the second derivative of $P_W(t)$ with respect to time we can work as follows. Equation (4.10) by differentiation yields

$$\ddot{y} = B\dot{y} - L\frac{d\varphi}{dy}\dot{y} = (B + LG(y))\dot{y} ;$$

since $\dot{y} < 0$, $\ddot{y}$ is negative. For the optimal reservation price curve from (4.9), by differentiating we obtain $\ddot{y} = -u'\ddot{P}_W + u''(\dot{P}_W)^2$. Hence:

$$\ddot{P}_W = -\frac{\ddot{y} - u''(\dot{P}_W)^2}{u'} . \tag{4.12}$$

From (4.12) we have: if the prospective buyer is risk-preferring ($u'' > 0$) or risk-neutral ($u'' = 0$), $\ddot{P}_W$ is positive. The individual's reservation price increases at an increasing rate. If, on the other hand, the prospective buyer is risk-averse ($u'' < 0$), the sign of $\ddot{P}_W$ cannot be determined in general.

For particular forms of the utility function more specific results can be obtained. For example, if we assume a utility function of the type $U = \rho(t)(W-P)^u$, where u is a constant less than one, it can be shown that $\ddot{P}_W$ is negative in the neighborhood of $t = T$ and it may change sign during search. Whether or not the second derivative of P_W with respect to time changes sign during search also depends upon the time-horizon of search.

<u>Asking price dispersion and reservation price</u>. We shall now examine the relationship between reservation price (and hence expected utility from the transaction) and dispersion characteristics of the asking price distribution. Let us consider two distributions f_1 and f_2 with the same mean and different dispersion characteristics. Distribution f_2 is exhibiting higher concentration around the mean than f_1. The monotone transformation $u = u(W-P)$ allows us to obtain the

utility distribution functions f_{u_1} and f_{u_2} that correspond to the asking price distribution functions f_1 and f_2. In general, f_{u_1} and f_{u_2} may not have the same dispersion relationship to each other as f_1 and f_2. In the particular case of risk-neutral individuals, the discussion in section 3.3 applies. Therefore, the expected utility of the search process is lower with greater concentration around the mean. Hence the optimal reservation price is higher in the case of greater concentration of sellers' asking prices around the mean than in the case of lower concentration. This result suggests that buyers may be better off by following a "naive" search rule rather than an optimal stopping rule in the case of low dispersion of asking prices.

Shift of the asking price distribution and reservation price. We can examine the effect upon the reservation price curve of an upward shift in the distribution of asking prices by using the results obtained in section 3.3. Since $u(W-P)$ is a decreasing function of P, an upward shift in the distribution of asking prices, f, will translate to a downward shift in the utility distribution, f_u. The latter will yield a lower expected utility from the process, $y(t)$. Finally, the optimal reservation price will be higher at all times except $t = T$.

Value of a housing unit to a buyer and reservation price. The value of a housing unit to a buyer, W, was defined earlier in this chapter as the maximum net benefits from owning and occupying the unit, net of maintenance costs and taxes, plus its value at the termination of ownership, all discounted to the time when ownership begins. W can vary among different individuals because of such factors as differences in preferences, income, cost of the technology of maintenance, sale value of a unit at the termination of ownership, taxes, etc. It is thus interesting to predict effects that differences in W might have upon the optimal reservation price.

Let us consider the case of a small increase in W. This will cause an upward shift in the utility distribution, f_u, which will yield a higher expected utility from the process, $y(t)$. Unfortunately, the direction of the change in $P_W(t)$ cannot be predicted.

$$dP_W = \left(1 - \frac{1}{u'} \frac{\partial y}{\partial W}\right) dW \ . \tag{4.13}$$

The partial derivative of P_W with respect to W can be obtained from (4.11) but the sign of the term in parentheses on the right-hand side of (4.13) cannot be obtained.

4.4 Buyers' Use of Informational Intermediaries and Search Costs

As in our discussion of a seller's decision problem, we shall restrict ourselves to the simple case of a buyer participating in a specific quality submarket under price competition and finite time-horizon of search. A buyer can vary the rate at which he makes contacts with sellers by searching the market on his own and/or making use of informational intermediaries (real estate brokers, advertising, housing opportunity centers, etc.). Again, we shall restrict ourselves to personal search. With a finite time-horizon of search and because it takes time to generate and make contacts, there is an upper bound upon the expected number of contacts, q, that a buyer can make from time $t = 0$ to time $t = T$. In order to draw a two-dimensional diagram, we can lump all inputs to activities for identifying vacancies and generating contacts into a single variable, i. We will not go into a description of how such a information production function is generated, but we should note that the number of vacancies enters as a parameter of the production function. For different values of V, we obtain a family of production functions. Let us assume that the family is of a form as in Fig. 4.2.

