

POLITICAL THICKET, MATHEMATICAL QUAGMIRE

how voting is and is not a math problem

Moon Duchin @ Stanford University, May 2019

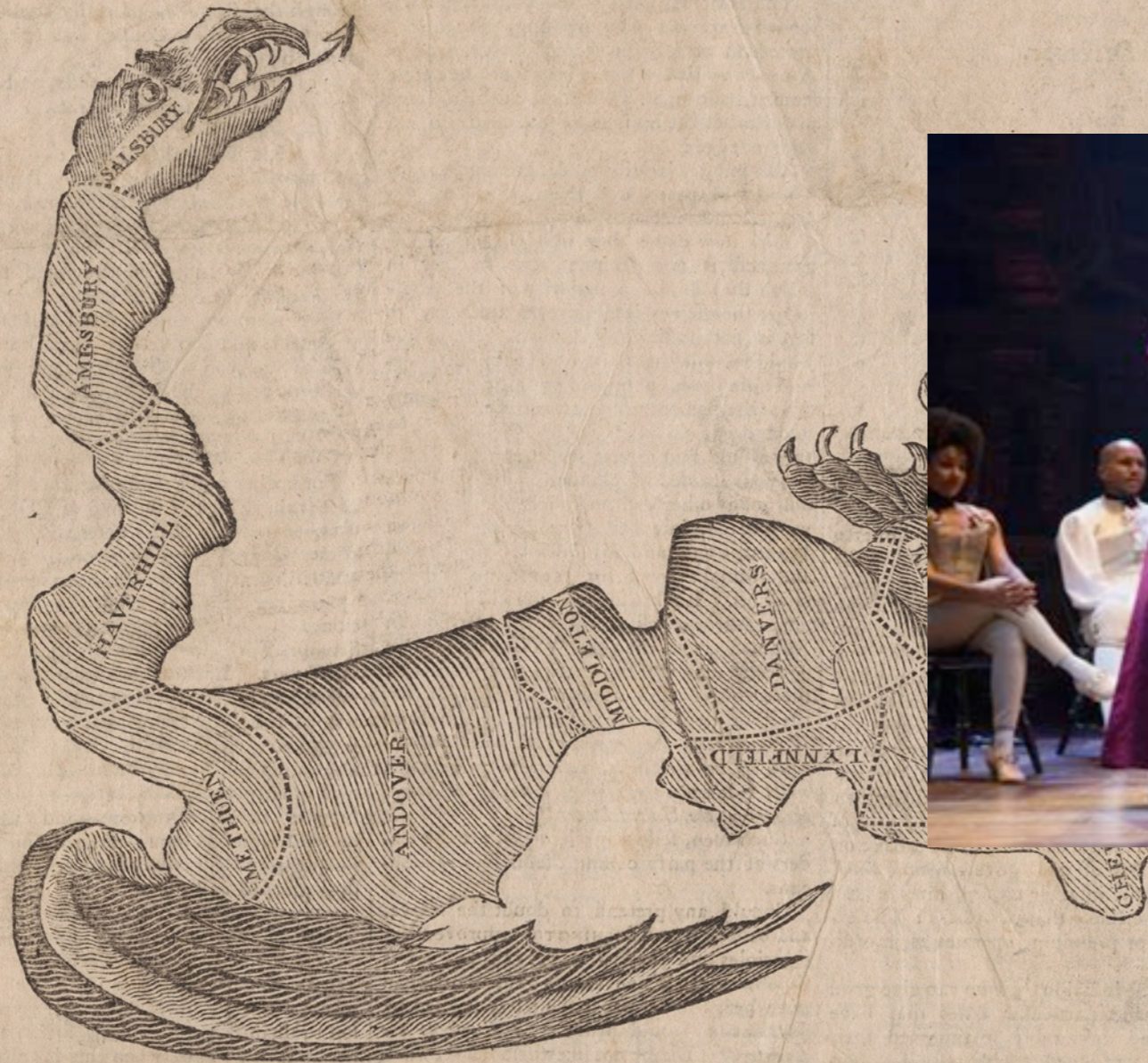
SALEM GAZETTE.

VOLUME XXVII.]

SALEM, FRIDAY, APRIL 2, 1813.

[No. 27.]

THE GERRY-MANDER, OR ESSEX SOUTH DISTRICT FORMED INTO A MONSTER!



requisition their utmost cunning to devise means expressly to accommodate their own views and to promote their own interest, should still be unwilling to adhere to the policy prescribed by themselves.

Such a man would have to learn, that violence and subordination must be employed to accomplish what dishonesty and trick had failed to secure. It is in fact the settled maxim of some, who, if the testimony of their own professions is to be credited, have much love for the people and a sacred regard for their rights, to contemn all legal provisions.



than he discovers a direct advancement of his own interest. It is greatly to be desired, that *honest* men would sometimes suffer themselves to be influenced as well by objects of *vision*, as *hearing*, and compare *actions* with *professions*; and then let judgment decide upon the worth of character, and the safety of reposing confidence. CIVIS.

Mr. PRINTER,—I am a native of *Great-Britain*, and some years ago was naturalized at *Gottenburg*; but for several months I have been engaged in trade in this State. I have seen very pressing notices of the Marshal for alien enemies to report themselves forthwith While I remain here I wish to comply with the laws and ordinances of the country, and

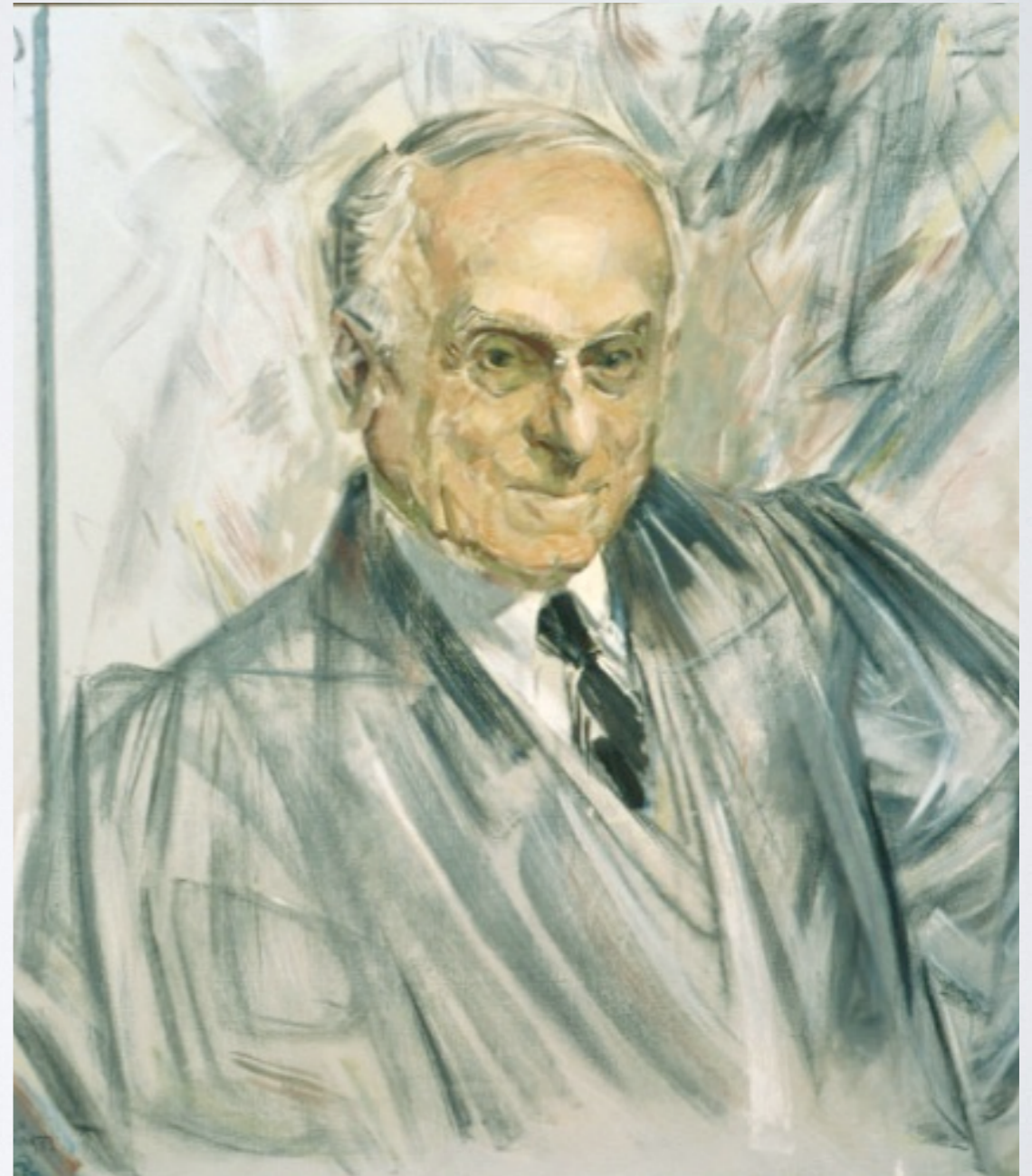
FEDERALISTS! FOLLOWERS OF WASHINGTON!

AGAIN behold and shudder at the exhibition of this terrific Dragon, brought forth to swallow and devour your Liberties and *equal* Rights. Unholy party spirit and inordinate love of power gave it birth—your patriotism and hatred of

FELIX FRANKFURTER

Colegrove v Green, 1946

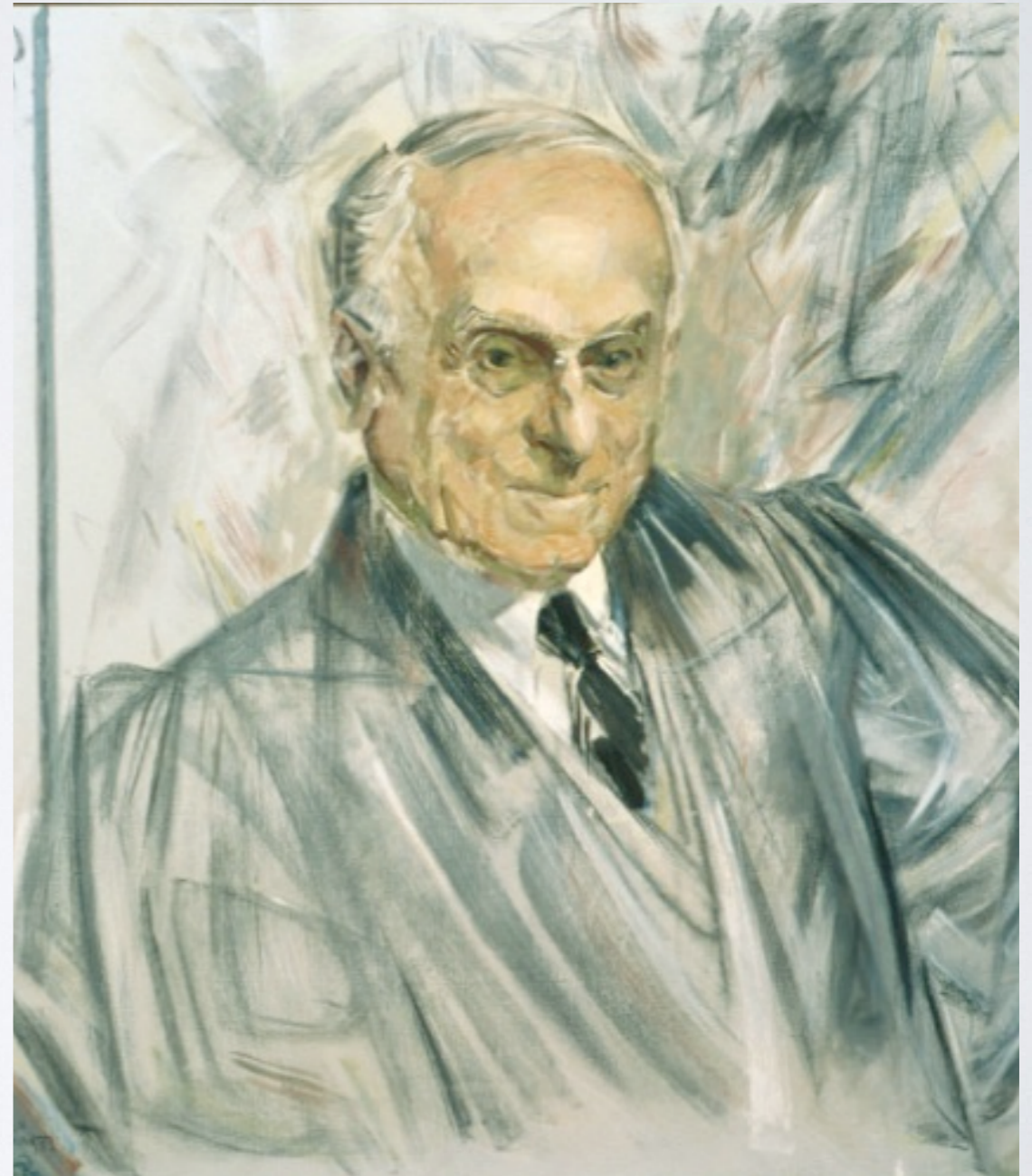
- “Throughout our history... the most glaring disparities have prevailed as to the contours and the population of districts.”
- “Courts ought not to enter this **political thicket.**”



