Prison-Based Gerrymandering

Prison-Based Gerrymandering

- "the practice of counting inmates at their prison address when apportioning populations and drawing electoral districts"

Impact of Prison-Based Gerrymandering

- inflates the population in districts that contain prisoners
- diminishes the population in the districts from which prisoners come

Violations:

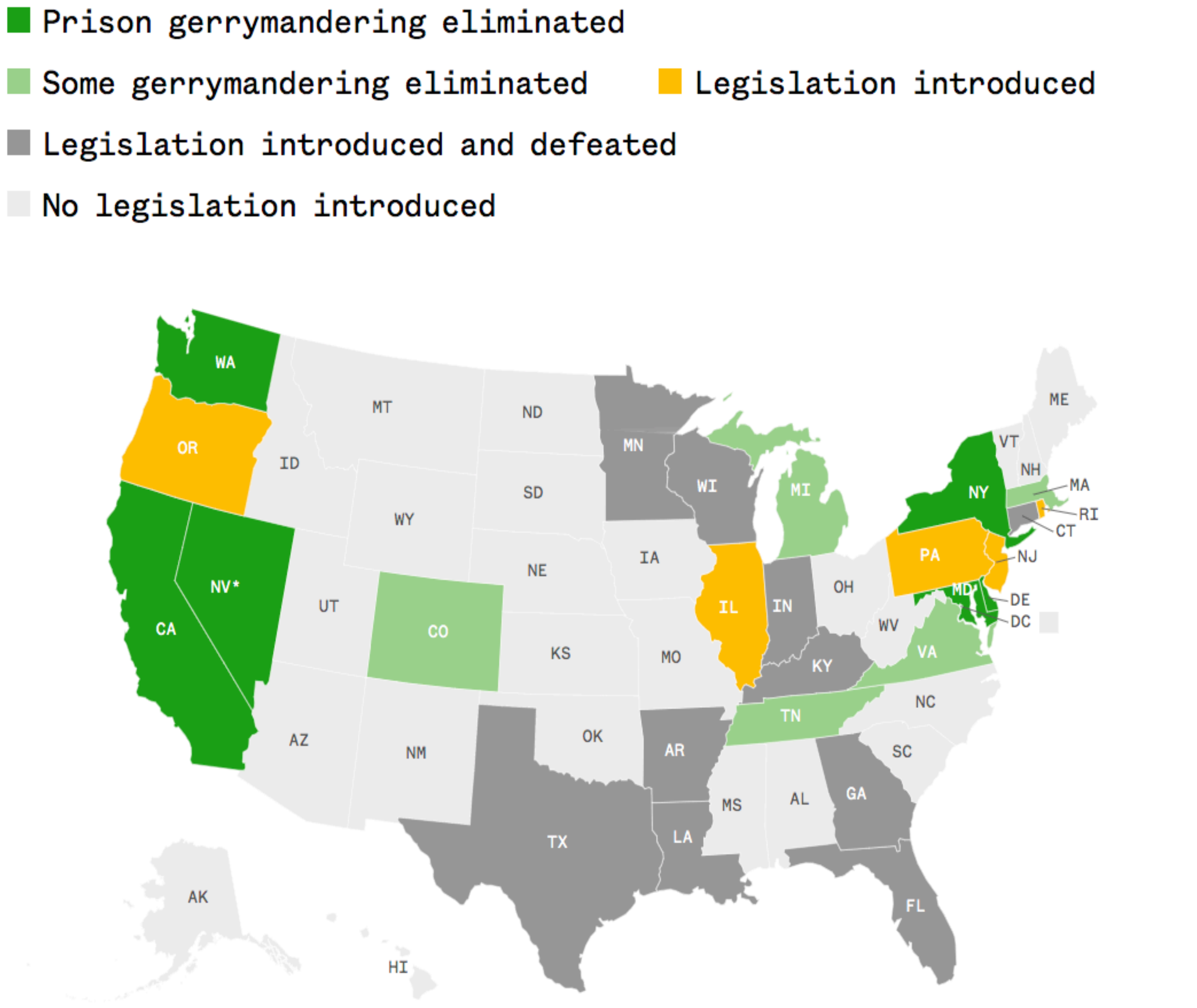
Section Two of the Voting Rights Act, the principle of “one-person-one vote

Equal Protection clause of the 14 th Amendment.

Sources of Prison-Based Gerrymandering

- Legislation
- Local practices
- Court decisions

Prison-Based Gerrymandering across states



* Nevada's legislation has passed the state house and senate, and is awaiting signature from Governor.

Source: Prison Policy Initiative

Graphic: Jiachuan Wu / NBC News

Shifting Context: Georgia and the Census

County of Origin

1. Fulton County(2,160 inmates) 12.3% of total inmate population
2. Dekalb County (1,157 inmates) 6.50% of total inmate population
3. Cobb County (1,075 inmates) 6.04% of total inmate population

