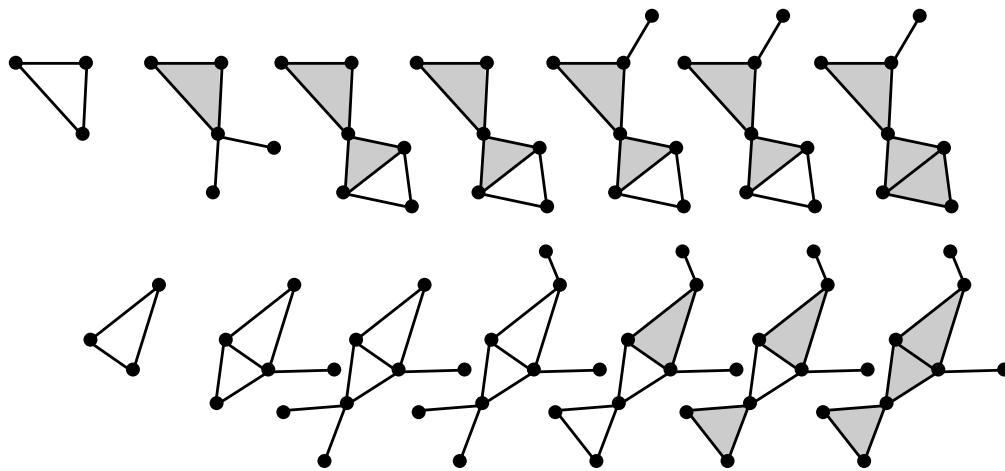
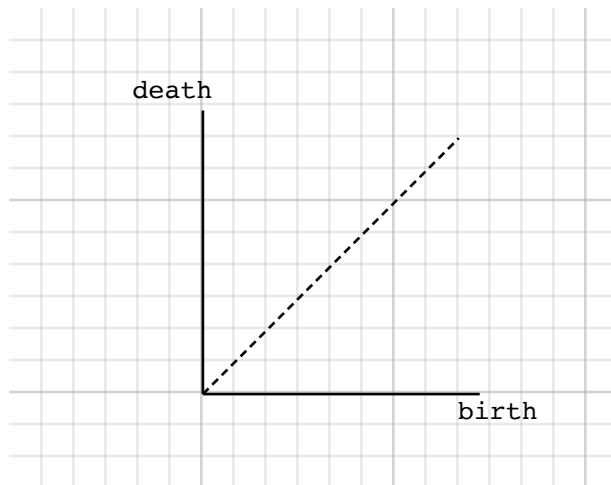


# TDA Breakout: Day 4 Worksheet

**Exercise.** Consider the filtered simplicial complexes shown below. Each row shows a different filtered simplicial complex. Assume that the time steps are all spaced equally, the leftmost figures each occurring at time 1 and the rightmost occurring at time 7.



Plot the degree-1 persistence diagram for each filtered simplicial complex on the same axes below (use different colors or X's and O's to distinguish them visually). Remember that the degree-1 persistence diagram should track "births" and "deaths" of loops in each filtered simplicial complex. Assume the grid lines are 1 unit apart.



Now compute the bottleneck distance between the persistence diagrams. For your convenience, a precise definition of bottleneck distance is included below.

## Appendix: Definition of Bottleneck Distance

Let  $D$  and  $D'$  be persistence diagrams. The *bottleneck distance* between them is given by

$$d_b(D, D') = \min_{\phi: A \rightarrow A'} \max \left\{ \max_{p \in A} c_m(p, \phi(p)), \max_{p \in D \setminus A} c_u(p), \max_{p' \in D' \setminus A'} c_u(p') \right\},$$

where the minimum is over *partial matchings*; i.e., bijections  $\phi : A \rightarrow A'$  where  $A \subset D$  and  $A' \subset D'$ . We use the *matching cost* between points  $p = (b, d)$  and  $p' = (b', d')$  given by

$$c_m(p, p') = \max\{|b - b'|, |d - d'|\}.$$

Another way to write this is

$$c_m(p, p') = \|p - p'\|_\infty,$$

where  $\|\cdot\|_\infty$  is the  $\ell_\infty$ -norm, defined on an arbitrary vector in  $\mathbb{R}^2$  by

$$\|(x, y)\|_\infty = \max\{|x|, |y|\}.$$

We also use a cost for each unmatched point  $p = (b, d)$  given by

$$c_u(p) = \frac{d - b}{2}.$$

This is just the  $\ell_\infty$  distance from  $p$  to the diagonal line  $y = x$ .