FELIX FRANKFURTER

Baker v Carr, 1962

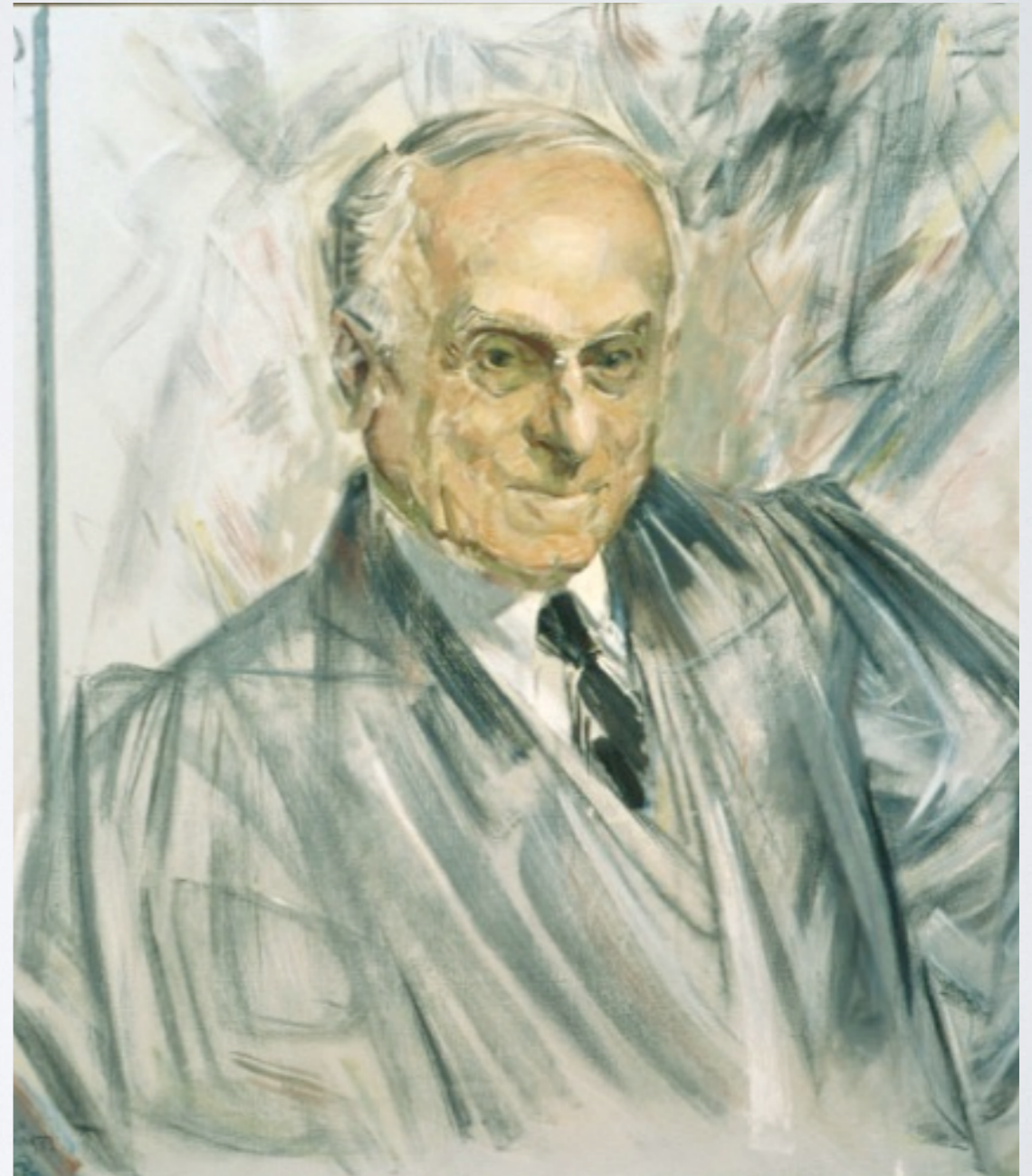
- “One of the Court's supporting opinions, as elucidated by commentary, unwittingly affords a disheartening preview of the **mathematical quagmire** (apart from divers judicially inappropriate and elusive determinants) into which this Court today catapults the lower courts of the country without so much as adumbrating the basis for a legal calculus as a means of extrication.”



FELIX FRANKFURTER

Baker v Carr, 1962

- “This [intervention], of course, implies that **geography, economics, urban-rural conflict**, and all the other non-legal factors which have throughout our history entered into political districting are to some extent not to be ruled out in the undefined vista now opened up by review in the federal courts of state reapportionments.”



8-1 MAJORITY

Reynolds v Sims, 1964

- “We are told that the matter of apportioning representation in a state legislature is a complex and many-faceted one. We are advised that States can rationally consider factors other than population in apportioning legislative representation. We are admonished not to restrict the power of the States to impose differing views as to political philosophy on their citizens. We are cautioned about the dangers of entering into political thickets and mathematical quagmires. Our answer is this: **a denial of constitutionally protected rights demands judicial protection; our oath and our office require no less of us.**”

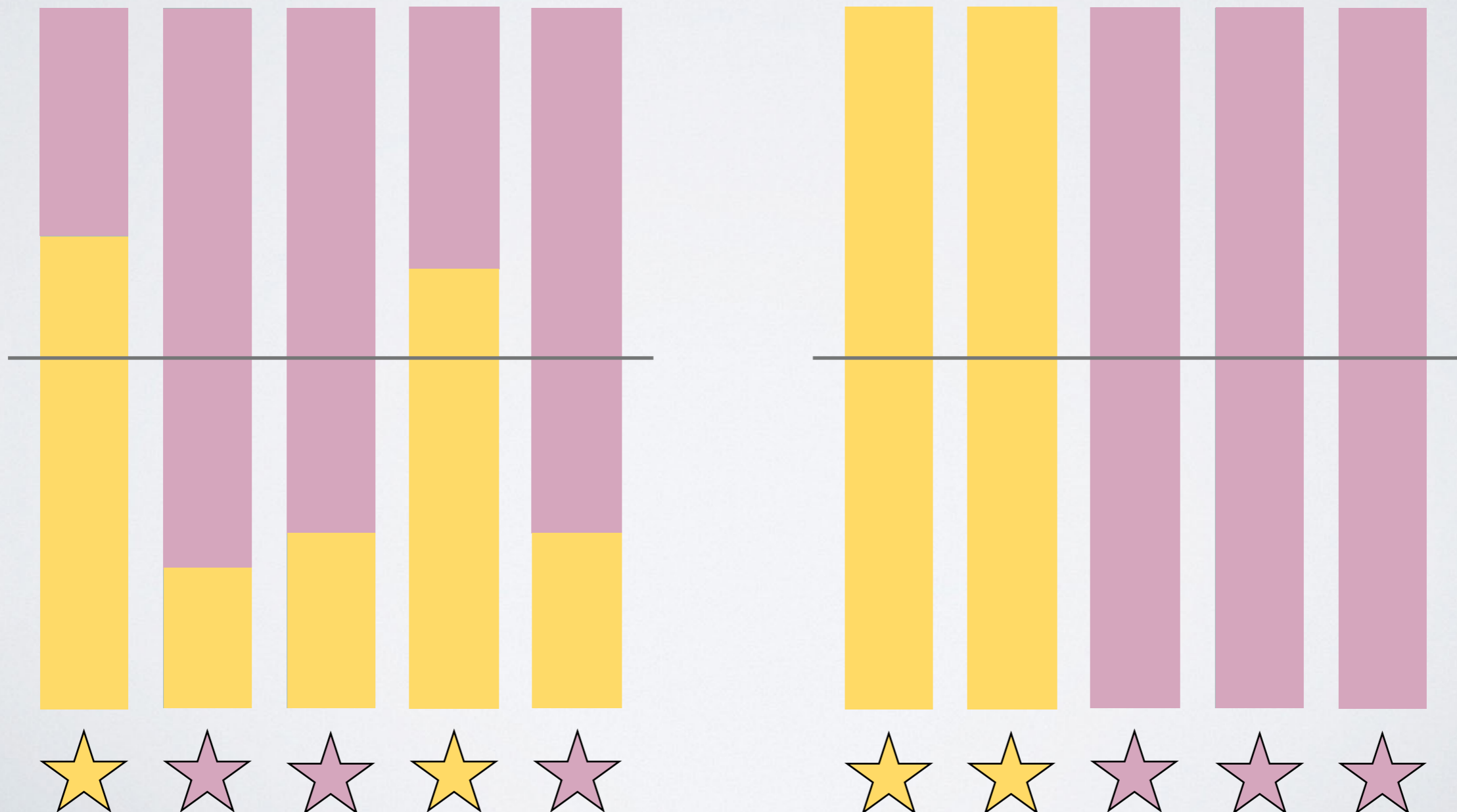


OK SO HOW HARD IS THE
MATH ANYWAY?

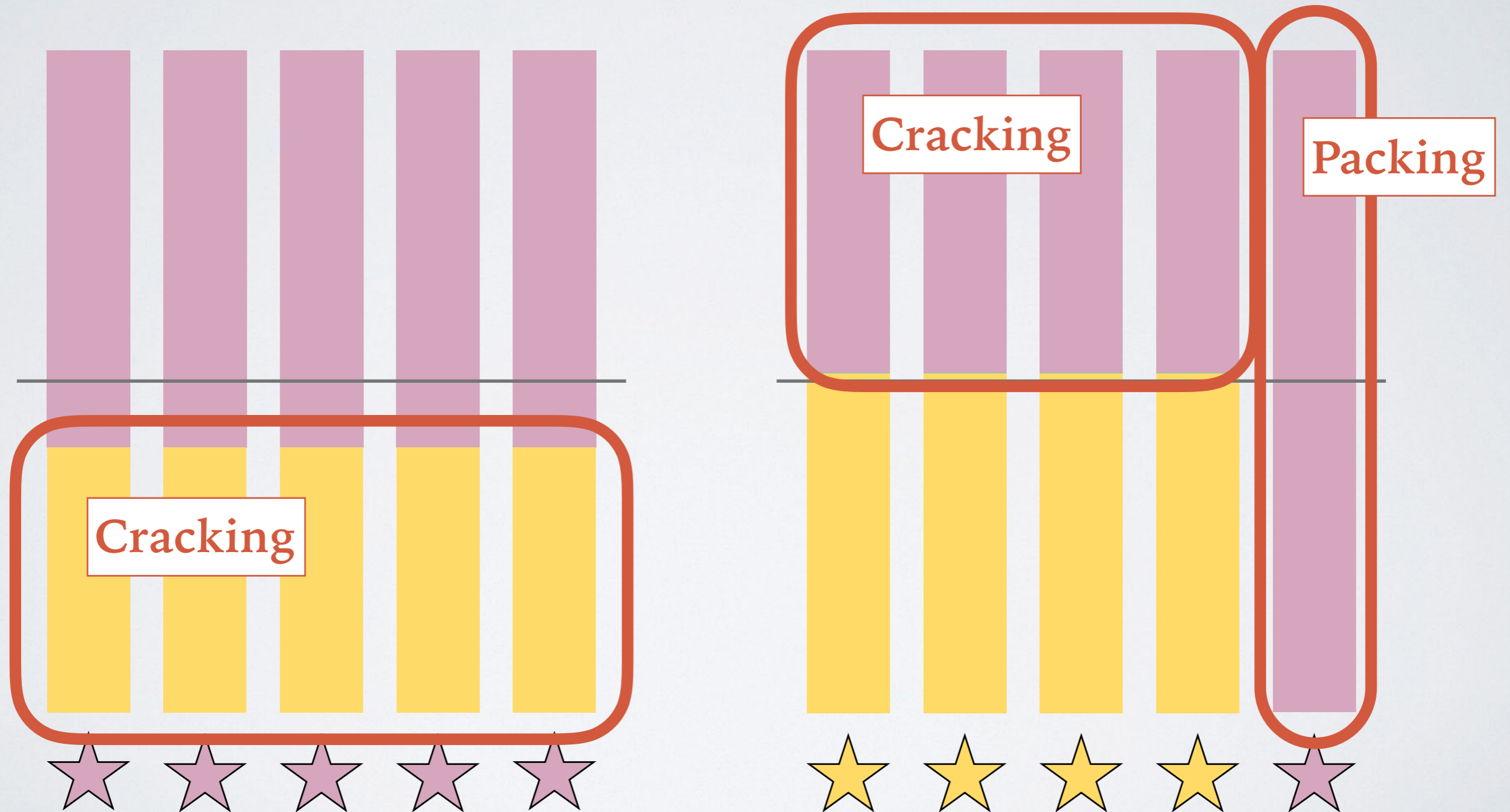
Suppose **orange** has about 40% of the vote vs **pink**.

