SOCIAL INTERACTIONS: THEORY AND EMPIRICS

Yannis M. Ioannides

ASSET 2001 RETHYMNO

October 26, 2001

1. SOCIAL INTERACTIONS: THEORY

- **Non-market interactions among agents**
- **Interdependent discrete decisions — Brock and Durlauf (2001)**
- **Interdependence has spatial structure: who trades with whom?**
  - **The overlapping generations model, a specific topology of interactions**
  - **Patterns of preferential trade arrangements among countries**

2. SOCIAL INTERACTIONS: EMPIRICS

- **Structural estimation of systems of discrete decisions**
- **Variance of community-level aggregates vs. variance of individual data [Glaeser and Scheinkman (2001)]**
- **Applications with Continuous Decisions**

SOCIAL STRUCTURE:

$\mathcal{I}$: **the set of individuals**

$G(V, E)$, **an undirected graph:** $V = \{v_1, v_2, \ldots, v_I\}$, **set of vertices, the individuals;** E, **is the set of edges, a subset of the collection of unordered pairs of vertices.**

Agent i **interacts with agent** j, **if there is an edge between nodes** i **and** j.

Let $\nu(i)$ **define the local neighborhood of agent** $i : \nu(i) = \{j \in \mathcal{I} | j \neq i, \{i, j\} \in E\}$.

STYLIZED TOPOLOGIES OF SOCIAL INTERACTIONS:

- **Complete** *pairwise* **interactions:** $\nu(i) = \mathcal{I} - \{i\}$,

  graph G **is complete, mean-field case**

- *Walrasian–star*, **a.k.a. "hub-and-spoke":**

  agent 1 interacts with each of all other agents in the economy, $\nu(1) = \mathcal{I} - \{1\}$,

  all other agents interact only with agent 1, $\nu(i) = 1, i \neq 1$.

- *Circular* **interaction: graph** G **is a cycle**

  $\nu(i) = \{i - 1, i + 1\}, \forall i \in \mathcal{I}$.

- *One-dimensional lattice case, line:* $\mathcal{I} = \{-L, \ldots, 0, \ldots, L\}$.

  $\nu(i) = \{i - 1, i + 1\}, \forall i \in \mathcal{I}, i \neq L, i \neq -L$.

  $\nu(L) = \{L - 1\},\ \nu(-L) = \{-L + 1\}$

Stylized Topologies

The Brock-Durlauf interactive discrete choice model

individual i is decision, state: $\omega_i,\ \omega_i \in S = \{-1, 1\}$.

$$\omega_i = \mathbf{argmax}\, U_i = U(\omega_i; \widetilde{\omega}_{\nu(i)}),$$

$\widetilde{\omega}_{\nu(i)}$ **denotes vector containing the decisions made by each of agent i's nearest neighbors,** $\nu(i)$.

$$U_i(\omega_i) \equiv u(\omega_i) + \omega_i \mathcal{E}_i \left\{ \frac{1}{|\nu(i)|} \sum_{j \in \nu(i)} J_{ij}\omega_j \right\} + \epsilon(\omega_i), \tag{1}$$

random variable $\epsilon(\omega_i)$, **I.I.D. type I extreme-value.**

$$\mathbf{Prob}(\omega_i = 1) = \frac{\exp\left[\beta\left(u(1) - u(-1) + 2\mathcal{E}_i\left\{\frac{1}{|\nu(i)|}\sum_{j\in\nu(i)} J_{ij}\omega_j\right\}\right)\right]}{1 + \exp\left[\beta\left(u(1) - u(-1) + 2\mathcal{E}_i\left\{\frac{1}{|\nu(i)|}\sum_{j\in\nu(i)} J_{ij}\omega_j\right\}\right)\right]}, \tag{2}$$

$\beta = 0:$ **purely random choice**

Mean Field Theory

$$\mathcal{E}_i(\omega_j) = m, \tag{3}$$

$$\omega_i \frac{1}{|\nu(i)|} \sum J\mathcal{E}_i(\omega_j) = \omega_i Jm.$$

Without loss of generality, $h := \frac{1}{2}(u(1) - u(-1))$, $h_0 := \frac{1}{2}(u(1) + u(-1))$. **Then:**

$$u(\omega_i) = h\omega_i + h_0,$$

$$m = \tanh(\beta h + \beta Jm), \tag{4}$$

where $\tanh(x)$, **hyperbolic tangent, defined as:**

$$\tanh(x) \equiv \frac{\exp(x) - \exp(-x)}{\exp(x) + \exp(-x)}, \ -\infty < x < \infty.$$

mft

Brock and Durlauf (2001) Theorem: Equ. (4) may have three or a unique root, depending on parameter values.