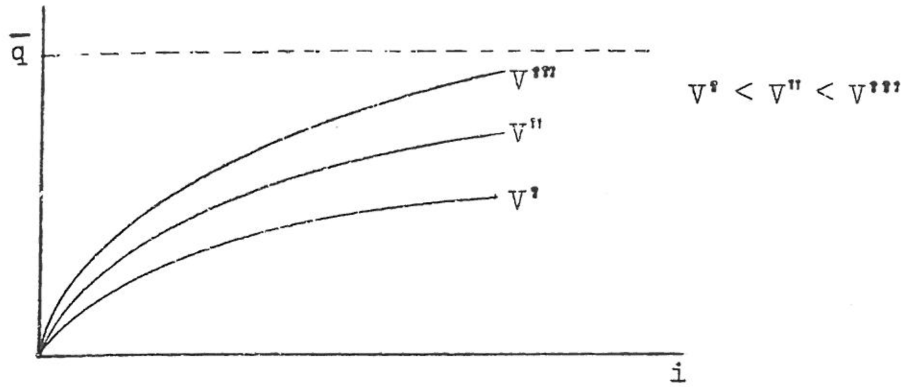

Fig. 4.2

Given prices for the various inputs that are lumped into i, one can obtain the efficient combinations and, thus, the minimum cost for obtaining any given level of expected number of contacts. As a result of these assumptions, the cost function exhibits the following properties:

$$\frac{dC}{dq} > 0, \quad \frac{d^2C}{dq^2} > 0 \tag{4.14}$$

i.e., marginal cost, $\frac{dC}{dq}$, is an increasing function of q.

A buyer is assumed to choose q, given $q = LT$, where L is the intensity of the process of contacts and T is the given time-horizon of search. Since T is fixed, L is what is implicitly chosen. Since the value of the transaction at $t = 0$, when an optimal policy is followed, depends upon L, the optimal L^* is the one that maximizes the utility form of the transaction, after having accounted for search costs. Furthermore, fixed transaction costs would not alter the results, while they would affect the participation decision. Transaction costs that depend upon the transaction price can be easily incorporated by adjusting the probability distribution function of asking prices. Search costs of the above type are assumed to be incurred in the beginning of a consumer's search. An alternative approach would have been to assume that they depend upon the actual duration of search.

In our behavioral model for a buyer, the discount factor $\rho(t)$ was assumed to reflect psychic costs of search. The search costs we are discussing here are pecuniary ones. Hence, they can be included in the argument of the utility function. It was shown earlier than an increase in the intensity of the process of contacts will increase the value of the search process. However, if search costs for ensuring an increased intensity are subtracted from the net economic surplus the value of the search process decreases. Therefore, intuitively, there is a trade-off. We will now proceed to extend the behavioral model for a buyer in order to include search costs.

For a (constant) intensity of the process of contacts, $\ell(t) = L$, and an exponential discount factor, $\rho(t) = e^{-Bt}$, let us consider the problem of maximizing y_o,

$$y_o = E\{e^{-B\tilde{T}}u(W - C(LT) - \tilde{P})\} , \tag{4.15}$$

with respect to L. $\tilde{T}$ and $\tilde{P}$ are, respectively, the time when the transaction takes place and the price at which it is carried out. For any given L, y_o--the optimal expected utility from the process--is given implicitly by

$$\int_0^{y_o} \frac{dy}{L\varphi(y) - By} = T . \tag{4.16}$$

Since the function φ, in the denominator of the integrand above, is now defined in terms of the distribution function of utilities, f_u, it depends upon L. The distribution function f_u is obtained from the asking price distribution by a change of variable according to $p = W - C(LT) - u^{-1}(y)$.

We can show that a solution to the problem of maximizing y_o with respect to L exists.

Theorem 4.3. There is a finite intensity of the process of contacts, L^*, that maximizes the expected utility from the search process, y_o, defined in (4.15) and given implicitly by equ. (4.16).

Proof. The proof is based on the boundedness of y_o if L increases and on the properties of the cost curve, $C(LT)$.

If we exclude search costs, y_o is an increasing but bounded function of L. This is apparent from equ. 4.11 and Fig. 4.2. The upper bound is given by $u(W)$. Also, we asserted earlier in this section that there is a finite upper bound, $\bar{q}$, upon the expected number of contacts, q, a buyer can make with sellers. Hence as q tends toward $\bar{q}$, the cost of ensuring q tends asymptotically toward infinity. Therefore, when we include search costs, the expected utility from the search process starts decreasing before q reaches its upper bound. Or alternatively, for any asking price, P_a, we can find an L large enough to make $u(W-C(LT)-P_a)$ negative. Therefore, the expected utility y_o can also be made negative. Let $\bar{L}$ be the (smallest) intensity of contacts that makes $y_o = 0$; y_o is a continuous function of L over the interval $[0,\bar{L}]$. Hence, according to the Weierstrass theorem, y_o attains a maximum on $[0,\bar{L}]$. Since y_o is positive on $[0,\bar{L}]$, maximizing L is feasible. Q.E.D.

The questions of the uniqueness of this solution will not be pursued here. However, it can be guaranteed by requiring that the utility function be concave.