You might expect a fair outcome to be 40% orange seats
– proportionality.

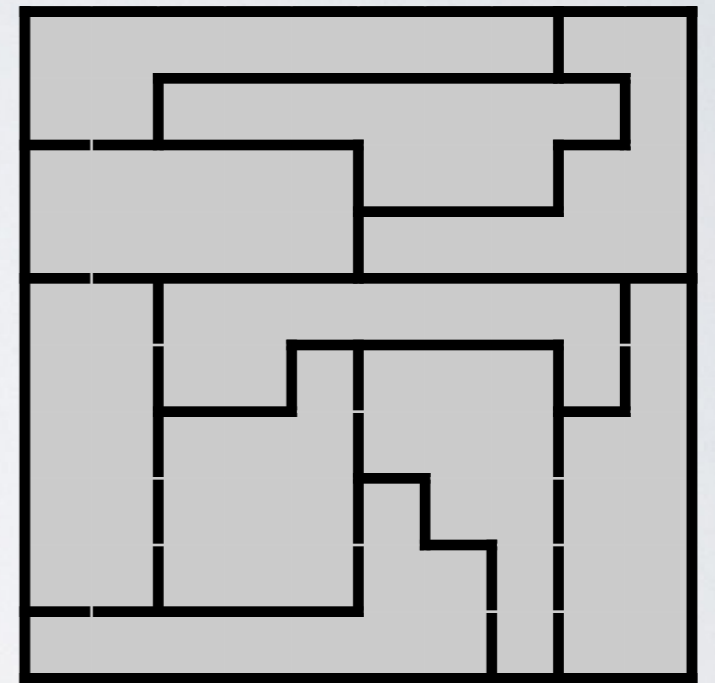
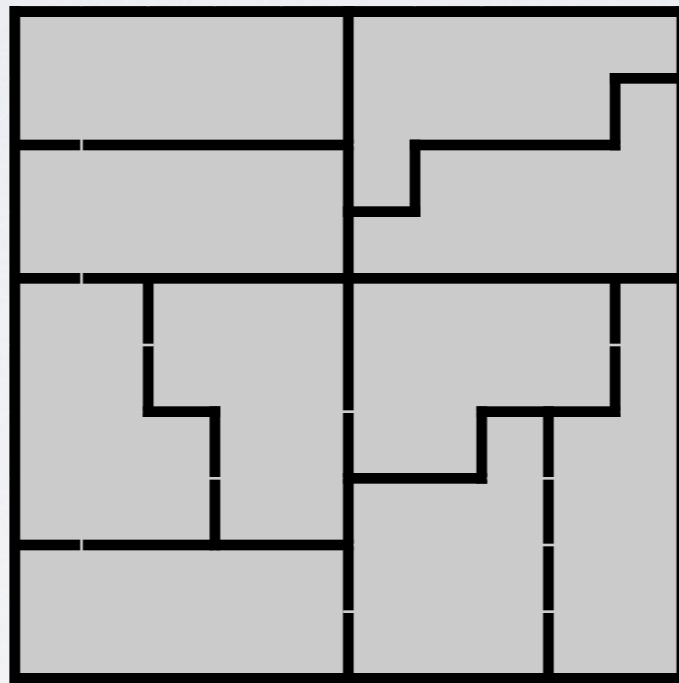
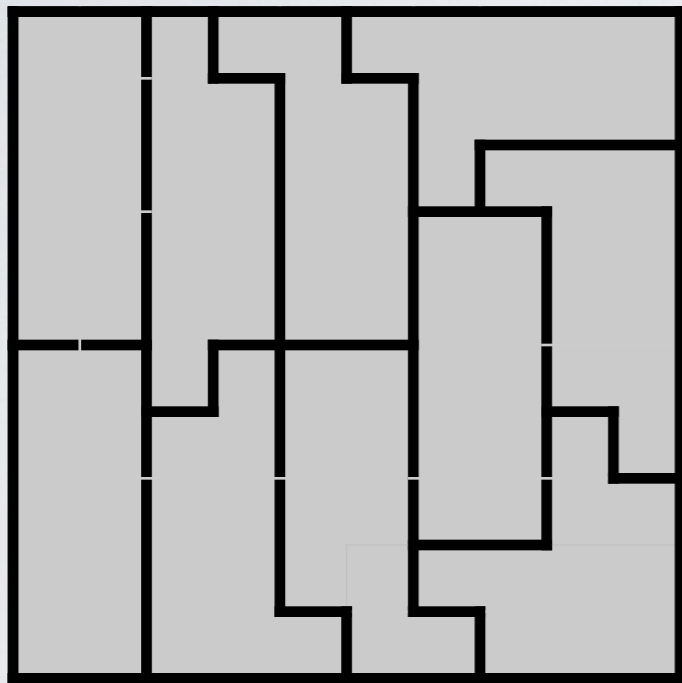
First note: proportionality can occur in many ways that are
qualitatively different to live in.



But it's hardly guaranteed—a roughly 40% orange vote can be arranged to get anywhere from 0-80% of the representation.



Maybe geometry will save us? – discreteness and geography might limit the power of the gerrymanderers, and bad shapes might give them away.



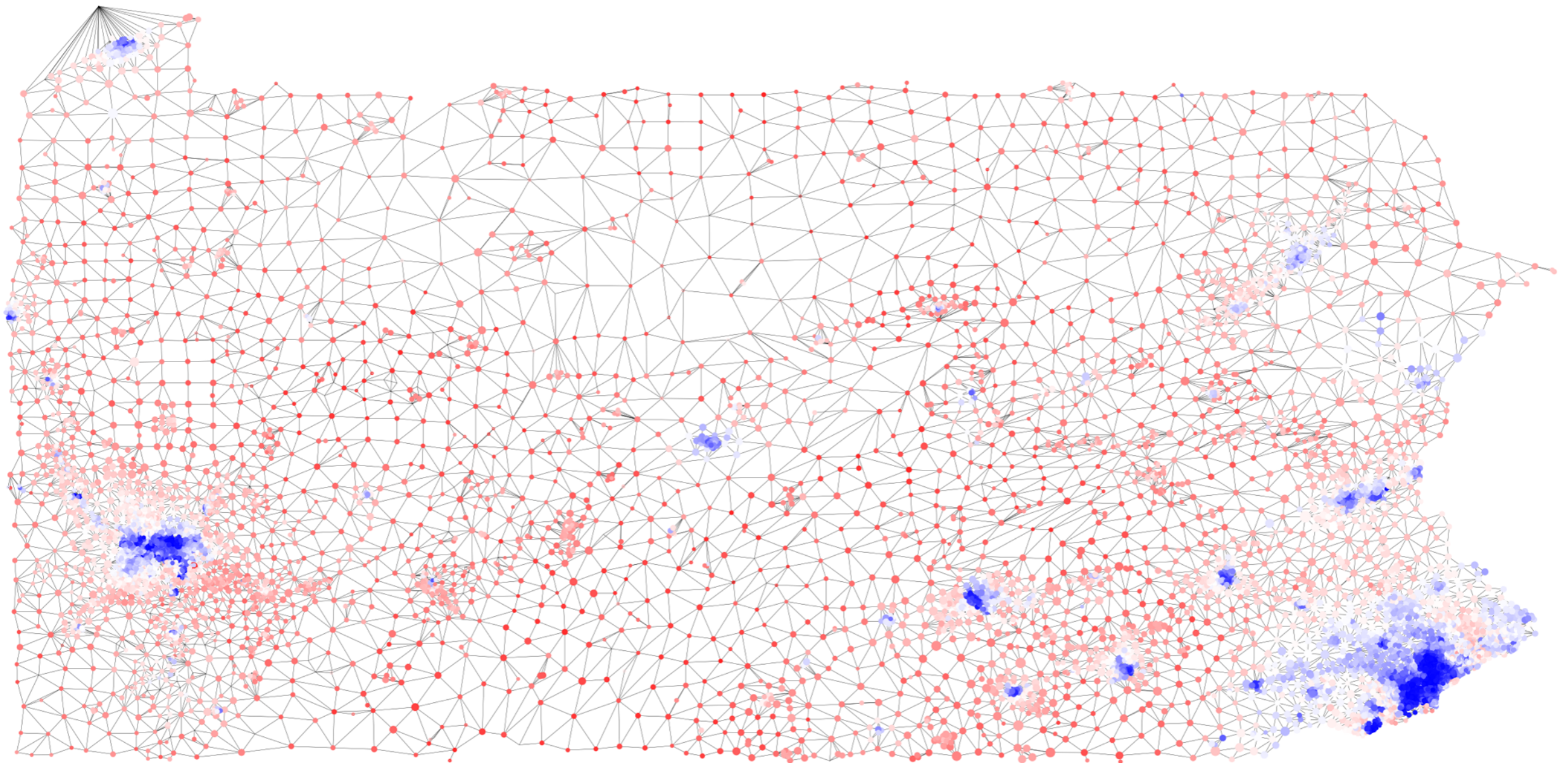
Real-sized problem: 9059 precincts of Pennsylvania

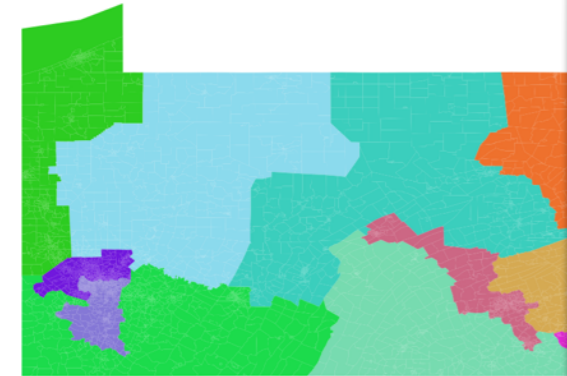
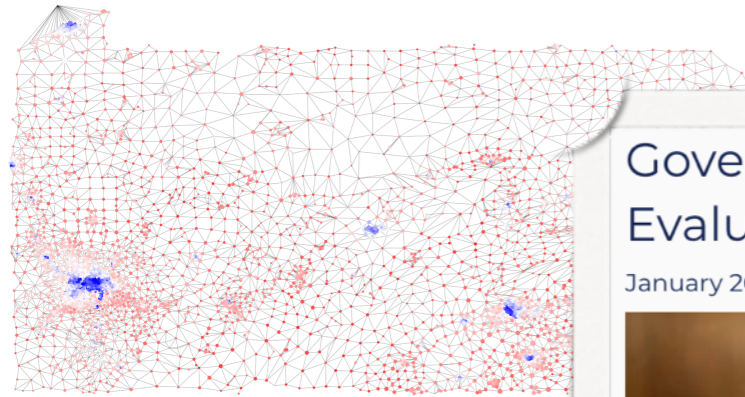
18 Congressional districts

*421,545 census blocks

50 State Senate districts

203 State House districts





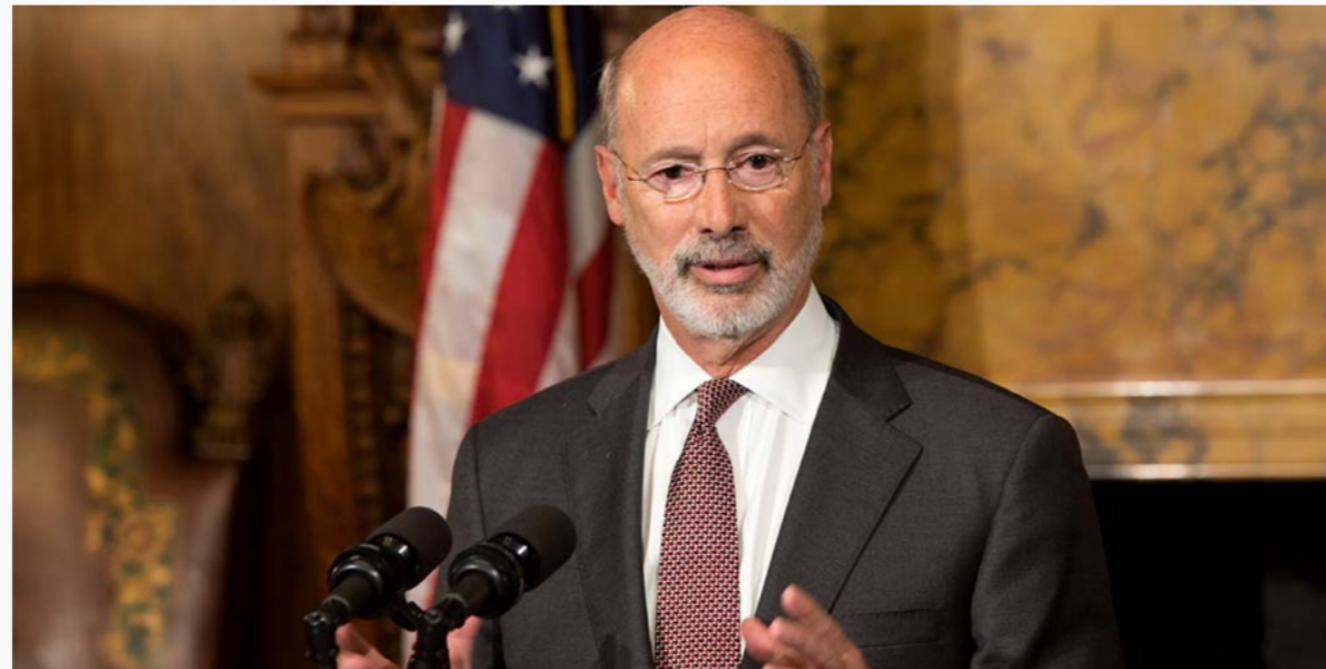
538 GOP



Gov

Governor Wolf to Enlist Non-Partisan Mathematician to Evaluate Fairness of Redistricting Maps

January 26, 2018



NATIONAL ISSUES, PRESS RELEASE, REDISTRICTING, VOTING & ELECTIONS

Harrisburg, PA – Governor Wolf today announced he will enlist a non-partisan mathematician, Moon Duchin, Ph.D. an Associate Professor of Mathematics from Tufts University, to provide him guidance on evaluating redistricting maps for fairness. Governor Wolf has made clear since the Supreme Court ruled the map unconstitutional that he saw this as an opportunity to eliminate partisan gerrymandering and deliver the people of Pennsylvania a fair Congressional map.

