The Metagraph

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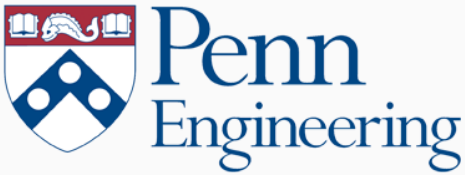

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Thanks to...

Metagraph
https://mggg.org/metagraph

Moon Duchin

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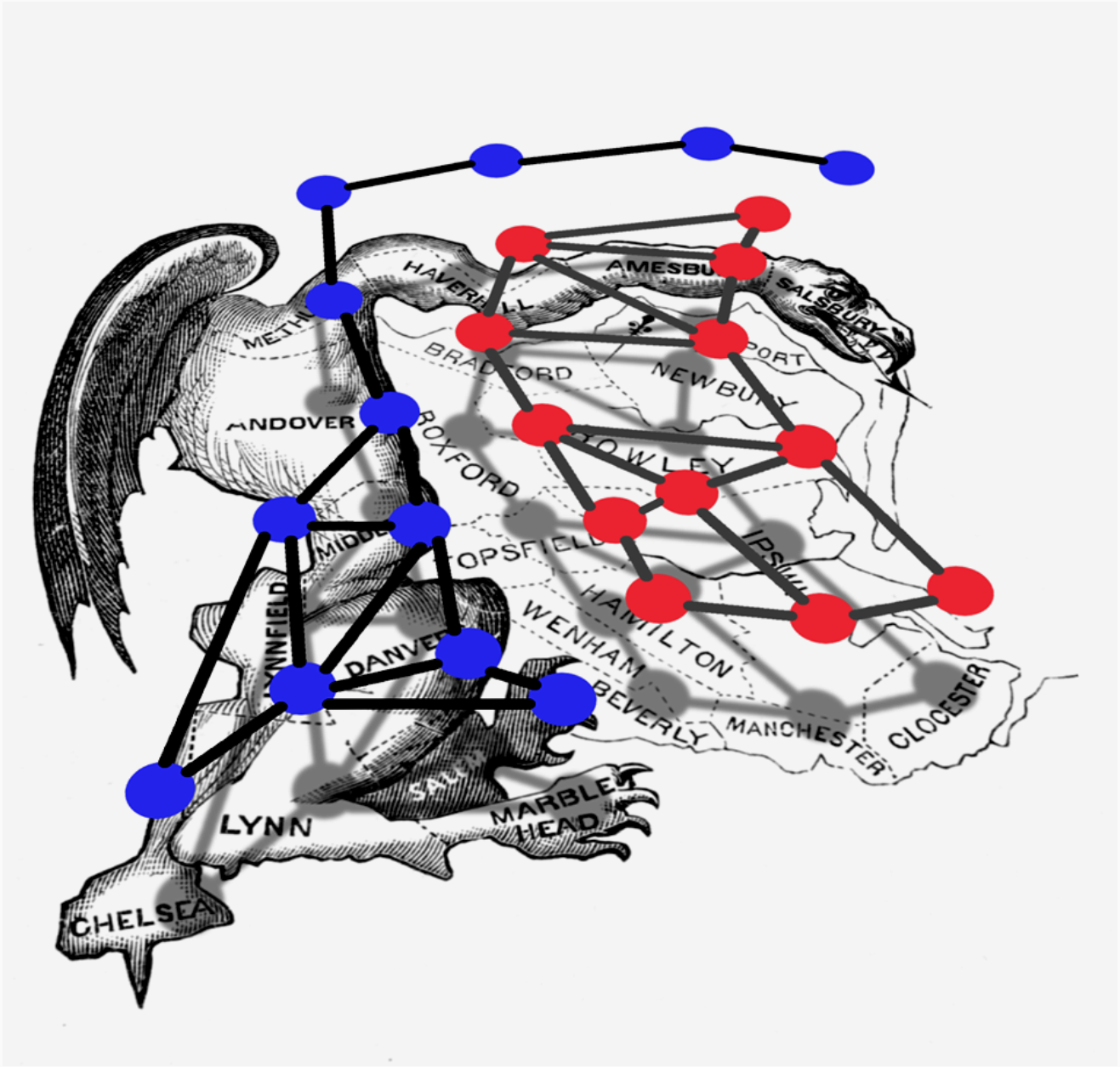
A Graph Theory Problem

Districts are formed from atomic geographic units (Census blocks)

Can we get anything by using the tools of *graph theory*?