Necessary and sufficient conditions for an interior maximizing point can be written as follows:

$$\frac{\partial y_o}{\partial L} = 0, \qquad \frac{\partial^2 y_o}{\partial L^2} < 0 . \tag{4.17}$$

Differentiating (4.16) with respect to L yields:

$$\frac{1}{L\varphi(y_o) - By_o} \frac{\partial y_o}{\partial L} - \int_o^{y_o} \frac{\varphi dy}{(L - By)^2} - \int_o^{y_o} \frac{L \frac{\partial \varphi}{\partial L} dy}{(L\varphi - By)^2} = 0 . \tag{4.18}$$

Since $L\varphi(Y_o) - By_o$ is positive, the first order condition from (4.17) becomes

$$\int_o^{y_o} \frac{(\varphi + L \frac{\partial \varphi}{\partial L}) dy}{(L\varphi - By)^2} = 0 . \tag{4.19}$$

We will now show that $\frac{\partial \varphi}{\partial L}$ is negative. The distribution of utilities, f_u, is obtained from the distribution of asking prices, $f(p)$, through the transformation $p = W - C(LT) - u^{-1}(y)$. $\varphi(y)$ can thus be written as follows:

$$\varphi(y) = \int_y^{u(W)-C(LT)} G(y)dy, \quad \text{where} \quad G(y) = \int_o^{W-C(LT)-u^{-1}(y)} f(p)dp .$$

Hence $\frac{\partial \varphi}{\partial L}$ is negative.

Unfortunately, due to the complexity of the functional forms, we cannot precisely characterize an interior solution to the problem of choosing an optimal intensity of the process of contacts. If search costs are high, compared to the net consumer surplus an individual derives from owning and occupying a housing unit, the corner solution

$L = 0$ may be the optimal one. At any rate, the above discussion suggests that an optimal intensity of the process of contacts exists and is bounded.

The optimal L^* depends upon the number of vacancies, V, that will prevail in the submarket at equilibrium. For the purposes of our discussion of submarket equilibrium we shall assert that $L^*(V)$ is an increasing function of V. Unfortunately, the complicated mathematics of the extension of the behavioral model for a buyer preclude us from proving this assertion. It should be noted, however, that the assumptions made earlier about the dependence of the information production functions upon the number of vacancies in the market and the properties of the expected utility of the search process, inclusive of search costs, suggest strongly that the above assertion is reasonable.

There are many other ways in which search costs can be incorporated into the behavioral model for a buyer. For example, we could explicitly consider a market for informational services, or we could allow search costs to vary with the duration of search. We chose this particular model because it is consistent with our specification of the informational structure of the market. Furthermore, as we shall see in the next chapter, this model describes well the behavior of buyers when the submarket is at equilibrium.

4.5 Summary and Conclusions

This chapter has been concerned with behavioral models for buyers under unit and price competition. Buyers were assumed to conduct their search so as to maximize a utility index of the von Neumann-Morgenstern type that accounts for psychic costs of search and the net consumer surplus derived from purchasing a housing unit for the buyer's occupancy.

A behavioral model for a buyer under price competition was thoroughly analyzed. A buyer's behavior was completely described in terms of a reservation price curve. Necessary and sufficient conditions for optimality were obtained. For the case of an exponential search cost factor, constant intensity of the process of contacts, and finite search-horizon it was shown that the optimal reservation price is an increasing function of time.

Our discussion in this chapter has emphasized the symmetry between the behavioral model for a seller under unit competition and the behavioral model for a buyer under price competition. We obtained most of our results by reference to corresponding sections in Chapter Three.

Some of the results obtained can be summarized as follows: An increase in the search cost coefficient will increase the reservation price while an increase in the intensity of contacts will decrease it, and a longer time-horizon of search and a downward shift of the asking price distribution will cause a downward shift of the optimal reservation price curve. For risk-preferring and risk-neutral individuals it was shown that the optimal reservation price increases at an increasing rate. It was also shown that for risk-neutral individuals the optimal reservation price is higher in the case of greater concentration around the mean of the distribution of asking prices.

Finally, the model was amended to account for the use of informational intermediaries and search costs prospective buyers incur in order to ensure a certain "stream of contacts" with sellers.

Chapter Five

QUALITY SUBMARKET EQUILIBRIUM

5.1 Introduction

The notion of equilibrium is of great importance in analyses of most dynamic systems that arise in the physical and social sciences. It should be remembered, however, that equilibrium is not interesting in itself, but rather for its usefulness in analyzing complex dynamic systems. Such systems may or may not have intrinsic equilibrium properties.

As M. Rothschild argues, "models of disequilibrium behavior do not make sense and cannot serve as reliable guides to further theorizing or to policy unless they meet certain standards of consistency and coherence."[1] Most disequilibrium models, however, provide for the existence of and/or convergence to an equilibrium which is characterized by a unique price. If, on the other hand, there is reason to believe that price dispersion persists, for reasons inherent in the market, then a new equilibrium concept is needed.