Conditional on a given private utility difference between the choices 1 and -1, which equals $2h$, there is a level which the conformity effect must reach in order to produce multiple self-consistent mean choice behavior. However, as βh increases in magnitude, the importance of the conformity effect βJ diminishes in a relative sense, and thus becomes unable to produce a self-consistent mean with the opposite sign.

If there exist three equilibria, we will refer to the "middle" one (m^*), as *symmetric* **and to the other two as** *asymmetric* (m^*_-, m^*_+) **[Figure 1].**

Economic fundamentals that drive private decisions and social norms play complementary roles. Even if private incentives, expressed by h, favor a particular decision, sufficiently strong social conformity effects can bring about equilibria, in which most teenagers conform, say by dropping out of school.

Individually optimal but collectively undesirable behavior.

Complete pairwise interaction: **a way to visualize mean field theory.**

“WALRASIAN-STAR” INTERACTION:

$$U_1(\omega_1) \equiv h\omega_1 + \omega_1 J_S \frac{1}{I-1}\mathcal{E}_1\left\{\sum_{i=2}^{I}\omega_i\right\} + \epsilon(\omega_1); \tag{5}$$

$$U_i(\omega_i) \equiv h\omega_i + \omega_i J \mathcal{E}_i\left\{\omega_1\right\} + \epsilon(\omega_i);\ i = 2,\ldots,I. \tag{6}$$

$$\mathcal{E}_i\left\{\omega_1\right\} = m_1, i = 2,\ldots,I; \quad m_{-1} = \mathcal{E}_1\{\omega_i\}, i \neq 1.$$

$$m_1 = \tanh(\beta h + \beta J_S m_{-1}); \tag{7}$$

$$m_{-1} = \tanh(\beta h + \beta J m_1). \tag{8}$$

The equilibria in this economy are the fixed points of the system of (7) and (8).

More possibilities than the mean field case. E.g., if both interaction coefficients are positive, there is always at least one root that is in the positive orthant of (m_1, m_{-1}) **space: conformism is an equilibrium.**

Walrasian Star

CIRCULAR INTERACTION:

$$U_i(\omega_i) \equiv h\omega_i + \omega_i \frac{1}{2} \mathcal{E}_i \left\{ J_B \omega_{i-1} + J_F \omega_{i+1} \right\} + \epsilon(\omega_i). \qquad (9)$$

Under common expectations, $m_j = \mathcal{E}_i \{\omega_j\}$

$$m_i = \tanh[\beta h + \frac{1}{2}\beta(J_B m_{i-1} + J_F m_{i+1})], \; i = 1, \ldots, I. \qquad (10)$$

I **equations in the** I **unknowns** $m_i, \; i = 1, \ldots, I$, **with "initial" conditions** $m_{I+1} = m_1$, **and** $m_{I+2} = m_2$.

Two classes of solutions, isotropic and anisotropic ones:

Isotropic solutions of (10) are like the mean field case.

Examination of anisotropic solutions similar to existence of periodic solutions to nonlinear difference equations. They exist if $I = 3$. **They are due to the nonlinearity of the problem and are "far" from the isotropic ones.**

One-dimensional lattice (interactions along a line).

Local interaction, but presence of two end agents: do they make a difference? Consider:

"If everybody needs 100 Watts to read by and a neighbor's bulb is equivalent to half one's own, and everybody has a 60-Watt bulb, everybody can read as long as he and both his neighbors have their lights on. Arranged on a circle, everybody will keep his lights on if everybody else does (and nobody will if his neighbors do not); arranged in a line, the people at the ends cannot read anyway and turn their lights off, and the whole thing unravels" [Schelling (1978)].

$$m_{-L} = \tanh[\beta h + \beta J_F m_{-L+1}]; \tag{11}$$

$$m_L = \tanh[\beta h + \beta J_B m_{L-1}]. \tag{12}$$

and Equ. (10), for $i = -L+1, \ldots, L-1$.

Again, two classes of solutions, isotropic and anisotropic ones:

Isotropic solutions of (10) are like the mean field case. Isotropic solutions for the end agents are obtained in the obvious way from the isotropic solutions for $i = -L + 1, \ldots, L-1$.