“Moon Duchin has been a leader in applying mathematics, geometry, and analytics to evaluate redistricted maps and work to eliminate extreme partisan gerrymandering,” Governor Wolf said. “The people of Pennsylvania are tired of partisan games and gridlock – made worse by gerrymandering – and it is my mission to reverse the black-eye of having some of the worst gerrymandering in the country. I am open and willing to work with the General Assembly but I will not accept an unfair map and enlisting a non-partisan expert is essential to ensure that is possible.”

Stanford



2018 Do-over



538 Compact



8th Graders

THE UNIVERSE OF POSSIBLE PLANS

How many are there?

Depends on the rules.

population balance

contiguity

compactness

racial fairness

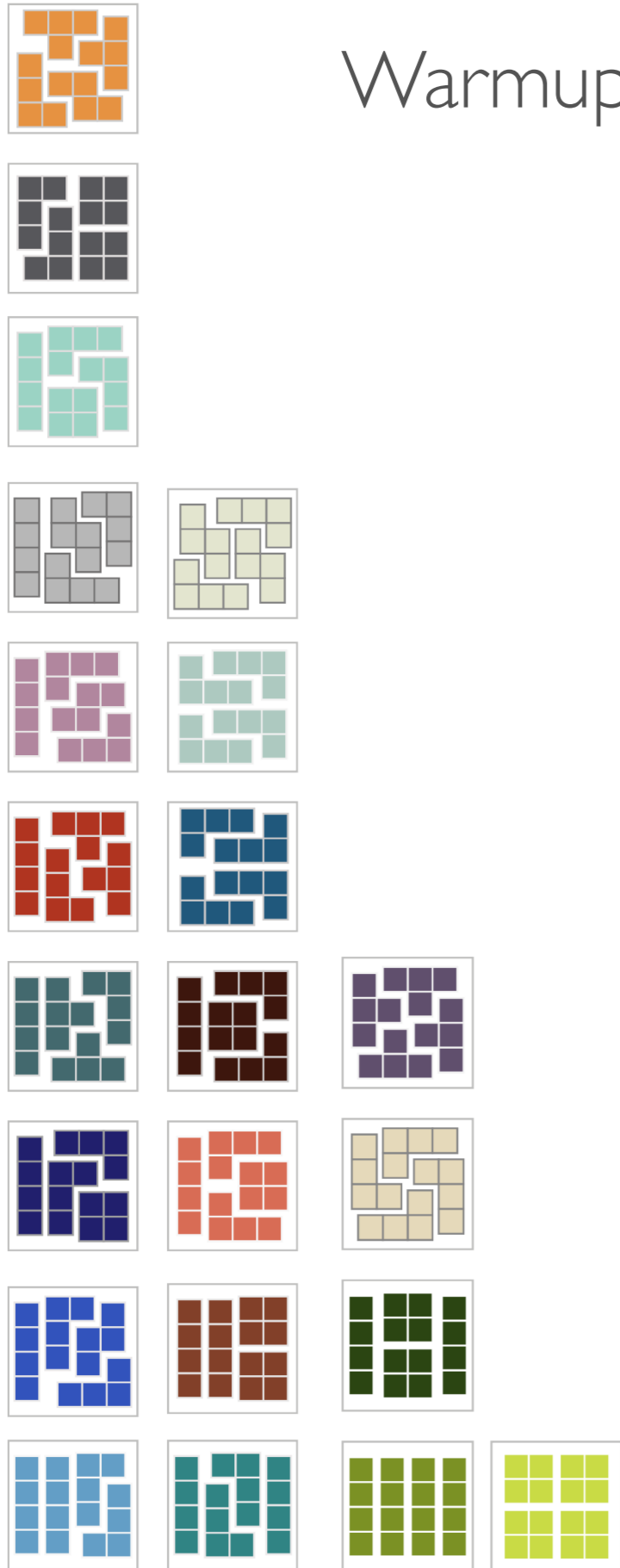
respect for boundaries

respect for communities

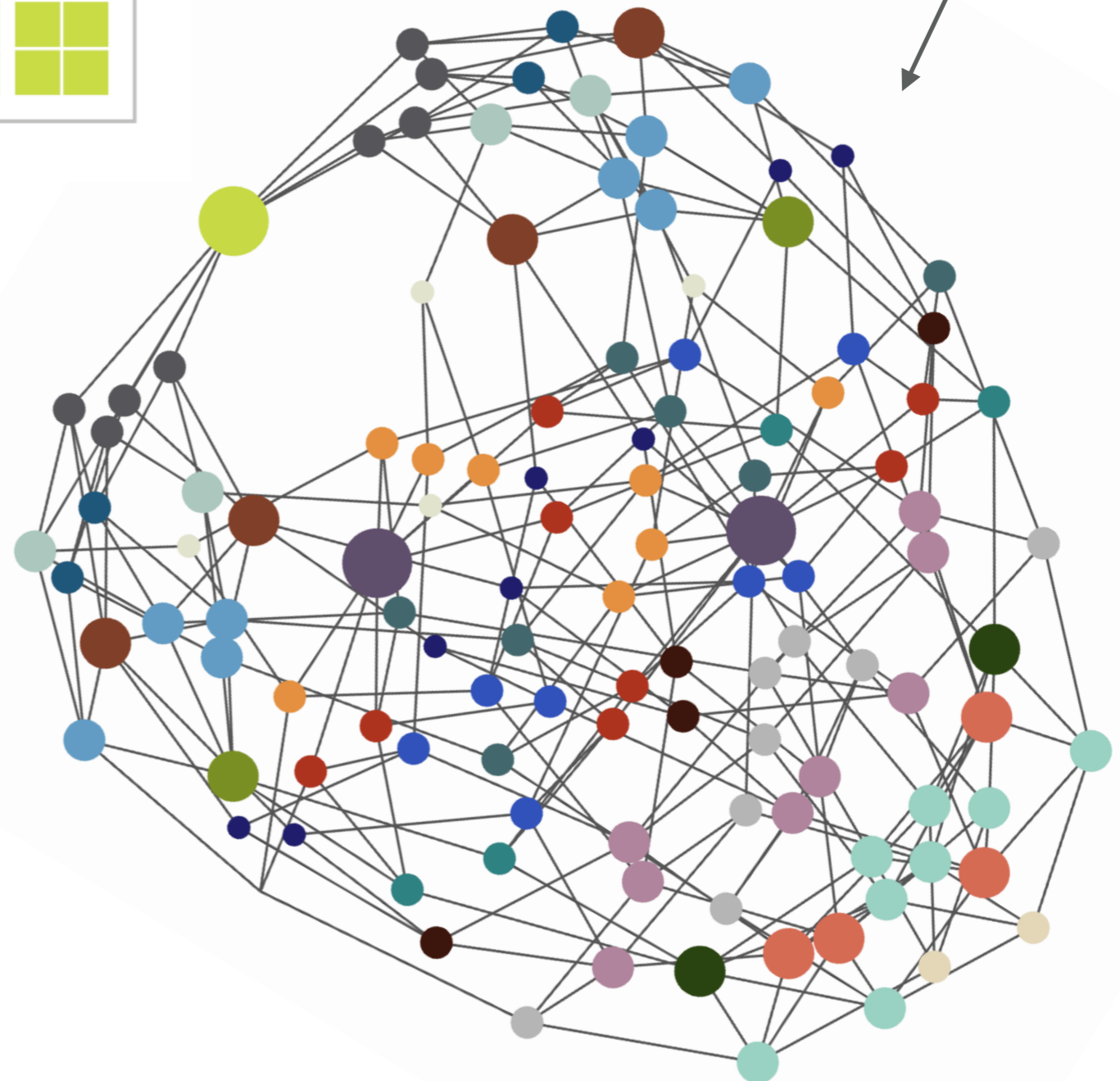
⋮

9 JUSTICE ALITO: No, no, but can you
10 just answer that -- that question, because it's
11 a real puzzle to me. So you've got -- let's
12 say you've got 100 maps or you might even have
13 25. I think you probably have thousands.