We shall explore such a concept in this chapter. The theoretical arguments which will be provided here are of a fairly general nature. With a few simple modifications they can be applied to markets other than housing markets.

In Chapter Three, we examined behavioral models for sellers. It was shown that sellers' behavior can be described in terms of critical and asking price curves, depending upon the form of market organization. Under the conditions of exponential discount factors, finite time-horizon

[1] Rothschild (1971), p. 5.

of search, and constant intensity of the process of contacts, critical and asking prices decrease over the time sellers stay in the market. In Chapter Four, it was shown that buyers' behavior can be described by means of reservation price and willingness to pay curves. Again, under the three conditions noted above, reservation prices and willingness to pay[2/] increase over the time buyers stay in the market in search of sellers. Prospective buyers and sellers who are in the market at any one time have entered the market at different times in the past. Therefore at any moment of time, we have a distribution of asking (critical) prices on the supply side of the market and a distribution of reservation prices (willingness to pay) on the demand side.

Our goals in this chapter are first to integrate the behavioral models for buyers and sellers into a theory of market equilibrium adjustment, next to characterize equilibrium for the submarket. We shall confine our attention to a price competition view of the market and use behavioral models under exponential discount factors and finite search horizons. The "duality" between unit and price competition ensures that similar results will be obtained if unit competition is assumed.

The discussion in this chapter will proceed as follows. First, a consistency condition regarding the specification of the informational structure of the market will be examined. Then by holding each side of the market at stationary equilibrium, we shall obtain differential equations describing the short-run dynamics of adjustment on the other side of the market. These adjustment equations will be used to

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2/ This property of an optimal willingness to pay curve is stated by analogy, since the behavioral model of buyers under unit competition was not thoroughly examined.

characterize equilibrium for the submarket to examine stability and to obtain comparative static results.

5.2 A Consistency Condition and the Informational Structure of the Market

Our behavioral models for sellers and buyers were analyzed separately. Both depend upon market conditions and this necessitates our examining the compatibility of relationships that were separately assumed for each of the two models. Let us assume for simplicity that the market is organized under price competition, and various factors are held fixed so that, as we noted earlier, the multigenerational composition of the stocks of prospective buyers and sellers remains as the only source of dispersion.

Let X^t be the number of all prospective buyers in the market at time t. Individuals numbered in X^t entered the market at different times in the past. The reservation prices of these individuals are characterized by a distribution function, $f_X(p;t)$, that reflects the intertemporal adjustment of reservation prices weighed by the relative distribution of individuals among generations. Also, let V^t be the number of all vacant units in the market, or, identically, all sellers in search of buyers, at time t. In like manner, vacancies numbered in V^t correspond to various generations of sellers. Similarly, the distribution of asking prices, $f_V(p;t)$, reflects the intertemporal adjustment of asking prices, weighed by the relative distribution of sellers among generations.

A basic requirement for the consistency of the model is that the expected rate at which consumer searchers find and buy housing units, $x(t)$, be equal to the rate, as seen from the supply side, at which housing units are bought, $v(t)$. In the remainder of this section we shall show that the requirement $x(t) = v(t)$ can be reduced to a condition that involves the intensities of the process of contacts for buyers and sellers, $\ell(t)$ and $z(t)$ respectively, and the numbers of prospective buyers and sellers in the market, X^t and V^t respectively. This result can be stated as a lemma.

Lemma 5.1. Under the assumptions of price competition and stationarity of the distributions of reservation and asking prices, the consistency condition $x(t) = v(t)$ is equivalent to

$$\ell(t)X^t = z(t)V^t \ . \tag{5.1}$$

Proof. To derive an expression for $x(t)$, we work as follows: ℓX^t is the rate at which contacts are made. The contribution to $x(t)$ from the interval $[p,p+dp]$ is $\ell X^t f_X(p)dp$ multiplied by the probability that the asking price of any seller would not exceed P, $F_V(p)$ (the cumulative of f_V). Hence:

$$x(t) = \ell(t)X^t \int_{p'}^{p''} F_V(p)f_X(p)dp \ . \tag{5.2}$$

In like manner, we can obtain an expression for $v(t)$:

$$v(t) = z(t)V^t \int_{p'}^{p''} (1 - F_X)f_V dp \ . \tag{5.3}$$

By integrating by parts the integral on the right-hand side of (5.3) becomes identical to the one on the right-hand side of (5.2). Hence, the consistency condition $x(t) = v(t)$ becomes: $z(t)V^t = \ell(t)X^t$. Q.E.D.

If we assume that market participants possess perfect information about market conditions, then equ. (5.1) clearly is a consistency condition. When equ. (5.1) is considered together with submarket equilibrium conditions, to be obtained later in this chapter, then it functions as a condition that guarantees informational equilibrium. That is, since different individuals make decisions about their use of informational intermediaries, which effectively determine z and ℓ, their estimates of z and ℓ have to be compatible in an equilibrium context. Alternatively, equ. (5.1) can be used to define z, given ℓ, if initiative in search is attributed to buyers. That is, the intensity of the process of contacts that sellers make with prospective buyers, which sellers use to calculate their optimal asking price policies, has to be compatible with buyers' expectations.