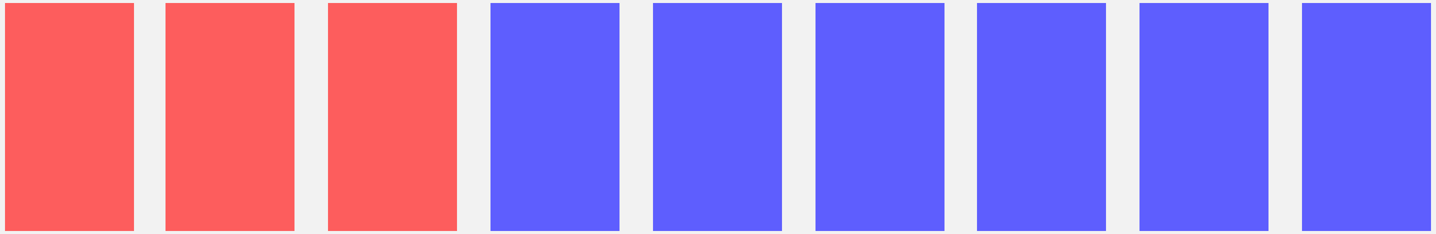
** 2,401 unreported counties of origin

Locating the Representational Baseline: Republicans in Massachusetts

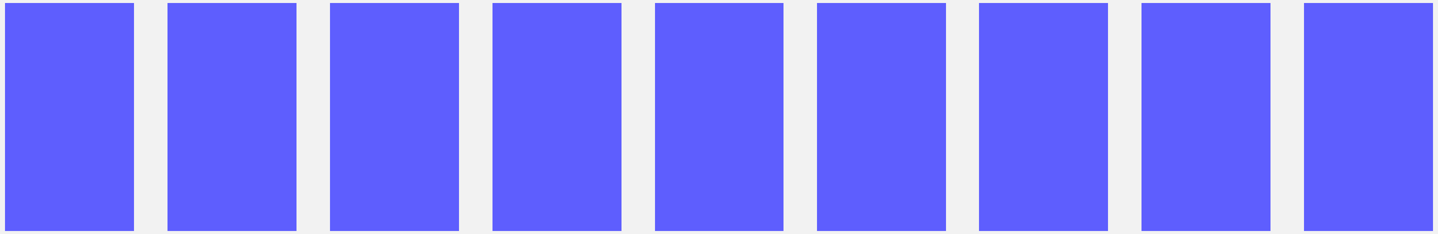
VRDI 2018

INTRO

Republicans often receive approximately 1/3 of the two way vote share in Massachusetts.



There has not been a single Massachusetts Republican in the U.S. House of Reps. Since 1994.



Is this a Democratic gerrymander?

CLAIM

No, it is not.

It is an artifact of the mathematical structure and distribution of Republicans in Massachusetts that leads to consistent Republican underperformance in proportional seat share.

NUMERIC FEASIBILITY

We found the maximum number of Republican seats when building districts out of towns and precincts.

No spatial constraints.

Suppose ideal district size is *l*. Numerically feasible to win *k* seats if there exists a collection of units (towns or precincts) of at least *kl* in which that party has a majority two-way vote share. Feasibility bound is largest such *k* where this majority exists. Infeasibility bound is smallest *k* where majority does not exist.

"Greedy Algorithm": creates largest R-majority collection of units by ordering them by maximum "Republican margin per capita" (shown below) and builds from this list.

$$\delta/p = (\#\text{R votes} - \#\text{D votes})/(\text{census population of unit})$$

Election	D Cand.– R Cand.	R Share	Seat Quota (9 seats)	R Feas/Infeas town	R Feas/Infeas prec	D Feas/Infeas town	D Feas/Infeas prec
Pres 2000	**Gore**–Bush	35.2%	3.2	0/1	—	9/-	—
Sen 2000	**Kennedy**–Robinson/Howell	25.4%*	2.3	0/1	—	9/-	—
Sen 2002	**Kerry**–Cloud	18.7%	1.7	0/1	0/1	9/-	9/-
Pres 2004	**Kerry**–Bush	37.3%	3.4	1/2	1/2	9/-	9/-
Sen 2006	**Kennedy**–Chase	30.6 %	2.8	0/1	0/1	9/-	9/-
Pres 2008	**Obama**–McCain	36.8%	3.3	1/2	1/2	9/-	9/-
Sen 2008	**Kerry**–Beatty	32.0%	2.9	0/1	0/1	9/-	9/-
Sen 2010	Coakley–**Brown**	52.4%	4.7	9/-	9/-	8/9	8/9
Pres 2012	**Obama**–Romney	38.2%	3.4	3/4	3/4	9/-	9/-
Sen 2012	**Warren**–Brown	46.2%	4.2	7/9	7/8	9/-	9/-
Sen 2013	**Markey**–Gomez	44.9%	4.0	7/9	7/8	9/-	9/-
Sen 2014	**Markey**–Herr	38.0%	3.4	3/4	3/4	9/-	9/-
Pres 2016	**Clinton**–Trump	35.3%	3.2	2/3	3/4	9/-	9/-

TABLE 2. If districts were to be made out of towns or out of precincts, with no regard to shape or connectedness, how many R or D districts could be formed? Feasibility and infeasibibilty bounds are shown in this table. Low-variance elections (see previous table) are marked in red. Election winners shown in boldface; R share with respect to 2-way vote; seat quotas for proportional representation out of 9 seats. (* Libertarian vote share added to R in 2000 Senate race)