REMARKS:

Social equilibrium may be characterized by aggregate uncertainty, even when individual states are purely random. That is, consider the case when $h = 0$, **the two states equally likely in terms of fundamentals. Then, even in the mean field case, the economy has three isotropic equilibria, associated with the roots of Equ. (4), for** $h = 0$.

$m = 0$, **no aggregate uncertainty.**

If $\beta J > 1$, **then the other two roots imply aggregate activity.**

Emergence of aggregate activity is due to the synergistic effects operating at the individual level.

"Aggregation is not summation!"

What if interactions are based on agents' actual environments? A number of interesting results:

- **Establish conceptual link with econometrics of systems of discrete choice models.**
- **One-dimensional Ising model from statistical mechanics admits a decentralized interpretation, along the lines of the circular interaction model.**

A GENERAL FRAMEWORK FOR DYNAMICS

The state of the economy at time t, $\tilde{\omega}_t$, depends upon the actual state of each agent's neighbors, $\tilde{\omega}_{\nu(i),t-1}$, that pertains to agent i's neighbors.

$$\textbf{Prob}\left\{\omega_{i,t} = 1 | \tilde{\omega}_{\nu(i),t-1}\right\} = \frac{\exp\left[\beta h + \beta \frac{1}{|\nu(i)|} \Sigma_{j\in\nu(i)} J_{ij}\omega_{j,t-1}\right]}{1 + \exp\left[\beta h + \beta \frac{1}{|\nu(i)|} \Sigma_{j\in\nu(i)} J_{ij}\omega_{j}\right]}. \tag{13}$$

Therefore, for each of the 2^I possible realizations of $\tilde{\omega}_{t-1}$, $\tilde{\omega}_{t-1} \in \underbrace{\{-1,1\} \times \ldots \times \{-1,1\}}_{I}$, Equ. (13) defines conditional choice probabilities for each agent in each of the models of social interaction.

Transition probabilities from Equ. (13) are in effect fixed transition probabilities for each of the 2^I possible realizations of $\tilde{\omega}_t$.

Transition probabilities for the dynamic counterparts of the mean field, Walrasian star, circular interaction and interaction along a line: follow by adapting Equ. (13) in the obvious way, according to the definition of agent i's neighborhood, $\nu(i)$.

State of the economy: evolves according to a Markov stochastic process, defined from the finite (but large, if I is large) sample space $\underbrace{\{-1,1\} \times \ldots \times \{-1,1\}}_{I}$, into itself and has fixed transition probabilities.

Dependence of the transition probabilities on the spatial details that characterize each of the topologies we are studying suggests that the process may be non-ergodic, in general: there may regions of starting values for each individual in the economy, (basins of attraction), for which different individuals may converge to different states.

DYNAMICS FOR THE STYLIZED TOPOLOGIES: under MYOPIC EXPECTATIONS.

Definition: each individual's expectation of her neighbor's choice at time t **is equal to that individual's** *expected choice* **at time** $t - 1$.

I.e., not *actual*, **as above.**

- **COMPLETE PAIRWISE: equivalent to mean field case, examined by Brock and Durlauf.**

  If there are three isotropic equilibria: symmetric is unstable; asymmetric ones stable.

- **WALRASIAN STAR: qualitatively similar to mean field case; some additional possibilities, depending on parameter values.**

- **CIRCULAR, ONE-DIMENSIONAL LATTICE (INTERACTION ALONG THE LINE): qualitatively different!**

  Let us see why!

- **DYNAMICS FOR THE CIRCULAR INTERACTION TOPOLOGY**

  Using tools proposed by Turing (1952), Glauber (1963):

  Local interaction plus circular symmetry introduce SPATIAL OSCILLATIONS, interpreted as SPATIAL CLUSTERING, with PERSISTENCE:

  if symmetric equilibrium disturbed, some agents will end up in one asymmetric equilibrium, others at the other.

- **DYNAMICS FOR THE INTERACTION ALONG THE LINE**

  Local interaction introduces transitory SPATIAL OSCILLATIONS, interpreted as SPATIAL CLUSTERING.

Conclusion: local interaction is responsible for clustering, closure is responsible for persistence.

NOTEWORTHY EXTENSIONS

- **Local interaction and global interaction: can be handled with similar tools.**

- **Extension clearly deserves future research: Link with economies with interacting agents literature, as studied by Allan Kirman: allow agents to choose whom to interact with; examine the speed of adjustment when agents do have such choice.**

SOCIAL INTERACTIONS: EMPIRICS
Recall:

- **A. Structural estimation of systems of discrete decisions**

  Dependence on expected vs. actual behavior of neighbors: reflects spatial structure.