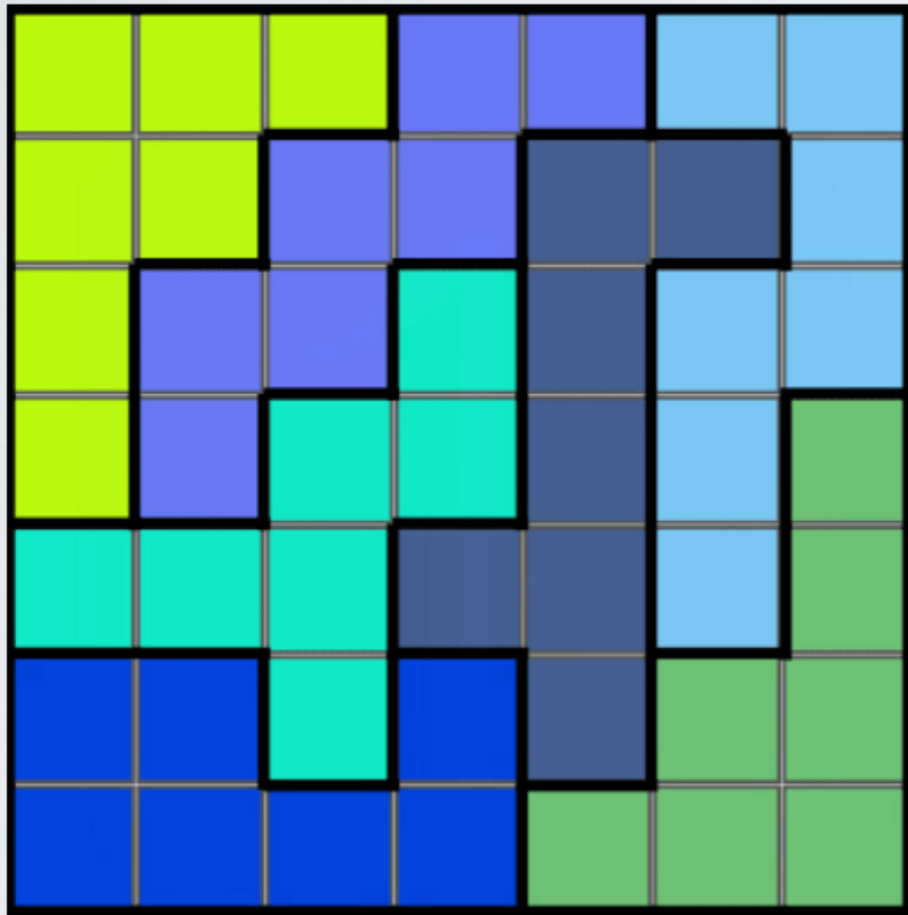
Warmup: 4x4 grid



the **swap**
network

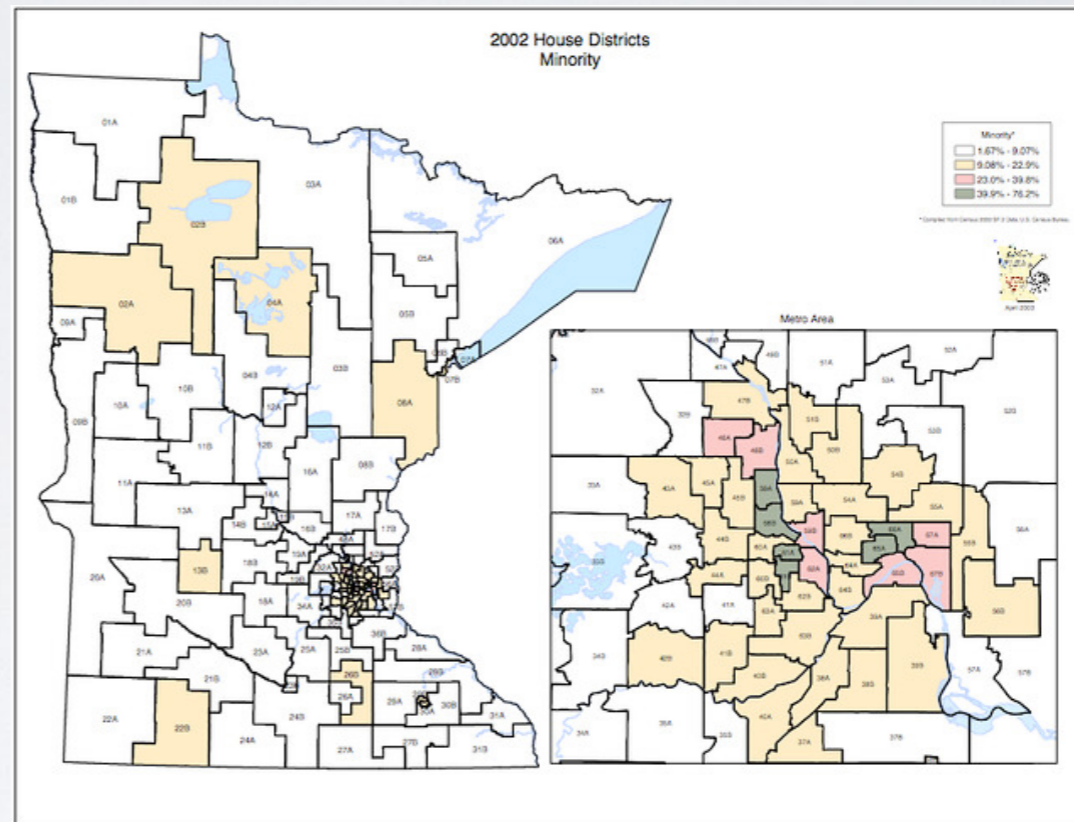


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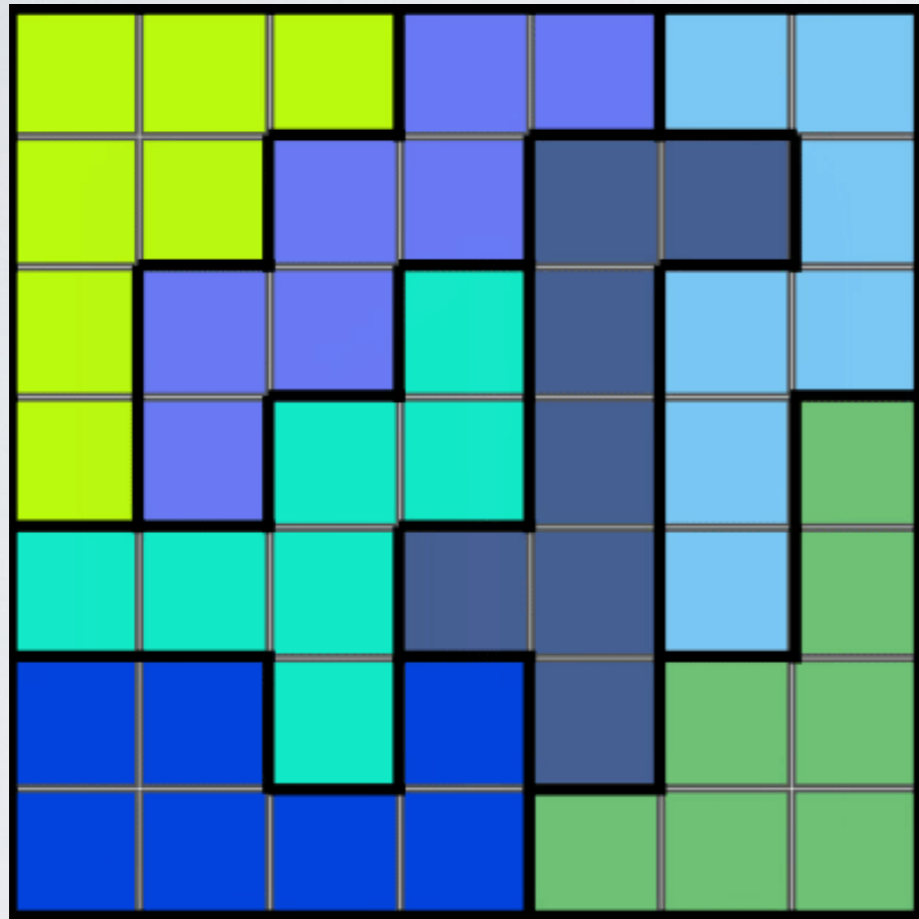
7x7 grid into 7 districts:
158,753,814 ways

How many ways to make 67 state Senate districts in Minnesota?



just from pairing:
6,156,723,718,225,577,984

EXPLORING AT RANDOM



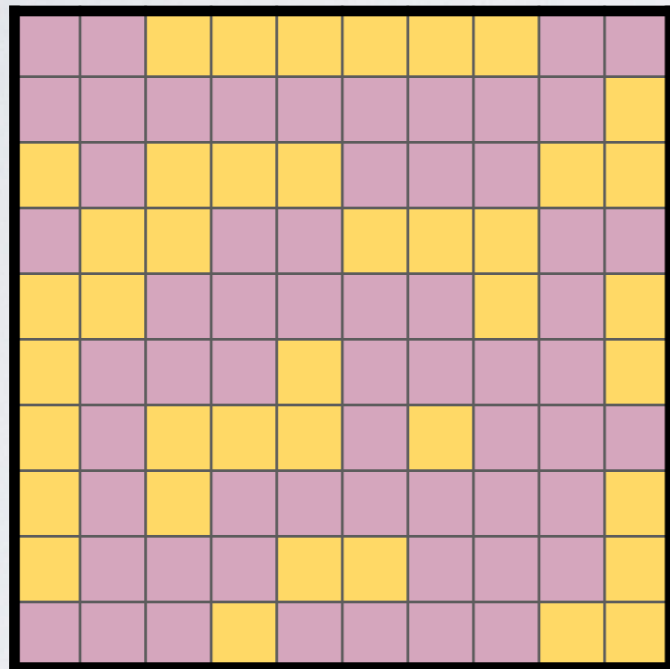
7x7 grid into 7 districts:
158,753,814 ways

explore with **Markov chain** that
makes swaps at random

Recall:

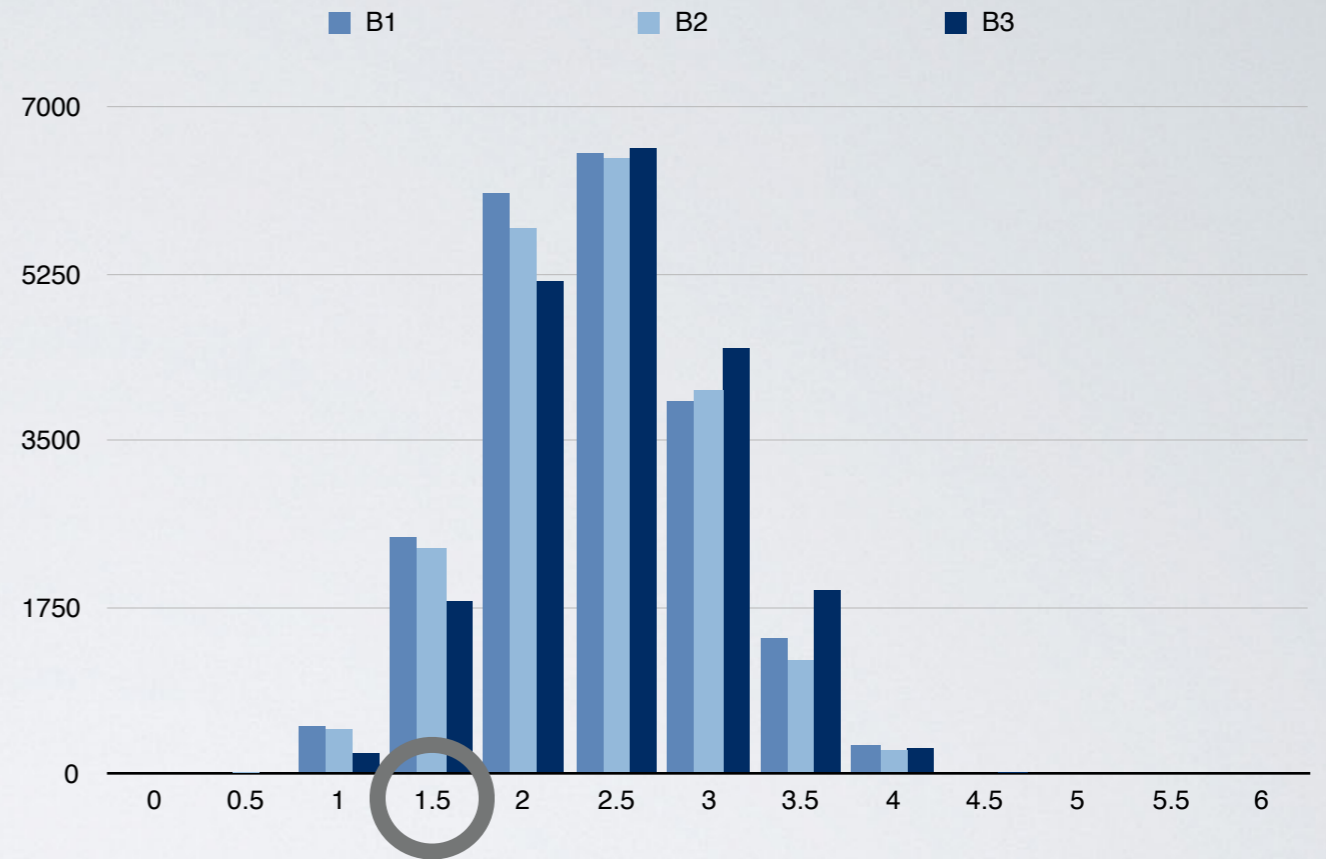
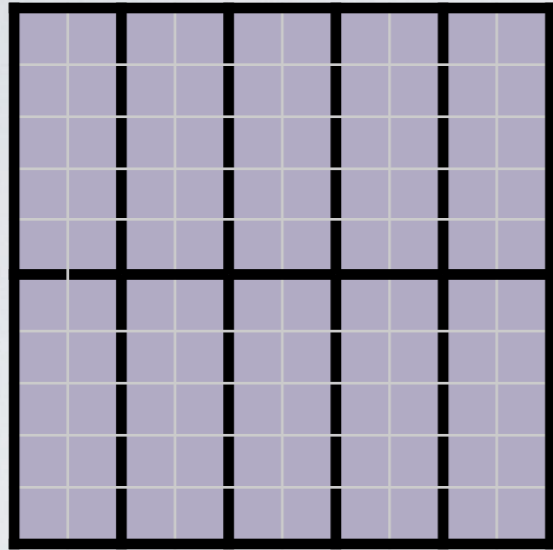
40% orange votes

How many seats to expect?