THE ROLE OF VARIANCE

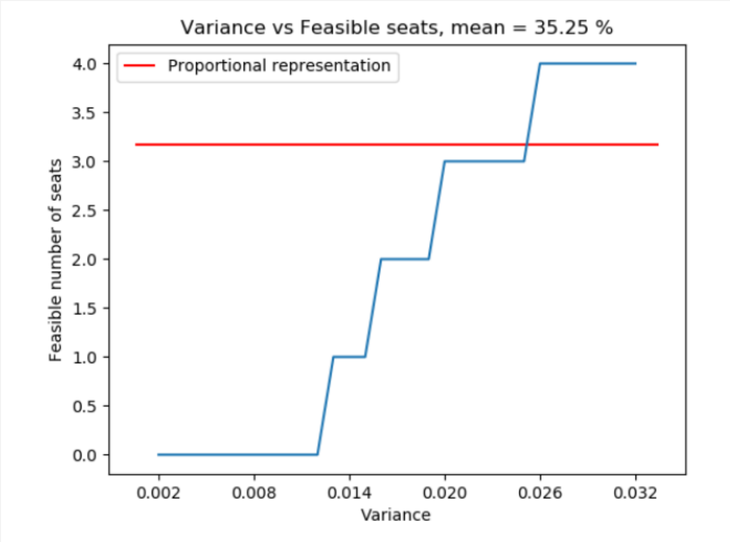
Election	R Share	R Share Mean		R Share Variance	
		town	prec	town	prec
Pres 2000	35.2%	39.70%	–	.0074	–
Sen 2000	25.4%*	29.15%	–	.0044	–
Sen 2002	18.7%	20.29%	17.43%	.0020	.0028
Pres 2004	37.3%	40.00%	34.53%	.0093	.0140
Sen 2006	30.6 %	33.24%	27.59%	.0077	.0119
Pres 2008	36.8%	39.00%	33.80%	.0117	.0181
Sen 2008	32.0%	34.40%	28.87%	.0094	.0142
Sen 2010	52.4%	53.79%	47.71%	.0202	.0310
Pres 2012	38.2%	41.06%	34.91%	.0146	.0228
Sen 2012	46.2%	49.20%	42.70%	.0169	.0275
Sen 2013	44.9%	48.89%	41.89%	.0217	.0312
Sen 2014	38.0%	41.15%	34.28%	.0141	.0206
Pres 2016	35.3%	40.18%	33.12%	.0165	.0236

TABLE 1. Statistics of Republican vote share in 13 statewide elections in Massachusetts. Lower-variance elections are marked in red.

Low variance

⇓

Lower upper bound on feasibility in comparison to proportional seat share



GEOMETRY: LACK OF REPUBLICAN ENCLAVES

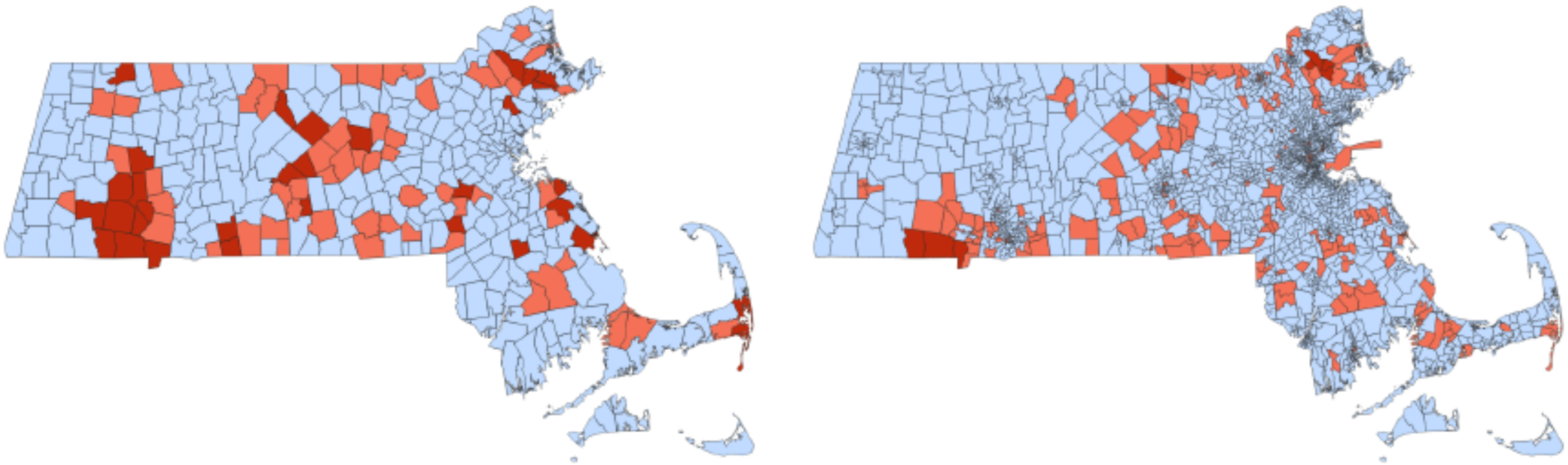


FIGURE 4. These figures show the voting pattern for Republicans George W. Bush in the 2000 Presidential race (left, by town) and Kenneth Chase in the 2006 Senate race (right, by precinct). The darkest red units favored the Republican outright, and the lighter red shade shows the most Republican-favorable units available in assembling enough population for a Congressional district. These quasi-districts still preferred Gore and Kennedy, respectively, by comfortable margins.

GEOMETRY: CONNECTEDNESS

How does level of segregation affects representation?

Changing Cluster Energy:

Observed Republican clustering scores are very close to uniformly clustered, as seen in lack of enclaves

This is directly related to variance:

low variance → all units similar → no spatial pattern

Election	R Share	uniform H	observed H	clustered H
Pres 2000	35.2%	.5001	.5135	.9456
Sen 2000	25.4%*	.5000	.5063	.9374
Sen 2002	18.7%	.5001	.5035	.8982
Pres 2004	37.3%	.5000	.5182	.9351
Sen 2006	30.6%	.5001	.5171	.9537
Pres 2008	36.8%	.5000	.5210	.9591
Sen 2008	32.0%	.5000	.5181	.9513
Sen 2010	52.4%	.5001	.5329	.9587
Pres 2012	38.2%	.5000	.5243	.9268
Sen 2012	46.2%	.5000	.5272	.9597
Sen 2013	44.9%	.5002	.5366	.9492
Sen 2014	38.0%	.5001	.5276	.9557
Pres 2016	35.3%	.5000	.5344	.9480

TABLE 3. Clustering scores for Republican versus Democratic voters at the town level in each of the elections discussed in this paper. We show the score $H = H(R, D)$ for a uniform trial, the actual observed votes, and a highly clustered trial, each with the statewide share that corresponds accurately to the given election. The numbers are truncated (not rounded) after four decimal places.