For the purposes of the discussion in this chapter, initiative in search will be attributed to buyers. Sellers are "passive" searchers. Hence, assuming stationary V^t, $V^t = V$, ℓ can be replaced by $L^*(V)$, which was obtained in section 4.4. Furthermore we shall assume that $L^*(V)$ is linear in V. (Some of the implications of relaxing this assumption will be examined later in the chapter.) Thus, if X^t is stationary, $X^t = X$, $z(t)$ is constant, independent of time. In sum, we have obtained a condition, involving only the size and the informational structure of the market, under which a basic requirement for the consistency of the model is satisfied. This consistency condition was also interpreted as an informational equilibrium condition. Finally an assumption was made regarding the informational structure of the market. Under this assumption, stationarity of the number of market participants on one side of the market implies constant intensity of the process of contacts for participants on the other.

5.3 The Short-Run Dynamics of Adjustment and the "Flow-Through" Equilibrium

Introduction. As noted by R. Radner[3/], J. R. Hicks distinguished two notions of equilibrium: temporary equilibrium, in which at a given date supply equals demand, and equilibrium over time, which he defined by "the condition that prices realized (on each date) are the same as those which were previously expected to rule at that date." The latter notion, simply put as "desired equals actual," is the cornerstone of much of microeconomic as well as macroeconomic equilibrium analysis.

In our opinion, this notion of equilibrium is a meaningful and analytically useful concept, if it can be associated with some adjustment process, which in turn is intimately related to the structure of information flow in the market. Adjustment processes of the tâtonnement-type have traditionally been most popular in the economic literature. In the recent literature on the economics of uncertainty and equilibrium under uncertainty one can find examples of adjustment processes that are essentially not of the tâtonnement-type. We discussed earlier why such models may not be applicable to housing markets.

In this section we shall examine equilibrium for a quality submarket under conditions of price competition. Individuals make decisions whether to participate in the market or not. Once they decide to participate they set their search policies. Because both prices and quantities change in a way too complicated to be analyzed directly, we shall decompose the problem as follows. We shall first discuss the short-run dynamics of adjustment for each side of the market while

3/ Radner (1973), p. 1.

the other side is held at equilibrium characterized by a stationary price distribution and number of market participants. Once we have obtained some results that could illuminate the workings of the market model, we shall examine an overall equilibrium. The emergence of distributions of reservation and asking prices is explained by the fact that market participants adopt optimal search policies.

The Short-Run Dynamics of Adjustment and the "Flow-Through" Equilibrium: The Demand Side

Under the assumption that the supply side is held at stationary equilibrium--that is, the distribution function of asking prices and the number of sellers in the market are stationary--we shall derive a set of differential equations which describe the adjustment over time of the numbers of prospective buyers with different reservation prices.

Under price competition, sellers set optimal asking price policies and buyers search by adopting optimal reservation price policies. A prospective buyer enters a particular quality submarket at $t = 0$. His reservation price, given a distribution of asking prices, is defined at any point in time while he remains in the market. By setting a finite time-horizon for his search, he leaves the market when he finds a suitable housing unit or at the end of his time-horizon if he does not find one. We shall assume that all prospective buyers possess perfect knowledge of the distribution of asking prices.

For simplicity, let us consider the price range (P', P''), which characterizes the particular quality submarket under examination as consisting of a finite number of prices p^i, $i = 1,m$. Generality is not lost by such a discretization because m can be very large. Let

X^t be the number of prospective buyers in the market at time t, and X_i^t, $i = 1, \dots, m$, the numbers of prospective buyers with reservation prices equal to p^i, $i = 1, \dots, m$. Clearly, by definition:

$$X^t = \sum_{i=1}^{m} X_i^t .$$

Similarly, let V_i^t be the number of vacancies, or sellers, with asking prices equal to p^i. Hence:

$$V^t = \sum_{i=1}^{m} V_i^t .$$

The intensity function of the process of contacts that prospective buyers make with sellers, $\ell(t)$ is constant (independent of time, $\ell(t) = L$) at equilibrium.[4/] If we also assume no differential access to information, then L is common among all participants on the demand side of the market. Finally, contacts are made continuously and transactions are finalized without any time lag for market adjustment purposes.