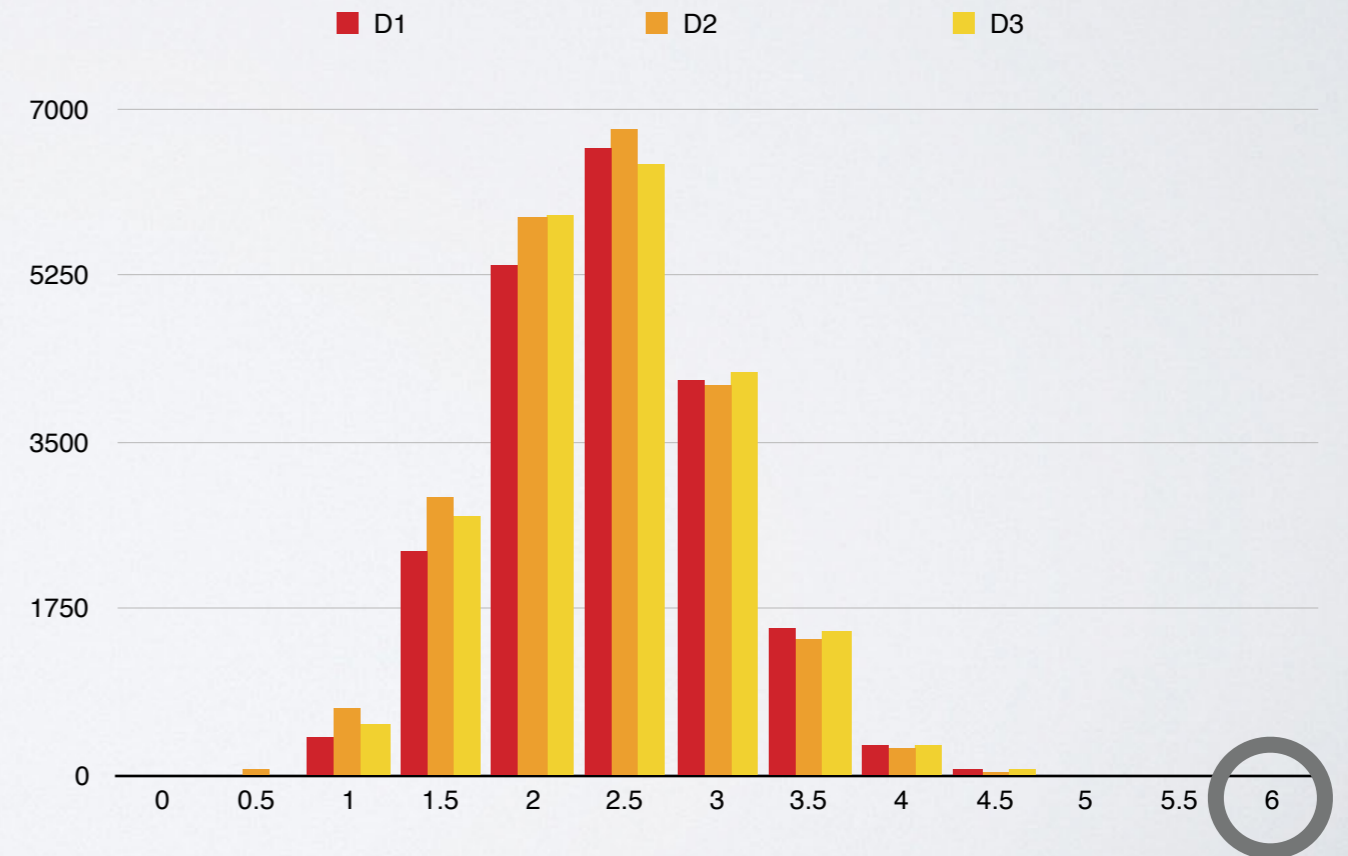
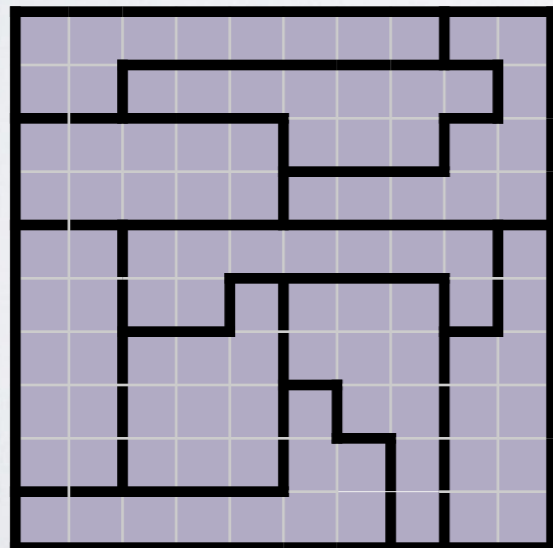


For the 10x10 grid, even though we have so far failed to fully enumerate the valid plans, we seem to **explore** effectively in just 10,000 Swap steps.

Start with rectangles (1.5 orange seats)

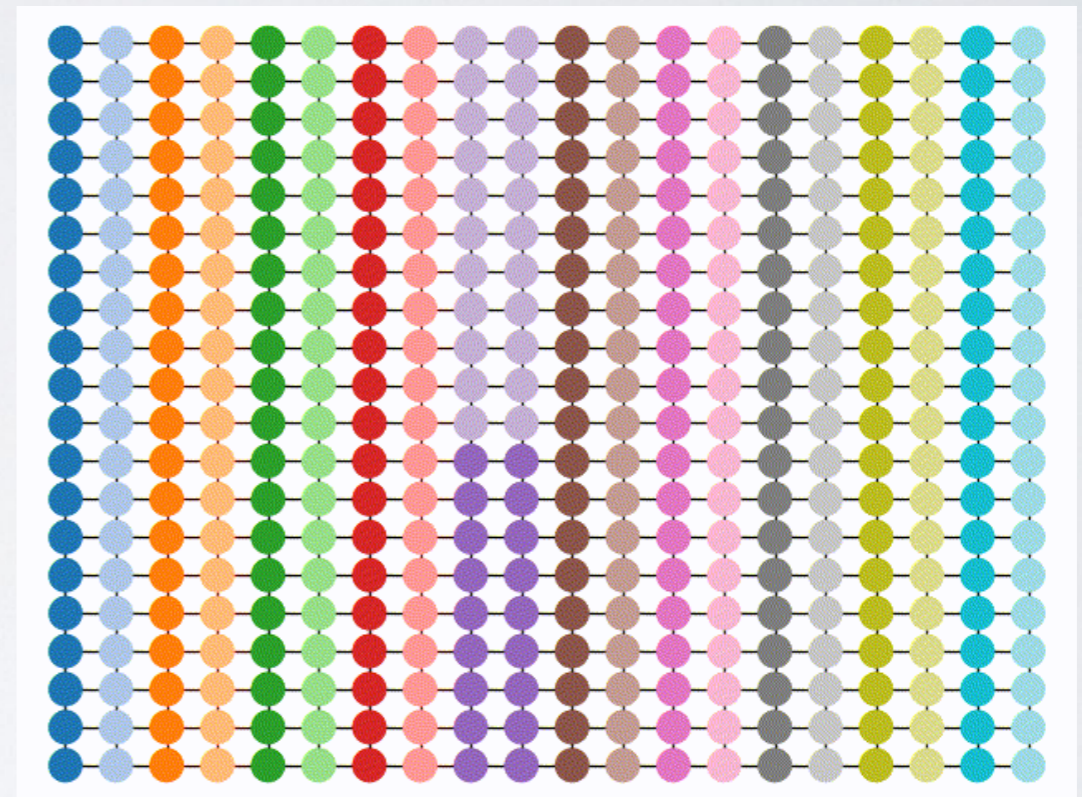
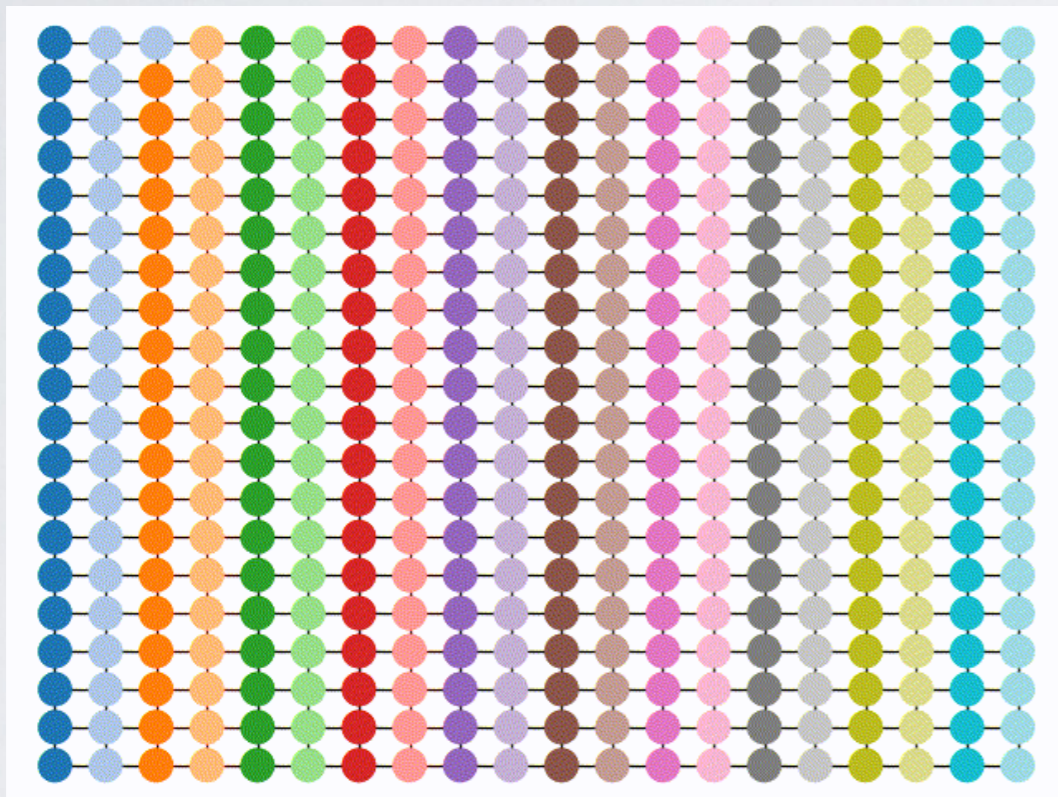


Start with "plan O" (6 orange seats)

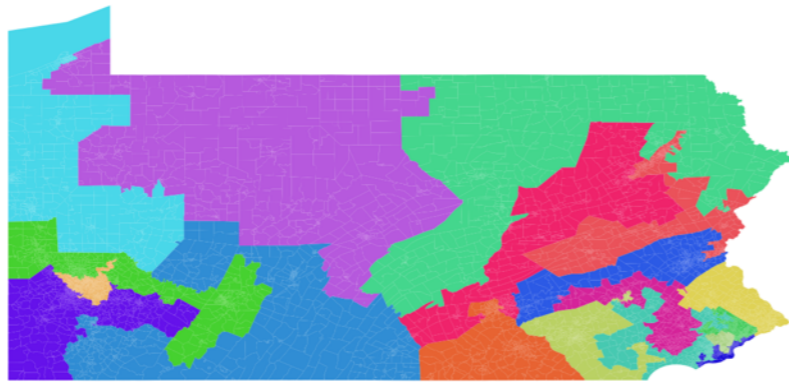


The bad news: Flip/Swap scales up badly

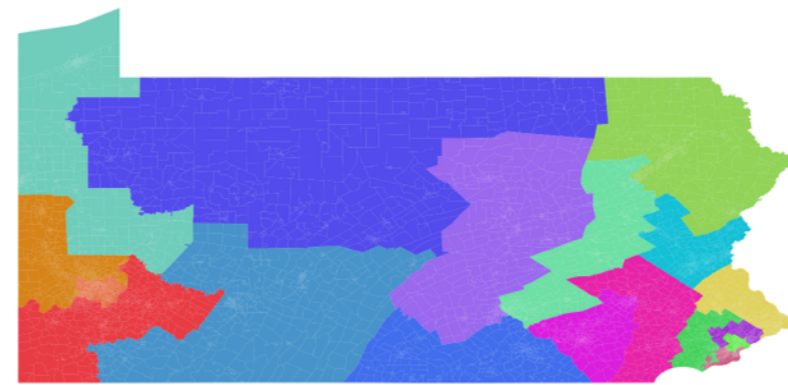
The good news: there are other Markov chains to try



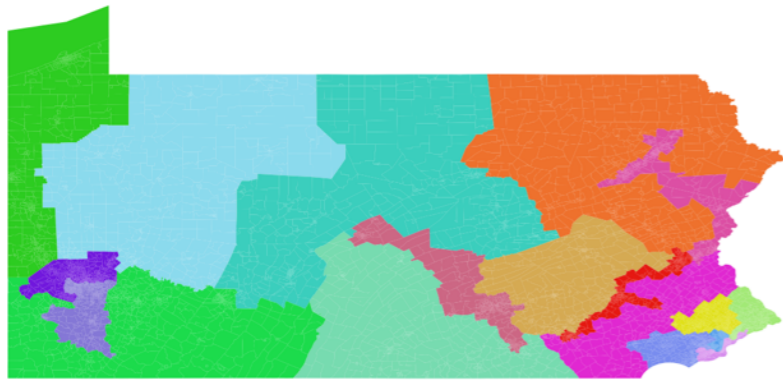
Two Markov chains:
Flip vs **Recombination**