Constructing the dual graph



The graph-theoretic definition

A **districting plan** (on a dual graph) is a *partition* of the vertices into k disjoint, connected pieces which satisfy the criteria we care about.

- equal population
- partisan, racial metrics
- not splitting towns

Hardness

Unfortunately, no matter how you slice it, redistricting is NP-Hard.

- Population constraints, racial and partisan metrics → SUBSETSUM
- Geometric constraints (minimize cut edges, e.g.) → MILP
- Optimization problems (find the ‘fairest’ plan …) → (combinatorial) *k*-KNAPSACK

However, NP-Hard doesn’t mean impossible.

The Dream

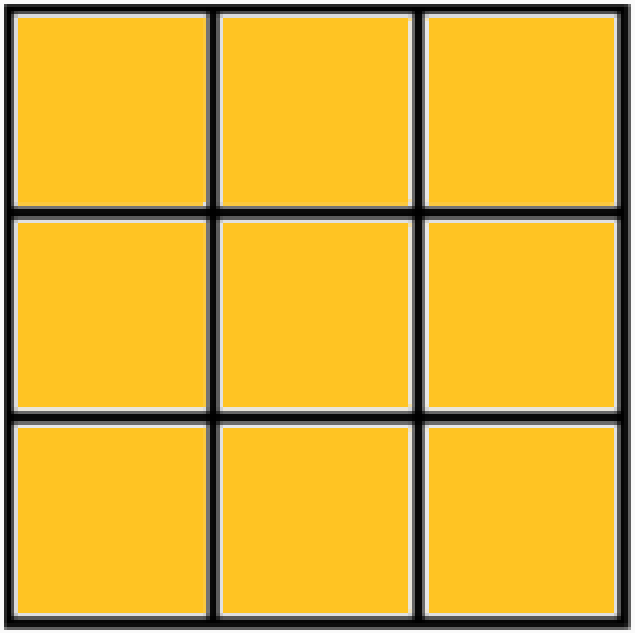
We’re working in a very particular corner of the universe of the problem.

Maybe [the problem] is easy for our setting?

Is there enough structure that we can just write down all of the plans and look at every one?