For each of the individuals with reservation prices equal to p^i, the probability that a contact will result in a transaction is equal to:

$$g_i = \sum_{k=1}^{i} \frac{V_k^t}{V^t} . \tag{5.4}$$

With a discretized price range, prospective buyers' optimal reservation price curves can be interpreted as follows: on the average, searchers "stay" at p^i, $i = 1, \dots, m$, i.e., keep their reservation prices equal to p^i, for T_i units of time. The T_i's are obtained

4/ This is a crucial feature of our equilibrium model; it facilitates aggregation and is also essential to our comparative statics analysis.

from the optimal reservation price curves by discretization (see Fig. 5.1). For a buyer, from the specification of the process of

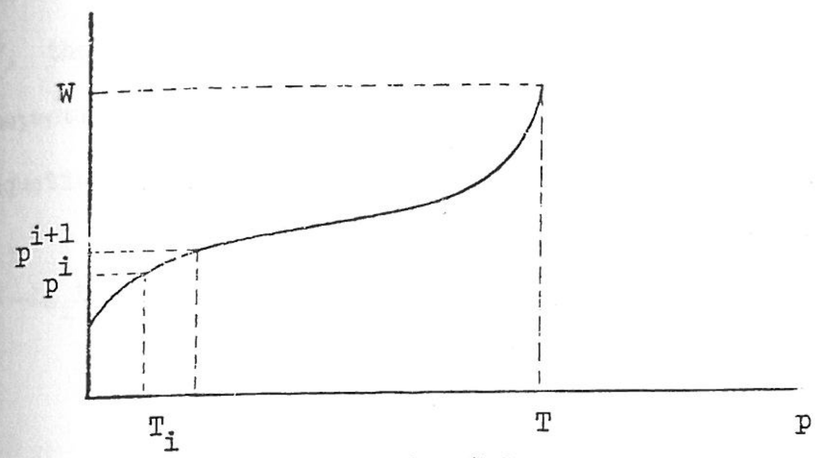

Fig. 5.1

contacts under price competition, we obtain the following: the probability that he will find a suitable housing unit and leave the submarket is equal to Lg_i per unit of time, the probability that he will make no contacts during his tenure at the level p^i is equal to e^{-Lg_i}, and the probability that he will make no successful contacts is equal to $e^{-Lg_i T_i}$. Individuals at level p^m who do not find houses during T_m leave the submarket. We shall assume that new participants start their search at the lower end of the price range.

At a price level p^i, on the average and per unit of time, $Lg_i X_i^t$ individuals leave the submarket, having found and bought houses; $\frac{1}{T_i} e^{-Lg_i T_i} X_i^t$ individuals adjust their reservation prices to p^{i+1} and thus "leave" the price level p^i, because they made no effective contacts with sellers during their tenure at p^i; and $\frac{1}{T_{i-1}} e^{-Lg_{i-1}T_{i-1}} X_{i-1}^t$ individuals "enter" the price level p^i from p^{i-1}, having made no contacts or no effective contacts during their tenure at the price level p^{i-1}.

New prospective buyers enter the submarket continuously over time at a rate $\bar{x}$. We shall make the important assumption that new prospective buyers start their search with reservation prices equal to p^1, the lower end of the price range. Having described the "flow" of prospective buyers through the price range, we can now write adjustment equations for the numbers of buyers in the market, $X_1^t, X_2^t, \ldots, X_m^t$.

$$\dot{X}_i^t = -Lg_i X_i^t - \frac{1}{T_i} e^{-Lg_i T_i} X_i^t + \frac{1}{T_{i-1}} e^{-Lg_{i-1} T_{i-1}} X_{i-1}^t, \qquad i = 2, \ldots, m$$

and

$$\dot{X}_1^t = -Lg_1 X_1^t - \frac{1}{T_1} e^{-Lg_1 T_1} X_1^t + \bar{x} .$$

Write:

$$\theta_i = \frac{1}{T_i} e^{-Lg_i T_i}, \qquad i = 1, \ldots, m ,$$

and the above equations can be written in a matrix form as follows:

$$\begin{bmatrix} \dot{X}_1^t \\ \dot{X}_2^t \\ \vdots \\ \dot{X}_m^t \end{bmatrix} = \begin{bmatrix} -Lg_1 - \theta_1 & & & 0 \\ \theta_1 & -Lg_2 - \theta_2 & & \\ & \ddots & \ddots & \\ 0 & & \theta_{m-1} & -Lg_m - \theta_m \end{bmatrix} \begin{bmatrix} X_1^t \\ X_2^t \\ \vdots \\ X_m^t \end{bmatrix} + \bar{x} \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} .$$

Let M be the $m \times m$ matrix in the right-hand side of the above equation. The elements of M, m_{ij}, are defined as follows:

$$m_{ij} = \begin{cases} -Lg_i - \theta_i, & j = i \\ \theta_{i-1} , & j = i - 1 \\ 0 , & \text{otherwise} ; \end{cases}$$

namely, elements above the diagonal are all zero, along the diagonal are all negative and below the diagonal are all zero except for those immediately below that are all positive, $m_{i,i-1} = \theta_{i-1}$, for $i = 2, \ldots, m$. Also, let $X^t]$ and $\overline{E}$ denote the m-dimensional column vectors $(X_1^t, X_2^t, \ldots, X_m^t)$ and $(1,0, \ldots, 0)$ respectively. The above differential equations can be written in compact form as follows:

$$\dot{X}^t] = MX^t] + \overline{x}\,\overline{E} \,. \tag{5.5}$$

To obtain a stationary reservation price distribution, it is sufficient to require $\dot{X}_i^t = 0$, $i = 1, \ldots, m$. Clearly, this is also sufficient for the stationary distribution of prospective buyers over the price range, $X^t]$. If the entry rate of new prospective buyers, $\overline{x}$, is independent of time, t, and of the distribution of buyers over the price range, $X^t]$--that is, $\overline{x}$ is exogenously given--then the (unique) equilibrium distribution $X^t]$ can be explicitly obtained from (5.5). That is, since the matrix M does not depend upon t and $X^t]$, setting $\dot{X}^t] = 0]$ in (5.5) yields $X^t] = \overline{x}M^{-1}\overline{E}$.