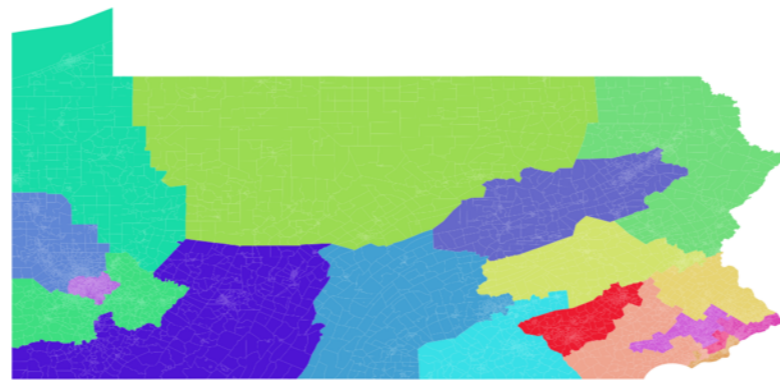
2011



2018 Do-over



538 GOP



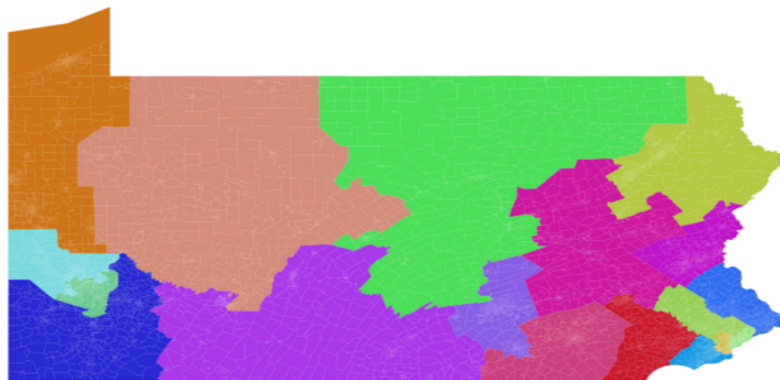
538 Dem



538 Compact



Gov

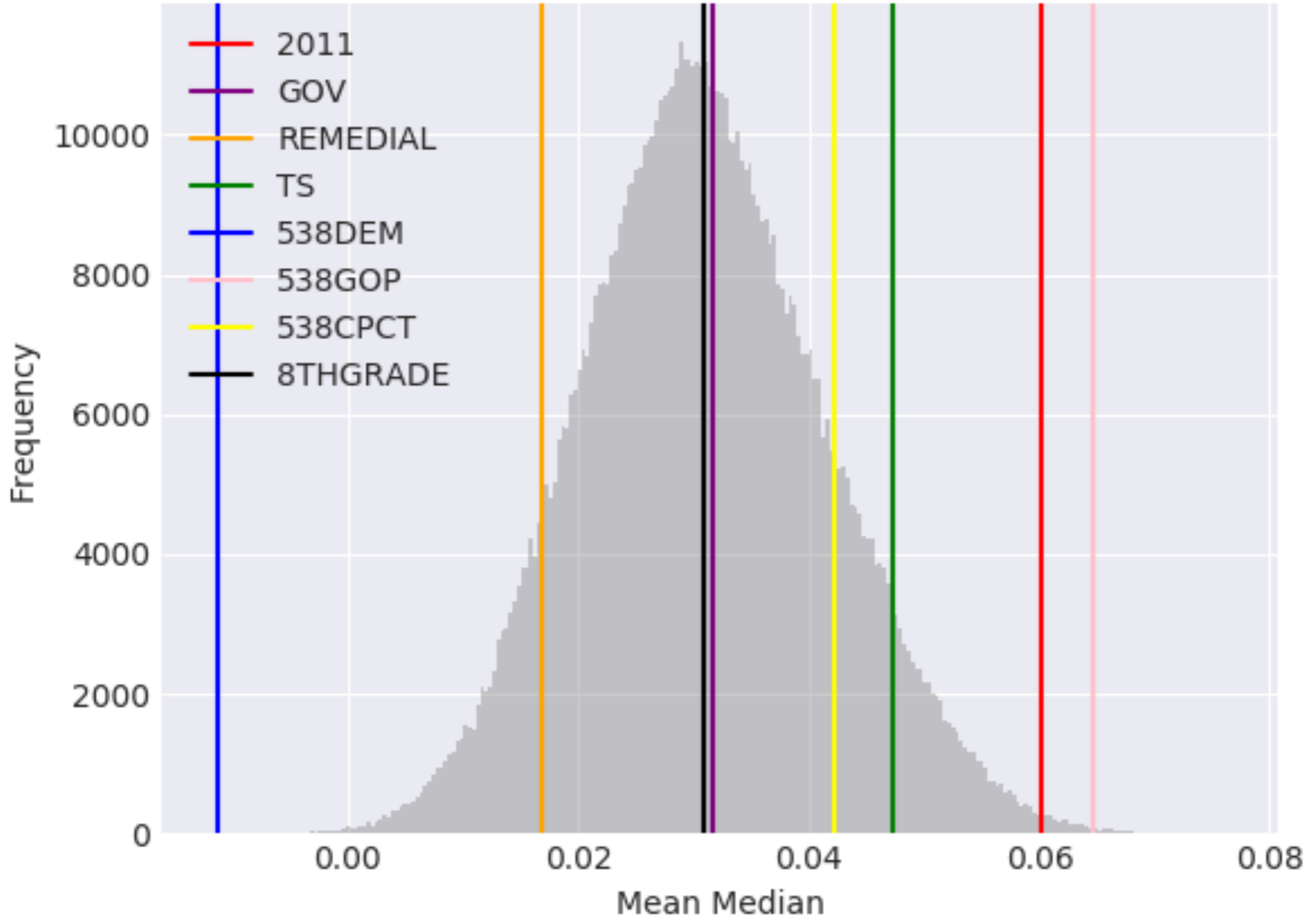


Stanford

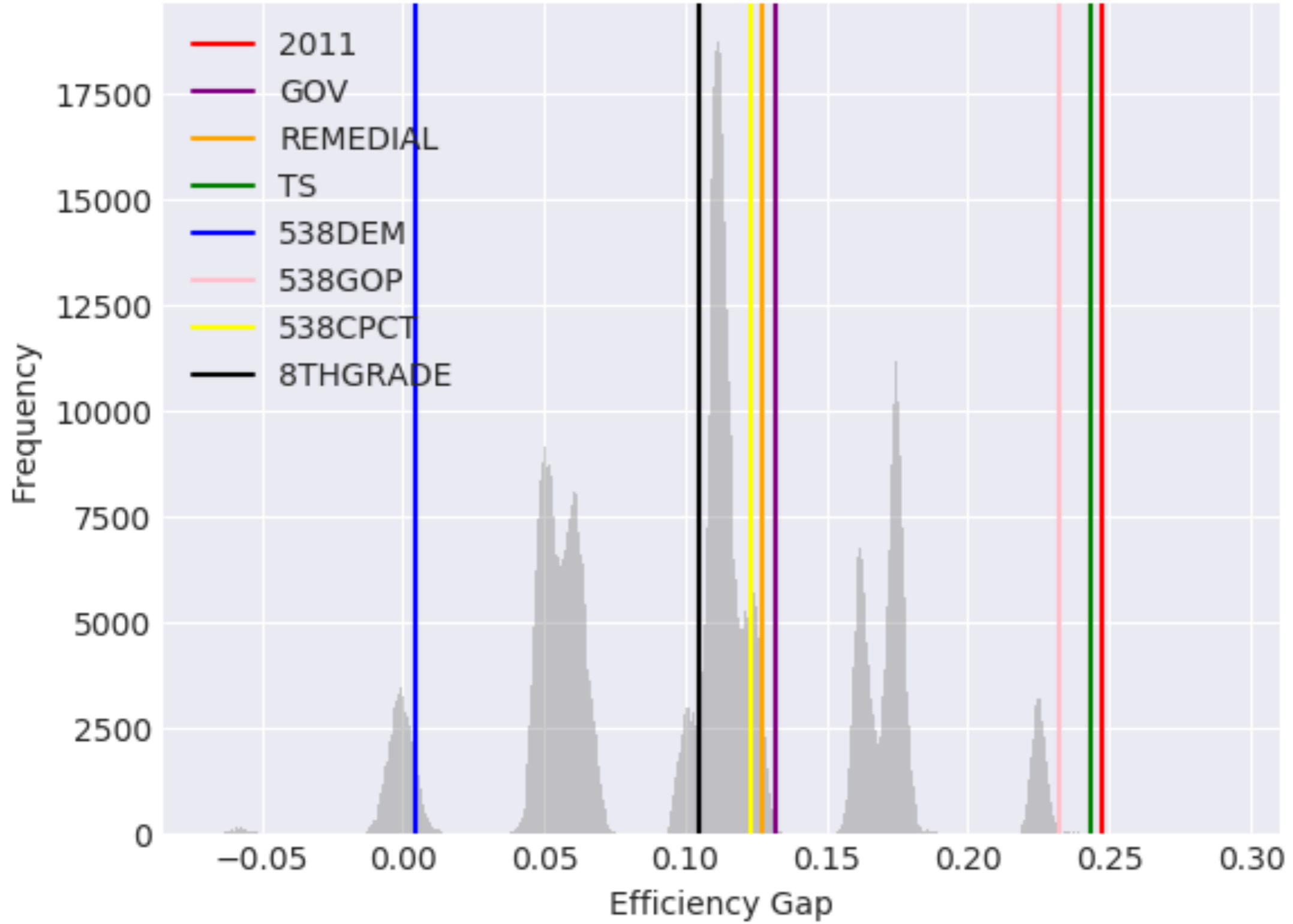


8th Graders

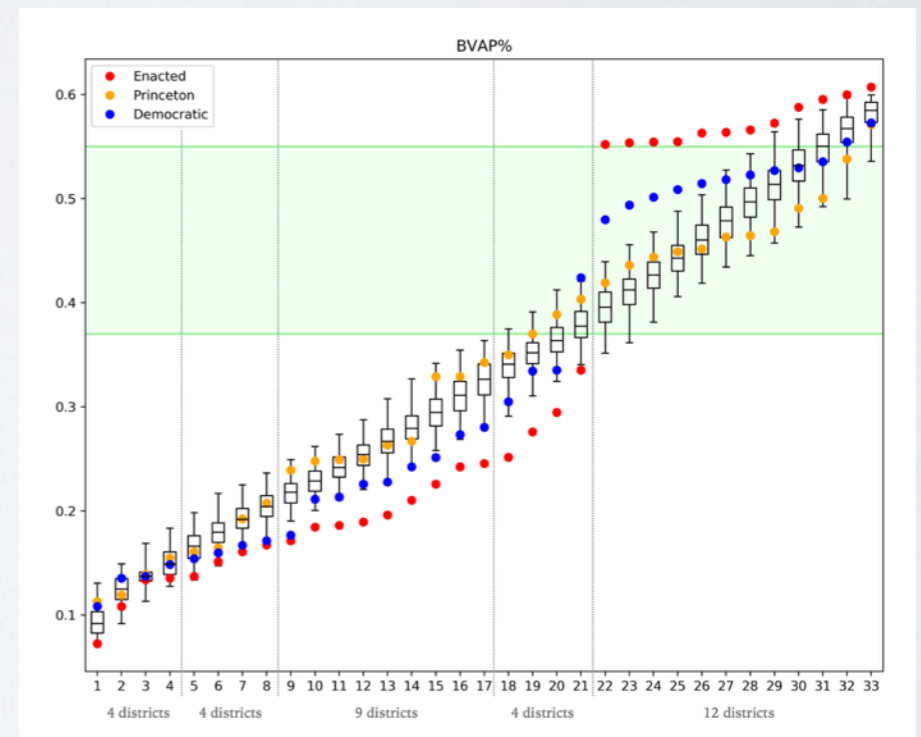
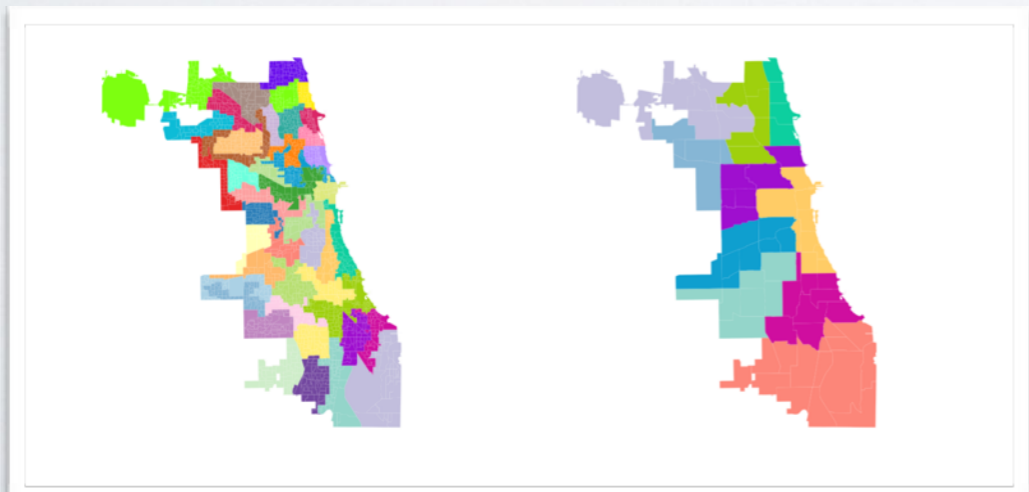
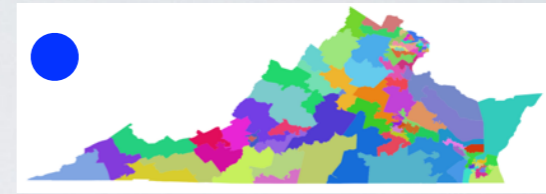
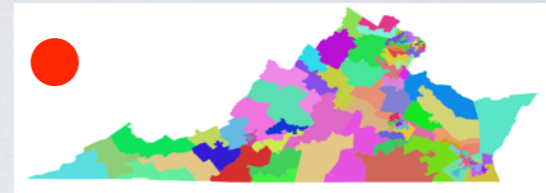
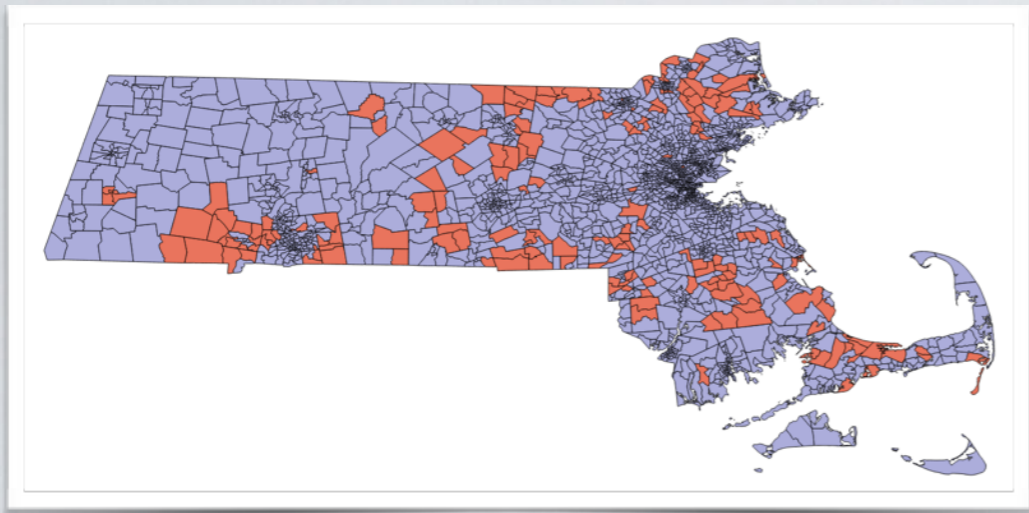
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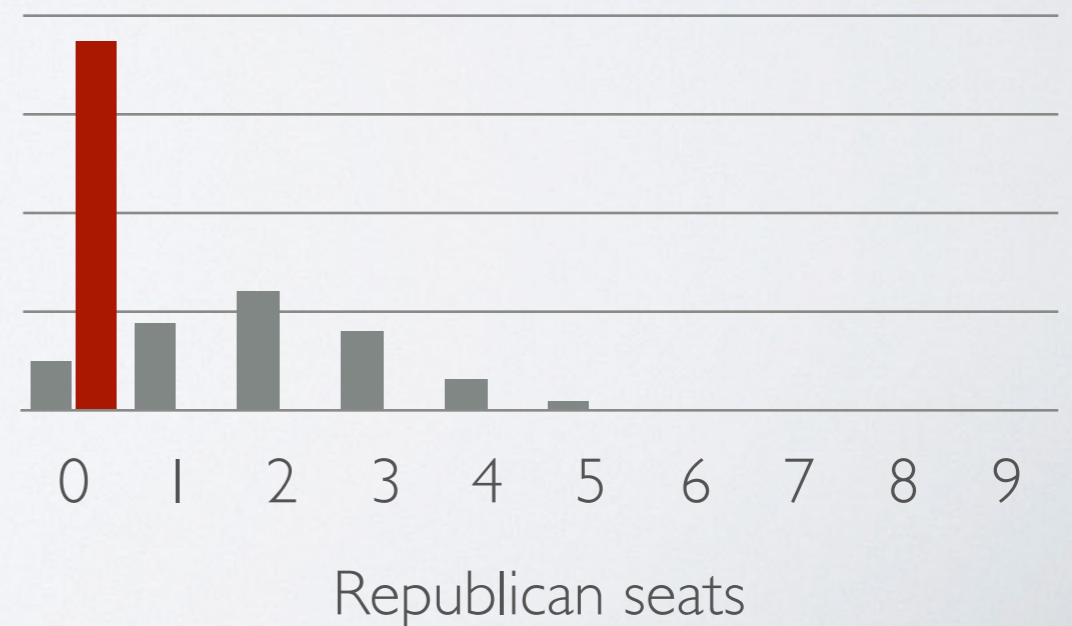
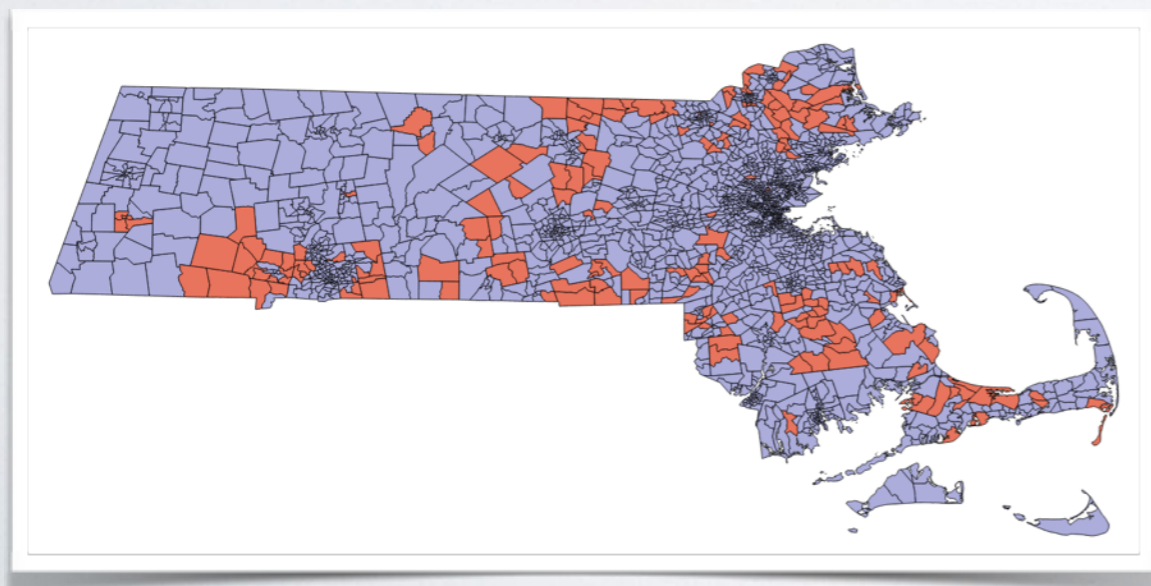


APPLICATIONS



SO ARE WE ANY CLOSER TO A LEGAL STANDARD?

- Some of us think so
 - see Mathematicians' Brief for the Supreme Court
- *Does a plan behave as though chosen only from the stated rules?*



PROPORTIONALITY

...not what we ordered

Jordan Ellenberg in Slate

,18,21	prong [5] 23:14 60:8,9,17 63:25	realistic
21	prongs [4] 22:5 58:14,16 59:22	reality [2]
	proof [1] 45:21	really [13]
	proportional [46] 9:20 10:18 26:	14 25:23
	10,16,23 27:8,13,24 28:10,18 29:8	64:6 69:1
	31:4,17 32:24 33:11,18,20 37:14,	reason [6]
	15 38:1,5 44:7,11 45:22 46:3,11,	12 54:15
	17 47:2,11 59:17,18 60:1,6,14 61:	reasonable
	6,18 62:24 63:16,19 65:5,24 66:4,	53:1 63:6
	8,12,20,21	reassign
	proportionate [1] 9:24	REBUTT
	proposed [1] 67:12	recall [1]
	proposing [3] 9:23 39:16 48:17	recent [1]



COUNSEL: I'd like a grilled cheese.

ALITO: OK, one tuna melt.

COUNSEL: No, I said grilled cheese.

KAVANAUGH: I hear the tuna melt's good.

GORSUCH: You want that tuna melt open face or closed?

COUNSEL: I don't want a tuna melt, I want a—

GORSUCH: It seems like you don't want to just come out and say it, but don't you want a tuna melt?

COUNSEL: No!