Perfectly uniform: H = ½, Perfectly clustered: H = 1

SO WHAT?

"Generally, counterintuitive limitations on representation can emerge from a complicated interplay of the numerical and spatial distribution of voter preferences; in the case of Massachusetts, the numerical distribution is so uniform that it makes the spatiality insignificant... Uniformity itself can block desired representational outcomes for a group in the numerical minority (like Republicans in Massachusetts)…"

"…it is only legitimate to compare an observed partisan outcome against the backdrop of actual possibility."

Transit Time Compactness

Suzie Tovar

Tarleton State University

July 17, 2019
VRDI

Acknowledgements

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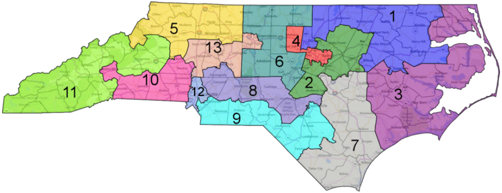
- Joseph Brown, Financial Recovery Technologies
- Dr. Scott Cook, Tarleton State Univ, Math
- Dr. Anne Egelston, Tarleton State Univ, Political Science
- Dr. Rob Muth, Washington & Jefferson College

1 Transit Time Compactness
- Method
- Validation of Method
- Today

Transit Time Compactness - Idea

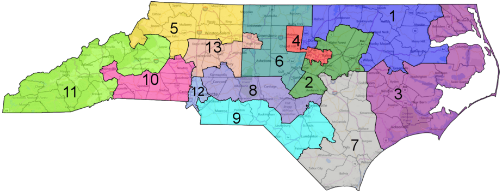
- Current compact measures measure cohesiveness **land**.
- We seek to measure the cohesiveness of the **people**.
- Water and mountains separate people who are geographically close.
- Roads and public transportation connect people who are geographically far.
- Metric: average transit time between citizens

Transit Time Compactness - Definition



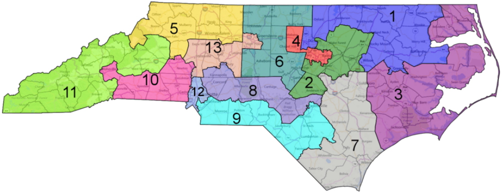
- Get centers $\vec{x}_i$ and populations P_i of all VTDs in a district

Transit Time Compactness - Definition



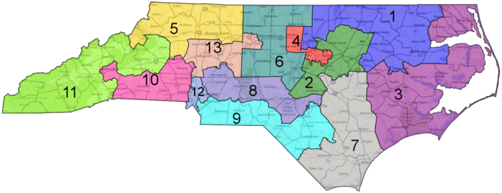
- Get centers $\vec{x}_i$ and populations P_i of all VTDs in a district
- Submit centers in pairs to Google Maps API

Transit Time Compactness - Definition



- Get centers $\vec{x}_i$ and populations P_i of all VTDs in a district
- Submit centers in pairs to Google Maps API
- T_{ij} = time of fastest transit mode from $\vec{x}_i$ to $\vec{x}_j$

Transit Time Compactness - Definition



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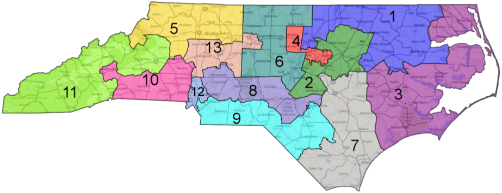
- T_{ij} = time of fastest transit mode from $\vec{x}_i$ to $\vec{x}_j$
- Weight by populations. Compute average.

Transit Time Compactness - Definition



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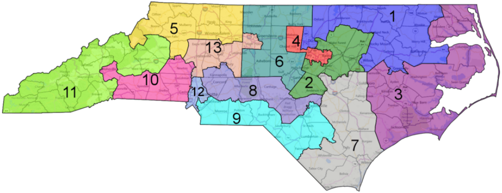
- Submit centers in pairs to Google Maps API
- T_{ij} = time of fastest transit mode from $\vec{x}_i$ to $\vec{x}_j$
- Weight by populations. Compute average.
- $C_{\text{transit}} = \frac{1}{P_{tot}^2} \sum_{i,j} T_{ij} P_i P_j$



- Get centers $\vec{x}_i$ and populations P_i of all VTDs in a district
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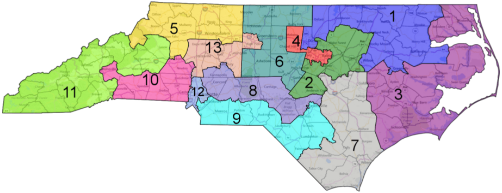
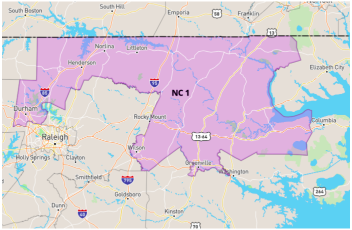
- Weight by populations. Compute average.
- $C_{\text{transit}} = \frac{1}{P_{tot}^2} \sum_{i,j} T_{ij} P_i P_j$
- Rewards skinny districts following transit routes



- Get centers $\vec{x}_i$ and populations P_i of all VTDs in a district
- Submit centers in pairs to Google Maps API
- T_{ij} = time of fastest transit mode from $\vec{x}_i$ to $\vec{x}_j$
- Weight by populations. Compute average.
- $C_{\text{transit}} = \frac{1}{P_{tot}^2} \sum_{i,j} T_{ij} P_i P_j$
- Rewards skinny districts following transit routes
- Punishes districts cut by water/mountains without crossings

Proof of Concept



North Carolina 1st

North Carolina 4th

C_{transit} : 77.15% difference

$C_{\text{Polsby-Popper}}$: 16.5% difference

4000 VTD pairs

Today

- We received a Google grant that will pay for the necessary volume of API calls.
  - Cannot cache Google Data
  - Lost grant had to delete all stored data
- Good news! Currently working on creating a GIS database on transit time using ESRI data sets.
  - Will allow us to wrap an API around the data set and test our model.
- Work on transit time compactness will restart very soon!

