TDA BREAKOUT: DAY 1 WORKSHEET

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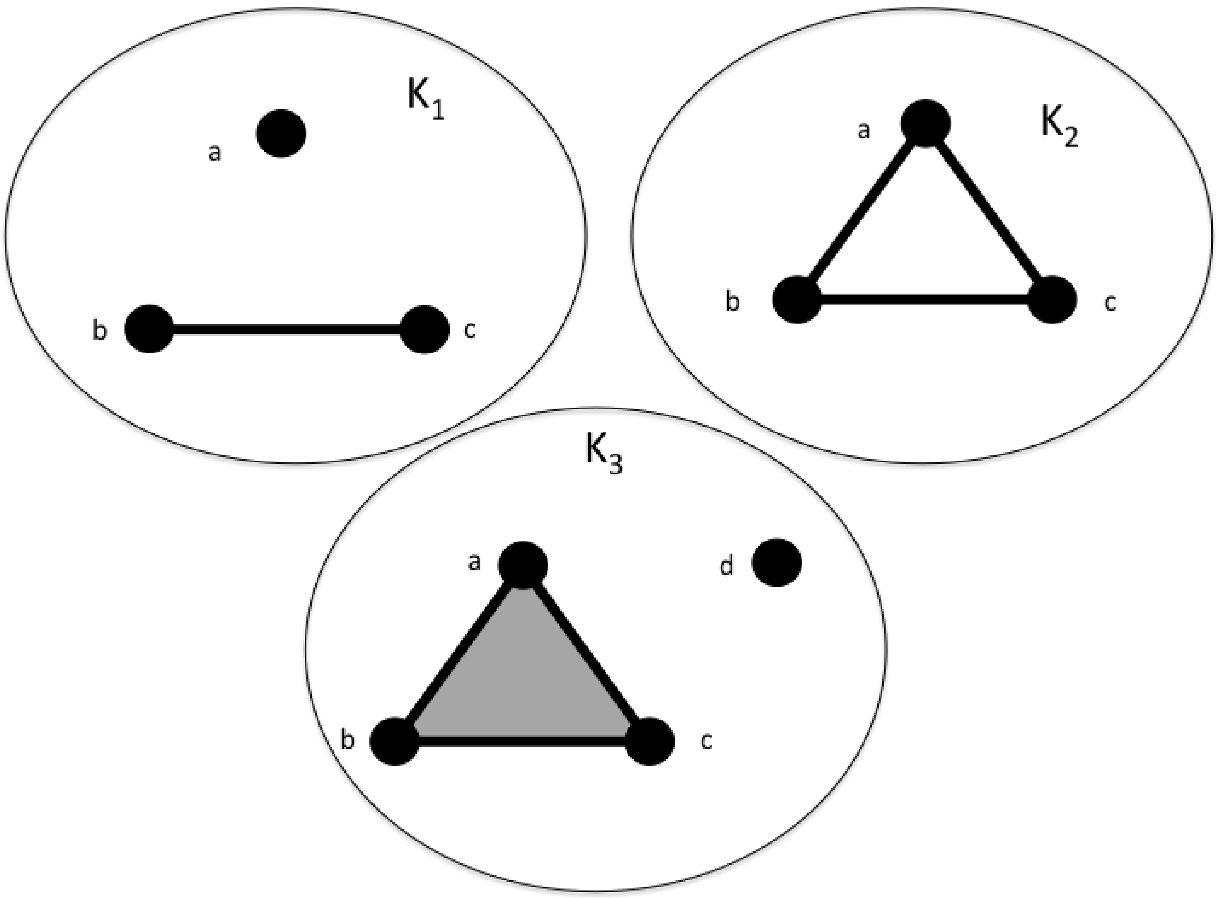
1. Computing simplicial homology



For each of simplicial complexes above:

(1) Write down the vertices, edges and faces (as subsets of the set of all vertices).

(2) For each dimension $d \leq 2$, write down the vector space C_d of all d-chains.

(3) For each dimension $d \leq 2$, write down the vector space Z_d of all cycles and the vector space B_d of all boundaries.

(4) For each dimension $d \leq 2$, compute the Betti number $\beta_d = \dim(Z_d) - \dim(B_d)$. (Does the answer agree with your intuition?)

(5) *(optional)* For each dimension d, write down the generators for the homology in dimension d.

Date: June 23, 2019.

2. Homomorphisms

Definition: A *simplicial map* from a simplical complex A to a simplicial complex B is a function f from the vertices of A to the vertices of B such that if $(v_1, v_2, \dots)$ is a simplex in A, then $(f(v_1), f(v_2), \dots)$ is a simplex in B.

Proposition: A simplicial map $f : A \to B$ always induces a linear map f_* from the homology of A to the homology of B.

Let's define two maps, one called f from the vertices of K_1 to the vertices of K_2, and one called g from the vertices of K_2 to the vertices of K_3. The definitions are easy to remember:

$$f(a) = a, \ f(b) = b, \ f(c) = c$$
$$g(a) = a, \ g(b) = b, \ g(c) = c$$

For each of these maps, answer the following three questions:

(1) Is this map a simplicial map?

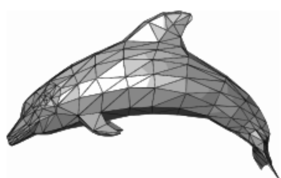
(2) Is the induced map on zeroth homology one-to-one? Is it onto?

(3) Is the induced map on first homology one-to-one? Is it onto?

(optional) Convince yourself that if f induces the map f_* on homology, g induces the map g_* on homology, then the composite $g \circ f$ induces the map $g_* \circ f_*$ on homology.

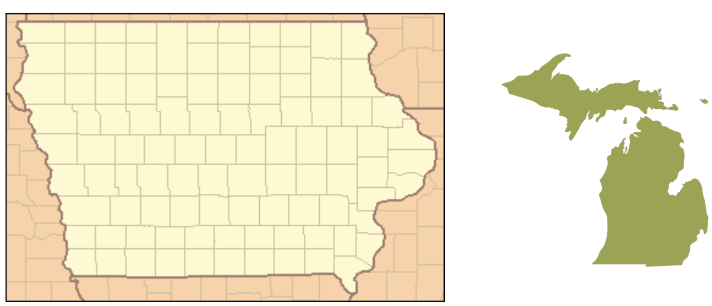
3. Modelling spaces via simplicial complexes

Not all spaces we care about are simplicial complexes, but we may want to approximate them by one. For example, here is one possible simplicial approximation of a dolphin from Wikipedia (such an approximation is called a "triangulation").



(1) Can you find good simplicial approximations for a circle? How about a (hollow) sphere, or a torus?

(2) Suppose I gave you a map of a state, say one of these *(which states are these?)*:



How would you go about making a simplicial complex which approximates it? You may use any extra information you like, such as GPS coordinates, census blocks or a dual graph. There are many possible answers.