KAGAN: She asked for a grilled cheese. That's not a tuna melt because there's no tuna in it.

GORSUCH: But if, as you say, you don't want a tuna melt, what *do* you want? Are we supposed to just make up a sandwich for you?

ALITO: You come in here, you ask for a hot sandwich on toasted bread with cheese on it. To me, that's a tuna melt.

BREYER: Nobody ever orders the chopped liver, but have they really given it a chance?

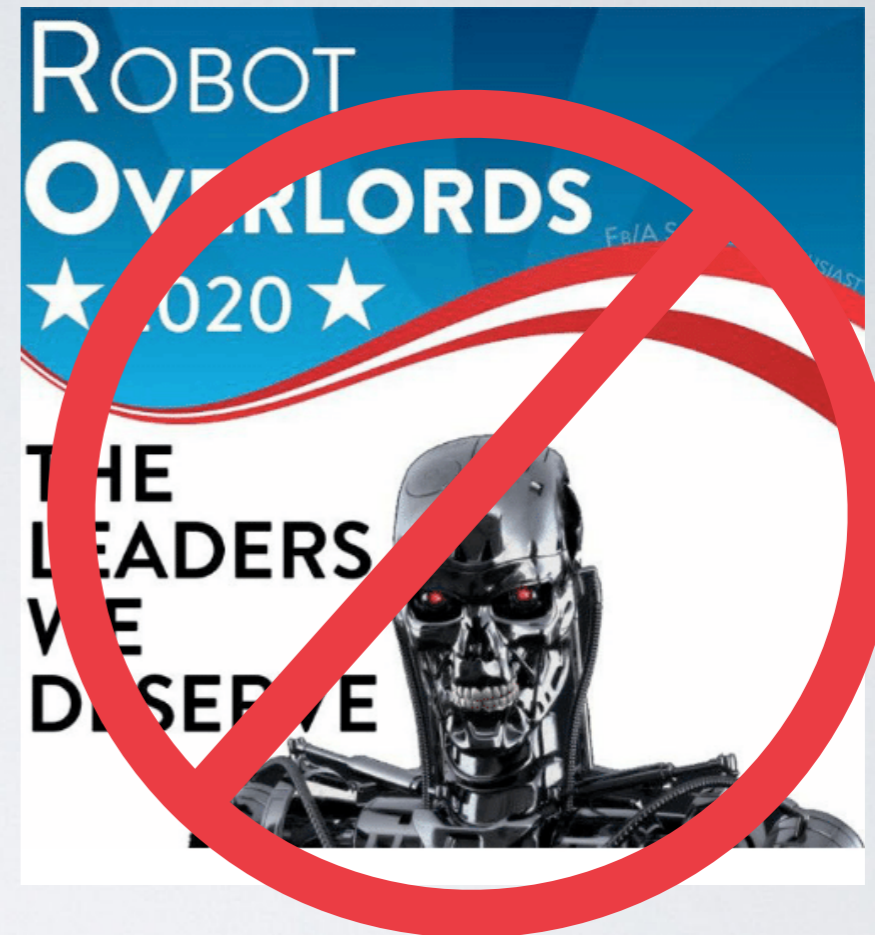
OPPOSING COUNSEL: The framers had every opportunity to make you a tuna melt, but they chose not to.

IN THE END,
WHAT IS A FAIR MAP?

10 JUSTICE ALITO: Can I take you back to
11 questions that were asked before? If you -- if
12 you make a list of the so-called neutral
13 criteria -- compactness, contiguity, protecting
14 incumbents, if that's really neutral,
15 respecting certain natural features of the
16 geography -- and you have a computer program
17 that includes all of those and weights them
18 all, and let's assume all that is neutral, and
19 at the end, what you get is a large number of
20 maps that satisfy all those criteria.

9 JUSTICE ALITO: No, no, but can you
10 just answer that -- that question, because it's
11 a real puzzle to me. So you've got -- let's
12 say you've got 100 maps or you might even have
13 25. I think you probably have thousands. So
14 you have all of these maps, and you have to
15 choose among them. The legislature chooses
16 among them.

17 And you've already programmed in all
18 of the so-called neutral criteria. How do you
19 -- how does the legislature go about choosing
20 among those maps? Would anything other than
21 just random choice be sufficient -- be
22 satisfactory?





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