Imposing Contiguity Constraints in Political Districting Models

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Austin Buchanan   Eugene Lykhovyd

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#vrdi2019

Overview

1. History
2. Contiguity models
3. Computational results
4. What about larger instances?
5. A redistricting puzzle

Section 1

History

Once upon a time . . .

NONPARTISAN POLITICAL REDISTRICTING BY COMPUTER*

S. W. Hess and J. B. Weaver

Atlas Chemical Industries, Inc., Wilmington, Dela.

H. J. Siegfeldt, J. N. Whelan, and P. A. Zitlau

E. I. Du Pont de Nemours & Co., Wilmington, Dela.

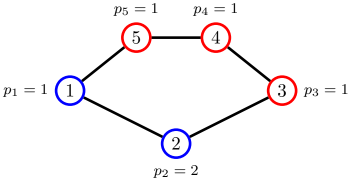
(Received January 28, 1965)

OR volunteers developed a compactness measure and a 'warehouse-location' heuristic to draw nonpartisan, Constitutional political districts. The heuristic maps compact and contiguous districts of equal population. The minimization criterion and compactness measure is population moment of inertia—the summed squared distances from each person to his district's center. The districting method is particularly useful when legislative impasse or indifference forces courts to intervene. Federal Courts have received a computer plan for possible use in Delaware and have asked for computer districts in Connecticut.

Terminology

Notations

$G = (V, E)$	*Undirected contiguity graph*
$n := \|V\|$	*Number of land parcels*
k	*Number of districts*
p_v	*Population of land parcel v*
d_{ij}	*Distance between (centroids of) parcels i and j*
$w_{ij} := p_i d_{ij}^2$	*Cost of assigning land parcel i to land parcel j*
L	*Minimum population allowed in a district*
U	*Maximum population allowed in a district*



A feasible redistricting plan on graph G with $n = 5$, $k = 2$, and $L = U = 3$.
Here, $d_{35} = 2$ and $w_{35} = 2$.

Hess formulation

$$x_{ij} = \begin{cases} 1 & \text{if vertex } i \text{ is assigned to (the district centered at) vertex } j \\ 0 & \text{otherwise.} \end{cases}$$

Hess formulation

$$x_{ij} = \begin{cases} 1 & \text{if vertex } i \text{ is assigned to (the district centered at) vertex } j \\ 0 & \text{otherwise.} \end{cases}$$

$$\min \sum_{i \in V} \sum_{j \in V} w_{ij} x_{ij} \tag{1a}$$

$$\sum_{j \in V} x_{ij} = 1 \qquad \forall i \in V \tag{1b}$$

$$\sum_{j \in V} x_{jj} = k \tag{1c}$$

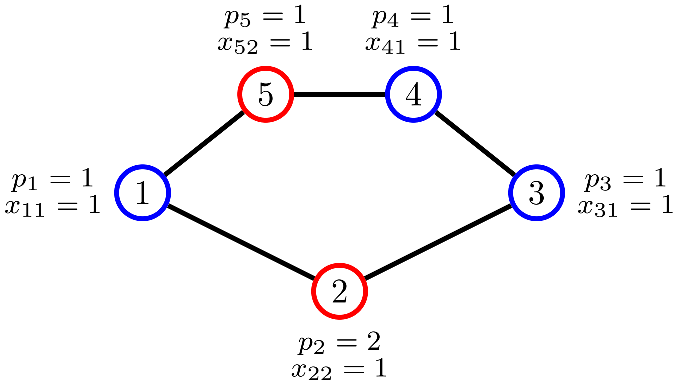
$$L x_{jj} \leq \sum_{i \in V} p_i x_{ij} \leq U x_{jj} \qquad \forall j \in V \tag{1d}$$

$$x_{ij} \leq x_{jj} \qquad \forall i, j \in V \tag{1e}$$

$$x_{ij} \in \{0, 1\} \qquad \forall i, j \in V. \tag{1f}$$

$$\text{HESS} := \left\{ x \in \mathbb{R}_+^{n \times n} \;\middle|\; x \text{ satisfies constraints (1b), (1c), (1d), (1e)} \right\}.$$

Hess formulation



A feasible solution for HESS with $k = 2$, and $L = U = 3$.