- **B. Variance of individual decisions as function of variance of individual shocks: depends on topology of interaction.**

- **C. Applications with Continuous Decisions**

I will emphasize items B and C.

B. Variance of individual decisions as function of variance of individual shocks: depends on topology of interaction.

Let W_i denote the response, ϵ_i an individual-specific shock, I.I.D. Consider linear response rule:

$$W_i = a + \frac{1}{|\nu(i)|} J \sum_{j \in \nu(i)} W_j + \epsilon_i, \tag{14}$$

- **Complete pairwise interaction:**

$$\mathbf{Var}(W_i) = \sigma_\epsilon^2 \left(1 + \left(\frac{J_P}{1 - J_P} \right)^2 \frac{3(I-1) - 2J_P(I-2) - J_P^2}{(I - 1 + J_P)^2} \right). \tag{15}$$

- **Circular interaction, with when $J_F = 0$:**

$$\mathbf{Var}(W_i) = \frac{1 + J^I}{1 - J^I} \frac{\sigma_\epsilon^2}{1 - J^2}. \tag{16}$$

I have successfully estimated such models with housing consumption data from the neighborhood clusters subsample of the American Housing Survey; for each respondent, residential neighbors are also sampled.

Papers may be found at my department's web page: http://ase.tufts.edu/econ

C. Applications with Continuous Decisions

1. INTERGENERATIONAL TRANSMISSION OF HUMAN CAPITAL

Fix ideas and set notation with a linear model [Kremer (1997); Ioannides (2001)] for the intergenerational transmission of human capital:

$$\Omega_{it+1} = a_0 + \frac{\alpha}{2}(\omega_{it} + \omega_{i't}) + \beta\omega_{\nu(i)t} + \epsilon_i, \quad (17)$$

where a_0 denotes an exogenous intercept, $\omega_{\nu(i)t}$, average education in the neighborhood of i's upbringing, $(\omega_{it}, \omega_{i't})$ parents' human capital, and ϵ_i a stochastic shock.

I have estimated this model with data from Panel Study of Income Dynamics, augmented with geocoded data on distribution of human capital within neighborhood of upbringing.

I have obtained significant nonlinear effects, in contrast to Kremer (1997), working with parametric and nonparametric models. Nonlinearity is critical for multiplicity of equilibria. The effects i have identified make the time map sigmoid.

My paper is forthcoming in the Drandakis *Festschrift,* edited by Bitros and Katsoulakos.

Simple model admits a broader interpretation: e.g., human capital of parents are interdependent through marital sorting, an important form of social interactions!

Residential sorting, marital sorting and educational outcomes have yet to be combined empirically.

2. RESIDENTIAL NEIGHBOR INTERACTIONS

Housing demand by individual i in neighborhood x:

$$\omega = \alpha + \beta E[\omega|x] + E[z|x]'\gamma + z'\eta + \epsilon, \quad E[\epsilon|x,z] = x'\delta. \quad (18)$$

Under Nash equilibrium,

$$E[\omega|x,z] = \frac{\alpha}{1-\beta} + E[z|x]'\frac{1}{1-\beta}(\gamma + \beta\eta) + z'\eta + x'\frac{1}{1-\beta}\delta. \quad (19)$$

The effect of the expected behavior of neighbors, β, may not be identified separately from the effect of the expected characteristics of neighbors, γ.

That is, two social effects may not be identified separately, unless:

- **instruments may be found, whereby the neighborhood average of a characteristic, $E[z|x]$, is not a causal variable;**
- **or, the model is nonlinear.**

The former is hard to do, except paradoxically, when individuals choose the neighborhoods [Brock and Durlauf (2001)].

So, we see how important nonlinearity is for the identification of the effects of social interactions.

I have successfully estimated such models with housing consumption data from the neighborhood clusters subsample of the American Housing Survey; for each respondent, residential neighbors are also sampled.

CONCLUSION

- **SOCIAL INTERACTIONS:**

  Theory

  Theory of economies with interacting agents: a rich paradigm.

- **SOCIAL INTERACTIONS:**

  Empirics

  Theory is critical for the establishing econometric identification of impact of social structure on economic decisions.

  Data sets with contextual information are increasingly available.