Let $-M^{-1}\overline{E}$ be denoted by $\mu]$. The matrix M can be inverted easily and the components of $\mu]$ are given[5] by

$$\mu_1 = \frac{1}{Lg_1 + \theta_1}, \quad \mu_i = \frac{\prod_{k=1}^{i-1} \theta_k}{\prod_{k=1}^{i} (Lg_k + \theta_k)} \quad \text{for } i = 2, \ldots, m \,. \tag{5.6}$$

5/ Our having assumed that new buyers "enter" the submarket at the price level p^1 is crucial for the simple expressions for the μ_i's.

The corresponding stationary (frequency) distribution of reservation prices can be easily obtained by normalization. In the following theorem, the existence and uniqueness of an equilibrium $X^t]$ is formally stated.

Theorem 5.1. Under the assumption of stationary supply side conditions and exogenous entry rate of new buyers, the adjustment process described by equ. (5.5) yields (1) a unique equilibrium distribution of prospective buyers over the price range, $X] = \bar{x}M^{-1}\bar{E} = \bar{x}\mu]$, where $\mu]$ is as defined in (5.6); and (2) a unique equilibrium distribution of reservation prices, f_X, obtained by normalizing $\bar{x}\mu]$.

In general, if $\bar{x}$ depends upon $X^t]$ and/or time explicitly, equ. (5.5) can be treated as a set of simultaneous differential equations. Now, by manipulating the expression for μ_i and using some approximations we can obtain an intuitive interpretation of the equilibrium $X]$. The expression for μ_i from (5.6) can be written as follows:

$$\mu_i = \frac{1}{(\frac{Lg_1}{\theta_1}+1)\cdots(\frac{Lg_{i-1}}{\theta_{i-1}}+1)(\frac{Lg_i}{\theta_i}+1)\theta_i} .$$

For a detailed discretization, the T_i's will be very small. Thus, for $k = 1, \ldots, i$,

$$1 + \frac{Lg_k}{\theta_k} = 1 + Lg_k T_k e^{Lg_k T_k} \approx 1 .$$

Hence, $\mu_i \approx T_i e^{Lg_i T_i} \approx T_i$. As a result, the equilibrium numbers of prospective buyers with reservation price p^i, $X_i = \bar{x}\mu_i$, $i = 1,\ldots,m$, are proportional to the times, T_i, prospective buyers "stay" at the price levels p^i.

The Short-Run Dynamics of Adjustment and the "Flow-Through" Equilibrium: The Supply Side

In this section, we shall derive a set of differential equations which describe the adjustment over time of the numbers of sellers with different asking prices. Then we shall discuss equilibrium conditions in terms of this adjustment process. We shall assume here, by analogy to the previous section, that the demand side is held at stationary equilibrium. That is, the distribution function of reservation prices and the number of buyers in the market are stationary.

Consistently with the treatment of demand side, we shall assume a finite time-horizon of search for sellers. Furthermore, we shall make the important assumption that new sellers enter the quality sub-market with initial asking prices equal to p^m, the upper end of the price range. Similarly with our treatment of the demand side, we can easily write the adjustment equations for the distribution of sellers over the price range according to their asking prices. Let us define G_j as follows:

$$G_j = \sum_{k=j}^{m} \frac{X_k^t}{X^t} \,. \tag{5.7}$$

The adjustment equations for the numbers of sellers $V_1^t, V_2^t, \ldots, V_m^t$ with asking prices $p^1, p^2, \ldots, p^m$ respectively are:

$$\dot{V}_j^t = -\lambda G_j V_j^t - \gamma_j V_j^t + \gamma_{j+1} V_{j+1}^t , \qquad j = 1, 2, \ldots, m-1 ,$$

and

$$\dot{V}_m^t = -\lambda G_m V_m^t - \gamma_m V_m^t + \bar{v} \, ;$$

where $\bar{v}$ is the rate at which new sellers (vacant units) enter the supply side of the submarket. The coefficients γ_j are given by:

$$\gamma_j = \frac{1}{S_j} e^{-\lambda G_j S_j} ;$$

The S_j's are obtained by discretizing the optimal asking price curve. The above equations can be written in matrix form as follows:

$$\begin{bmatrix} \dot{V}_1^t \\ \dot{V}_2^t \\ \vdots \\ \dot{V}_m^t \end{bmatrix} = \begin{bmatrix} -\lambda G_1 - \gamma_1 & \gamma_2 & & 0 \\ & -\lambda G_2 - \gamma_2 & \gamma_3 & \\ & & \ddots & \\ 0 & & & -\lambda G_m - \gamma_m \end{bmatrix} \begin{bmatrix} V_1^t \\ V_2^t \\ \vdots \\ V_m^t \end{bmatrix} + \bar{v} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} .$$