Section 2

Contiguity models

Shirabe's formulation

f_{ij}^{v} = the amount of flow, originating at district center v, that is sent across edge (i, j).

$$x \in \text{HESS} \tag{2a}$$

$$f^{j}(\delta^{-}(i)) - f^{j}(\delta^{+}(i)) = x_{ij} \qquad \forall i \in V \setminus \{j\},\ \forall j \in V \tag{2b}$$

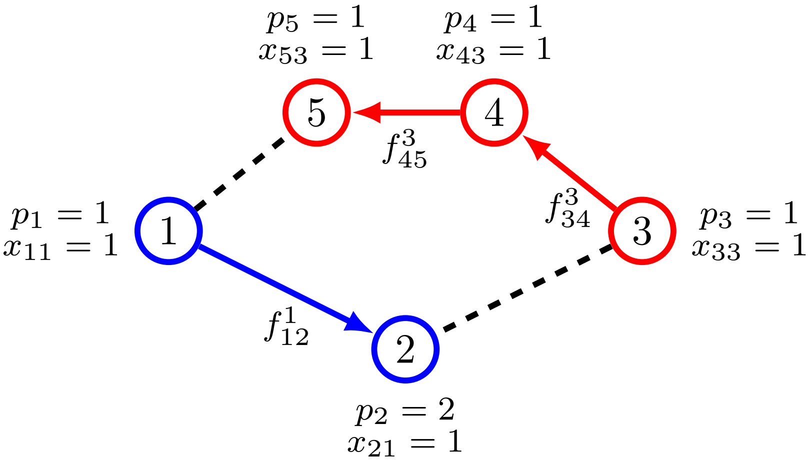
$$f^{j}(\delta^{-}(i)) \leq (n-1)x_{ij} \qquad \forall i \in V \setminus \{j\},\ \forall j \in V \tag{2c}$$

$$f^{j}(\delta^{-}(j)) = 0 \qquad \forall j \in V \tag{2d}$$

$$f_{ij}^{v} \geq 0 \qquad \forall (i, j) \in A,\ \forall v \in V. \tag{2e}$$

$$\text{SHIR} := \left\{ (x, f) \in \mathbb{R}^{n \times n} \times \mathbb{R}^{2mn} \;\middle|\; (x, f) \text{ satisfies constraints (2a)} - \text{(2e)} \right\}.$$

Shirabe's formulation



SHIR formulation

MCF formulation

$$f_{ij}^{ab} = \begin{cases} 1 & \text{if edge } (i,j) \in A \text{ is on the path to vertex } a \text{ from its district's center } b \\ 0 & \text{otherwise.} \end{cases}$$

$$x \in \text{HESS} \tag{3a}$$

$$f^{ab}(\delta^+(b)) - f^{ab}(\delta^-(b)) = x_{ab} \qquad \forall a \in V \setminus \{b\},\ \forall b \in V \tag{3b}$$

$$f^{ab}(\delta^+(i)) - f^{ab}(\delta^-(i)) = 0 \qquad \forall i \in V \setminus \{a,b\},\ \forall a \in V \setminus \{b\},\ \forall b \in V \tag{3c}$$

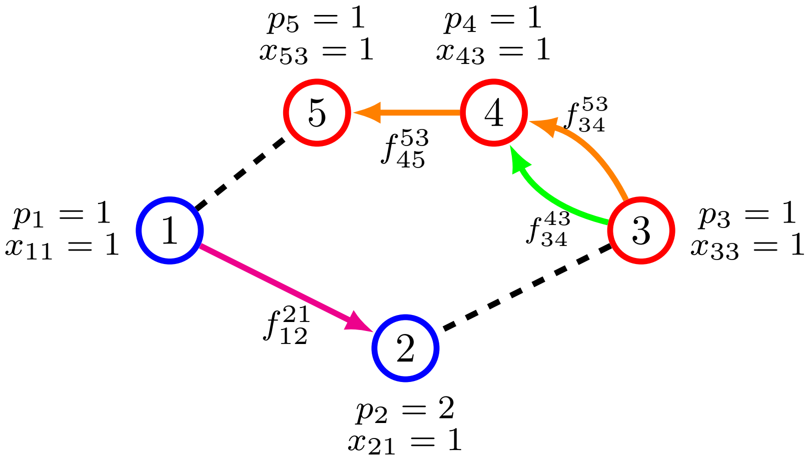
$$f^{ab}(\delta^-(b)) = 0 \qquad \forall a \in V \setminus \{b\},\ \forall b \in V \tag{3d}$$

$$f^{ab}(\delta^-(j)) \leq x_{jb} \qquad \forall j \in V \setminus \{b\},\ \forall a \in V \setminus \{b\},\ \forall b \in V \tag{3e}$$

$$f_{ij}^{ab} \geq 0 \qquad \forall (i,j) \in A,\ \forall a \in V \setminus \{b\},\ \forall b \in V. \tag{3f}$$

$$\text{MCF} := \left\{ (x,f) \in \mathbb{R}^{n \times n} \times \mathbb{R}^{2mn(n-1))} \;\middle|\; (x,f) \text{ satisfies constraints (3a)} - \text{(3f)} \right\}.$$

MCF formulation



MCF formulation

CUT formulation

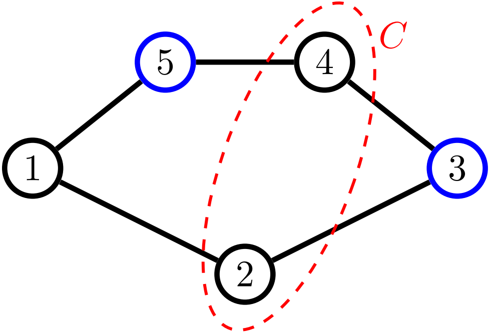
Definition (a, b-separator)

A subset $C \subseteq V \setminus \{a, b\}$ of vertices is called an a, b-separator for $G = (V, E)$ if there is no a, b-path in $G - C$.

CUT formulation

Definition (a, b-separator)

A subset $C \subseteq V \setminus \{a,b\}$ of vertices is called an a,b-separator for $G = (V,E)$ if there is no a,b-path in $G - C$.



Here, $C = \{2,4\}$ is a $3,5$-separator.

