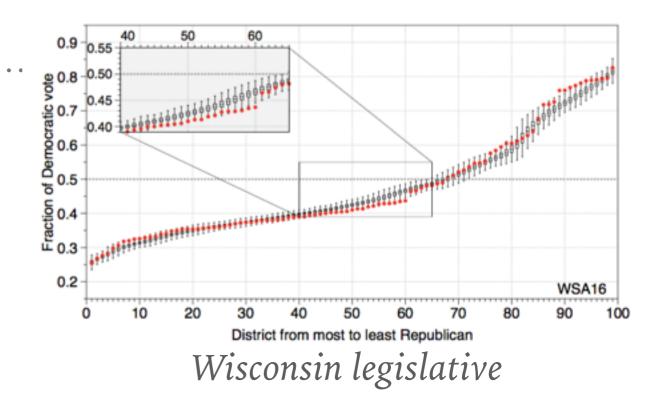


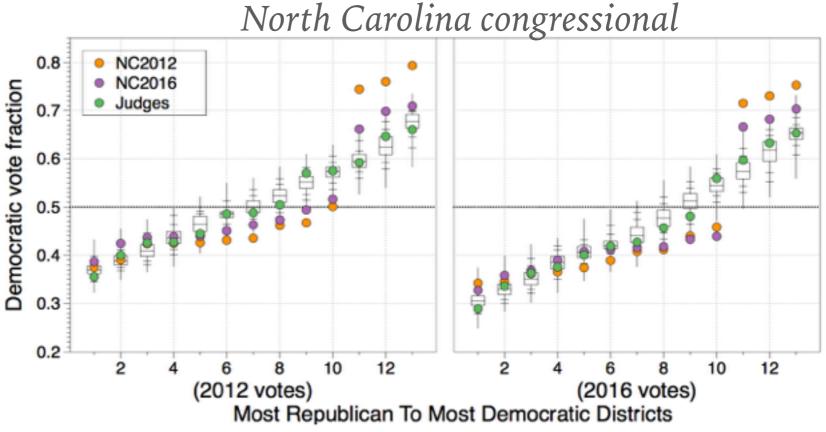
Wisconsin legislative



➤ Idea: if you look at the vote share district-by-district, then the "signature of gerrymandering" is that the other side has some wastefully high vote shares (packing) and others that are conspicuously depressed below 50% (cracking)

Create a score by measuring distance from the box plot of the comparison ensemble





DUKE GERRYMANDERING INDEX, CONTINUED

➤ The 2012 and 2016 Legislature maps are outliers against the ensemble, while the bipartisan Judges' map hits the middle of the curve—this gives another partisan metric.