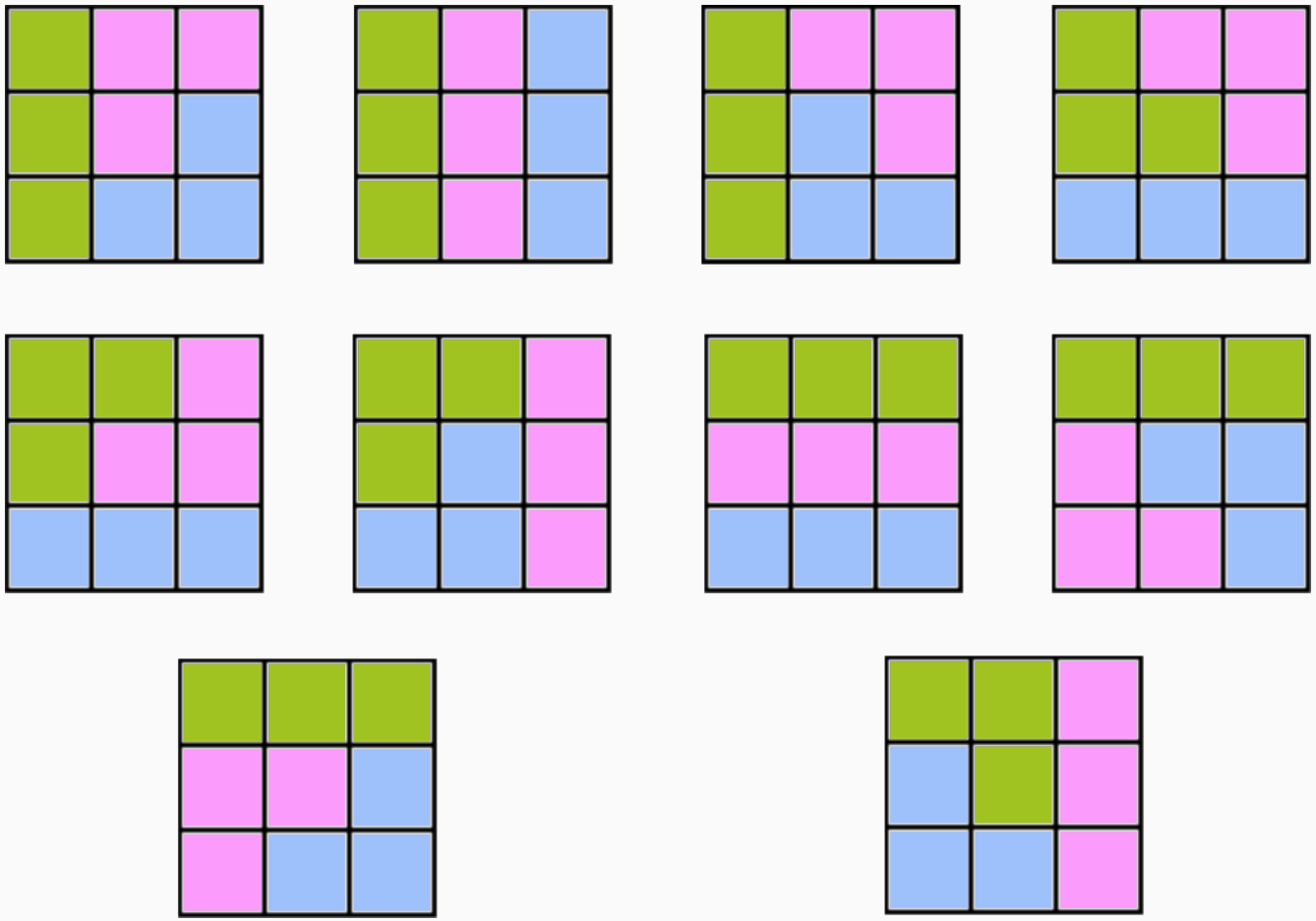
A Combinatorial Warmup

Let's take the fictional state of Gridlandia.



How many ways are there to divide Gridlandia into **3** connected pieces of size **3**?

Gridlandia Enumeration



And larger?

4x4 into 4 pieces of size 4? → 117

5x5 into 5 pieces of size 5? → 4006

6x6 into 6 pieces of size 6? → 451206

7x7 into 7 pieces of size 7? → 158753814 (10^8)

8x8 into 8 pieces of size 8? → 187497290034 (10^{11})

9x9 into 9 pieces of size 9? → 706152947468301 (10^{14})

10x10 into 10 pieces of size 10? → Open Problem!

Since we can't write down all the plans, we need some way of *sampling* them.