CUT formulation

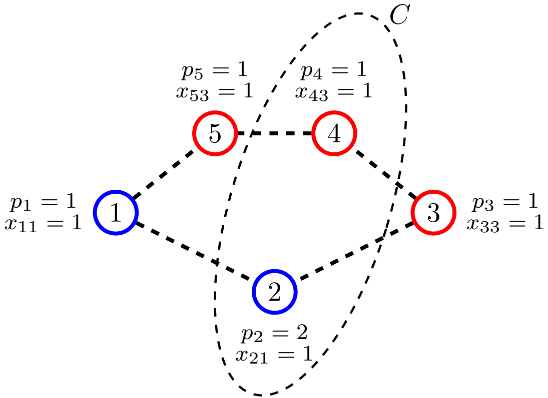
$$x \in \text{HESS} \tag{4a}$$

$$x_{ab} \leq \sum_{c \in C} x_{cb} \qquad \forall (a, b, C). \tag{4b}$$

CUT formulation

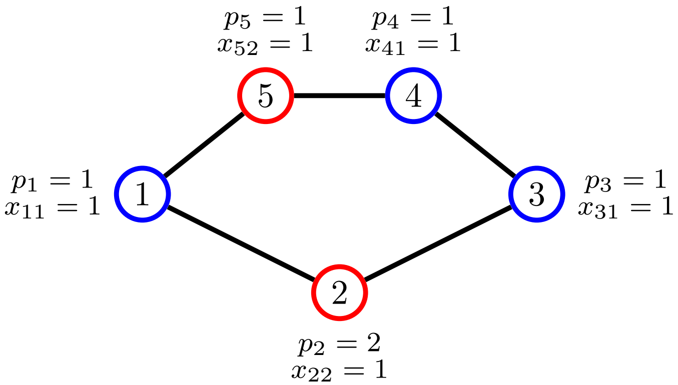
$$x \in \text{HESS} \tag{4a}$$

$$x_{ab} \le \sum_{c \in C} x_{cb} \qquad \forall (a, b, C). \tag{4b}$$



CUT formulation

Find a violated cut!

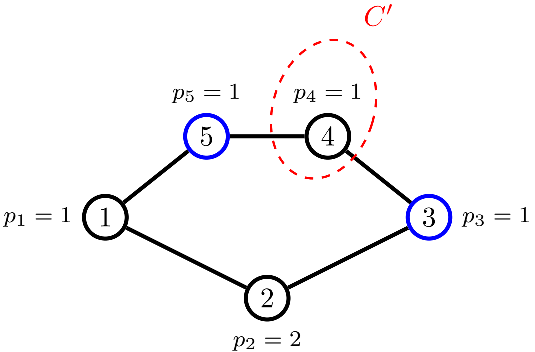


A feasible solution for HESS with $k = 2$, and $L = U = 3$.

LCUT formulation

Definition (length-U a,b-separator)

A subset $C' \subseteq V \setminus \{a,b\}$ of vertices is called a length-U a,b-separator in $G=(V,E)$, with respect to vertex weights p, if $\text{dist}_{G-C',p}(a,b) > U$.

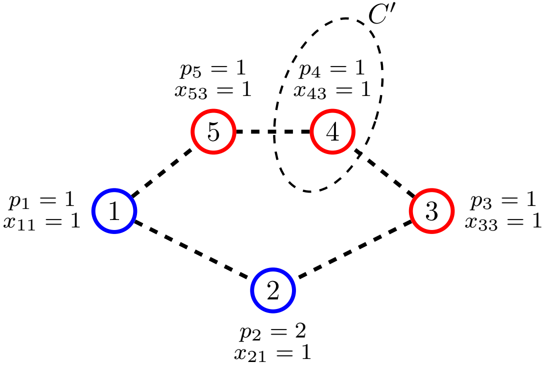


Here, $C' = \{4\}$ is a length-3 3, 5-separator.
Note that $\text{dist}_{G-C',p}(3,5) = 5 > 3$.

LCUT formulation

$$x \in \text{HESS} \tag{5a}$$

$$x_{ab} \leq \sum_{c \in C'} x_{cb} \qquad \forall (a, b, C'). \tag{5b}$$



LCUT formulation

Section 3

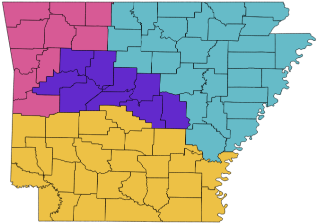
Computational results

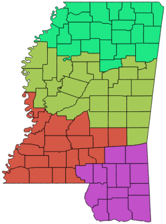
Computational results at county level

state	n	k	SHIR time (sec)	MCF time (sec)	CUT time (sec)	LCUT time (sec)
AL	67	7	75.04	867.94	49.88	55.86
AR	75	4	4.60	261.82	3.60	4.21
CO*	64	7	1.96	354.49	3600.00	0.02
IA	99	4	7.07	crash	2.08	1.84
ID	44	2	0.44	22.03	0.10	0.12
KS	105	4	17.26	crash	6.65	6.02
LA	64	6	13.20	461.30	15.05	16.77
ME	16	2	0.70	3.59	0.75	0.96
MS	82	4	1.35	152.85	0.48	0.53
NE	93	3	7.19	411.04	1.39	1.47
NH*	10	2	0.13	0.19	0.07	0.06
NM	33	3	0.35	8.41	0.06	0.06
OK	77	5	21.30	326.62	21.92	7.45
OR*	36	5	0.27	102.43	6.66	0.06
SC	46	7	3600.00	3600.00	3600.00	3600.00
WV	55	3	7.93	202.19	3.22	3.42

* Instances are infeasible.

Computational results at county level





Section 4

What about larger instances?

Handling larger instances

We can also solve 20 redistricting instances at the census tract level, including Indiana ($n = 1511$). To solve these large instances, we use Lagrangian arguments to safely fix most of the variables to zero (e.g., 96.7% fixed for Indiana).

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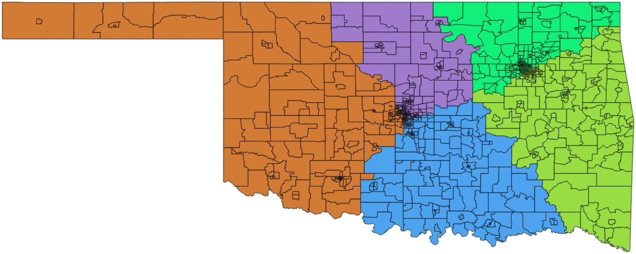

Section 5

A redistricting puzzle

Computational results at county level

state	n	k	SHIR time (sec)	MCF time (sec)	CUT time (sec)	LCUT time (sec)
AL	67	7	75.04	867.94	49.88	55.86
AR	75	4	4.60	261.82	3.60	4.21
CO*	64	7	1.96	354.49	3600.00	0.02
IA	99	4	7.07	crash	2.08	1.84
ID	44	2	0.44	22.03	0.10	0.12
KS	105	4	17.26	crash	6.65	6.02
LA	64	6	13.20	461.30	15.05	16.77
ME	16	2	0.70	3.59	0.75	0.96
MS	82	4	1.35	152.85	0.48	0.53
NE	93	3	7.19	411.04	1.39	1.47
NH*	10	2	0.13	0.19	0.07	0.06
NM	33	3	0.35	8.41	0.06	0.06
OK	77	5	21.30	326.62	21.92	7.45
OR*	36	5	0.27	102.43	6.66	0.06
SC?	46	7	3600.00	3600.00	3600.00	3600.00
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* Instances are infeasible.

? Win $20 by finding a feasible solution or proving infeasibility; see Hamid for printed handout.

A winner from #mip2019 at MIT!

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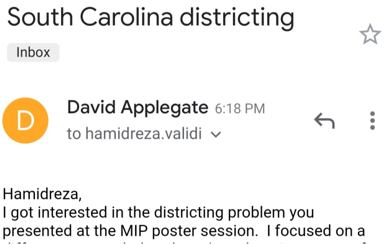

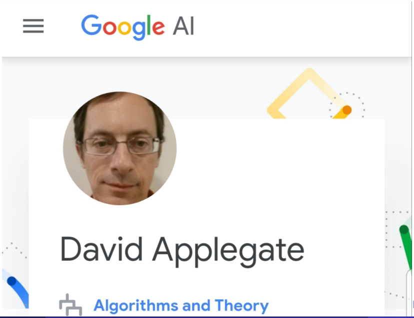

References

1 SW Hess, JB Weaver, HJ Siegfeldt, JN Whelan, and PA Zitlau. Nonpartisan political redistricting by computer. Operations Research, 13(6):998–1006, 1965.

2 Takeshi Shirabe. Districting modeling with exact contiguity constraints. Environment and Planning B: Planning and Design, 36(6):1053–1066, 2009.

3 Johannes Oehrlein and Jan-Henrik Haunert. A cutting-plane method for contiguity-constrained spatial aggregation. Journal of Spatial Information Science, 2017(15):89–120, 2017.

4 https://lykhovyd.com/files/public/districting/

Challenge Time!!

Referendum Elections & Question Interdependence

Colby Brown

The University of Arizona

Grand Valley State University

July 17, 2019

It's Election Day!

It's election day for the VRDI! Here's what's on the ballot:

1. Build a permanent MGGG building
2. Solicit donations through donors
3. Sell commissioned ~~gerrymanders~~ reports

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We're going to build a new building, but not pay for it!
The questions were *not separable*. The results for some questions affected how people may have voted on others.

Preference & Separability

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Definition

A *preference* is an ordering of the 2^n outcomes of an election with n questions.

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	Y	Y	Y
	N	Y	Y
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N	N	N
N	Y	Y
N	Y	N
N	N	Y

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Y	N	Y
Y	N	N
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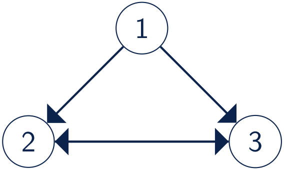
Y	Y	Y
N	N	N
N	Y	Y
N	Y	N
N	N	Y



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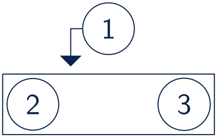
- **One-to-Many: One question q influences a set of questions T.**

Hypergraphs



# 1	# 2	# 3
Y	Y	Y
Y	Y	N
Y	N	Y
Y	N	N
N	Y	Y
N	N	Y
N	Y	N
N	N	N

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- One-to-One and One-to-Many can describe admissibility for different sets.
- One-to-Many is more descriptive, but harder to work with.

Future Work

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- *Conjecture:* Almost no sparability profiles are admissible as $|Q| \to \infty$.

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- Necessary & sufficient conditions for admissibility?
- *Conjecture:* Almost no sparability profiles are admissible as $|Q| \to \infty$.
- Applying influence to referendum simulations.

Acknowledgements

This work was partially supported by National Science Foundation grant DMS-1659113 and National Security Agency grant H98230-18-1-0011 which fund a Research Experiences for Undergraduates program at Grand Valley State University. We thank the NSF, NSA, and GVSU for their support.

Special thanks to Dr. Jonathan Hodge and my fellow group members Lee Trent, Benjamin Becker, and Kaleigh Roach.

Questions?

Answers!