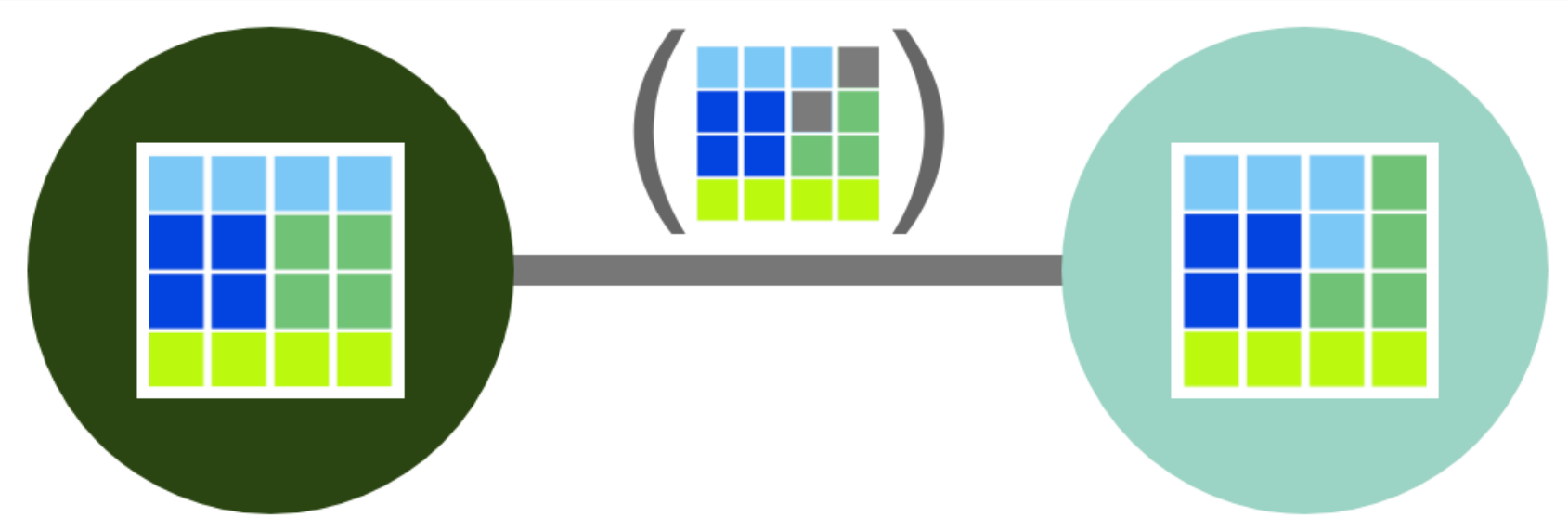
Constructing The Metagraph

Let's imagine a graph $\mathcal{M}$

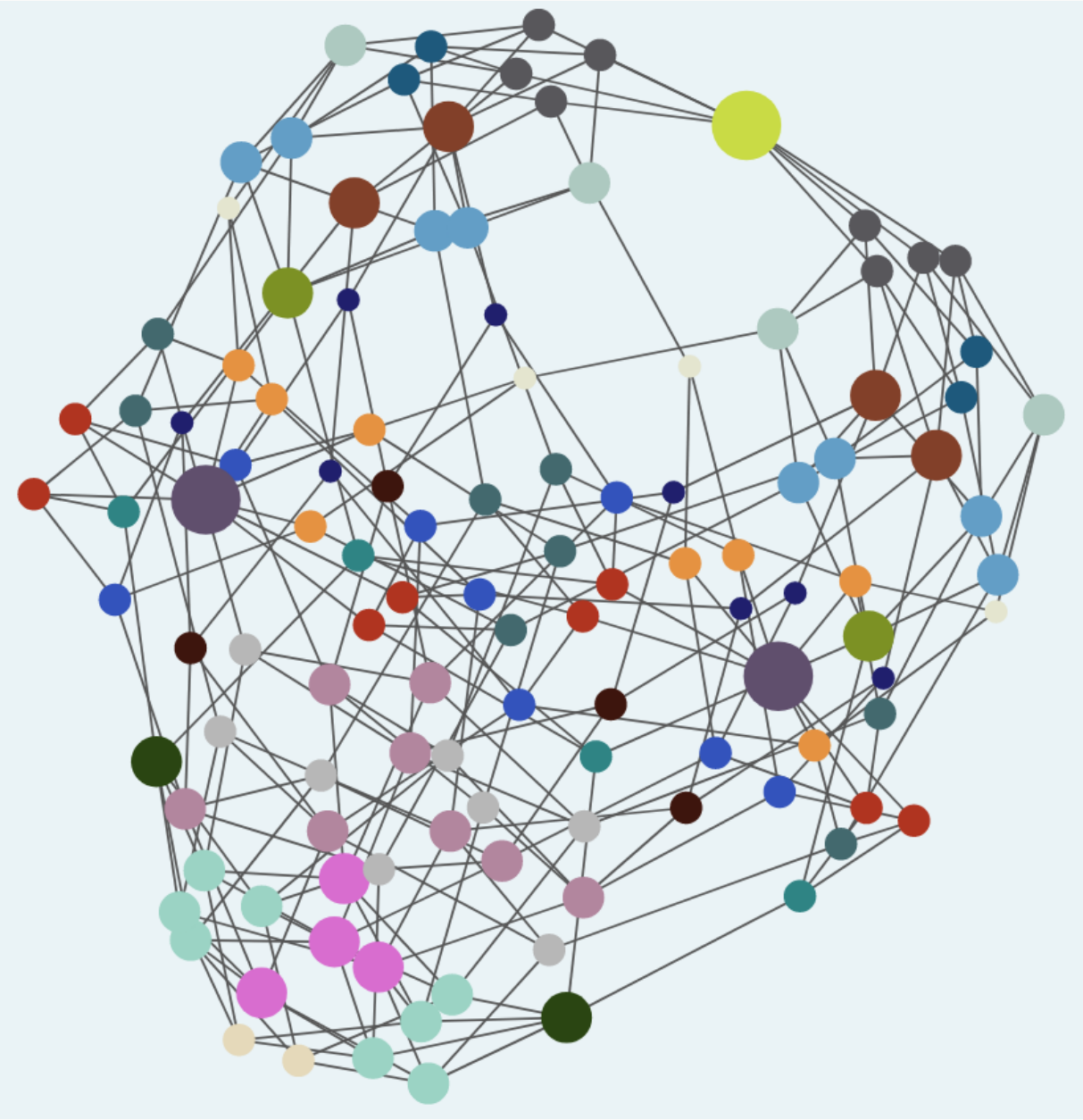
There is a vertex for each (valid) districting plan.

The edges are interesting. How do we define 'adjacent'?

A quick illustration



A quick illustration



INTERACTIVE

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