Let N be the $m \times m$ matrix in the right-hand side of the above equations. Its elements, n_{ij}, are defined as follows:

$$n_{ij} = \begin{cases} -\lambda G_i - \gamma_i , & j = i, \\ \gamma_{i+1} , & j = i + 1, \\ 0 , & j < i, \; j > i + 1 ; \end{cases}$$

namely, N has negative elements along the diagonal, zero below the diagonal and above the diagonal except for the elements $(i,i+1)$, $i = 1, \ldots , m - 1$ that are all positive. Let $V^t]$ and $\underline{E}$ denote the m-dimensional column vectors $(V_1^t, V_2^t, \ldots , V_m^t)$ and $(0, \ldots , 0, 1)$ respectively. The above differential equations can now be written in compact form as follows:

$$\dot{V}^t] = NV^t] + \bar{v}\underline{E} . \tag{5.8}$$

We should note that the similarities between the demand and supply adjustment processes are reflected in equs. (5.5) and (5.8), describing those processes. Similarly to the demand side, if the entry rate of new sellers, $\bar{v}$, is independent of time, t, and of the distribution of sellers over the price range, $V^t]$--that is, $\bar{v}$ is exogenously given--then the (unique) equilibrium distribution $V^t]$ can be explicitly obtained from equ. (5.8). The equilibrium $V^t]$ is equal to $\bar{v}N^{-1}\underline{E}$.

Let $-N^{-1}\underline{E}$ be denoted by $\nu]$. The components of $\nu]$ are given by

$$\nu_j = \frac{\prod_{k=j+1}^{m} \gamma_k}{\prod_{k=j}^{m} (\lambda G_k + \gamma_k)}, \quad j = 1, \ldots, m-1; \quad \nu_m = \frac{1}{\lambda G_m + \gamma_m} . \qquad (5.9)$$

The existence and uniqueness of an equilibrium $V^t]$ is formally stated in the following theorem.

<u>Theorem 5.2</u>. Under the assumptions of stationary demand side conditions and an exogenous entry rate of new sellers, the adjustment process described by equ. (5.8) yields (1) a unique equilibrium distribution of sellers over the price range, $V] = -\bar{v}N^{-1}\underline{E} = \bar{v}\nu]$, where $\nu]$ is as defined in (5.9); and (2) a unique equilibrium distribution of asking prices, f_V, obtained by normalizing $\bar{v}\nu]$.

Again, if the entry rate of new sellers, $\bar{v}$, depends upon $V^t]$ and/or time explicitly, equ. (5.8) can be treated as a set of simultaneous differential equations. An intuitive interpretation of the numbers of sellers at different asking price levels, at equilibrium, can also be obtained.

Summary

In this section, we have examined adjustment processes of the numbers of market participants towards equilibrium. Prospective buyers were assumed to enter the submarket with initial reservation prices at the lower end of the price range and then "flow through" the market by moving upwards over the price range--that is, adjusting their reservation prices upwards with time they stay in the market in search of houses. Some of them find and buy housing units and leave the market; others do not and thus leave the market at the end of their search horizon. Transactions at different prices take place simultaneously. Sellers, on the other hand, were assumed to enter the market with initial asking prices at the upper part of the price range. Once in the market they move downward over the price range.

Holding each side of the submarket at stationary equilibrium--stationary price distribution and constant number of market participants--we described the short-run dynamics of the adjustment process on the other side of the market. In terms of these dynamics we obtained equilibrium conditions for the "free" side of the submarket.

Ideally, one would like to examine an overall quality submarket, i.e., by solving simultaneously the two sets of equations

$$\dot{X}] = MX] + \bar{x}\,\bar{E}\ , \tag{5.10a}$$

$$\dot{V}] = NV] + \bar{v}\underline{E}\ , \tag{5.10b}$$

and the consistency condition, obtained from lemma 5.1,

$$\ell X = LV\ ; \tag{5.10c}$$

where the superscript t has, for simplicity, been suppressed. Let us recall, however, that the adjustment equations have been obtained assuming constant intensity of the process of contacts and stationary price distribution. Therefore the adjustment equations (5.10a) are valid, if the total number of sellers in the market and the distribution of asking prices are stationary. However, for $\dot{V} = 0$ to be consistent with a non-trivial (frequency) distribution we must also require $\dot{V}] = 0$. Since a symmetrical argument can be made for equ. (5.10b), the above adjustment equations cannot be construed as describing an overall equilibrium adjustment. That is, equ. (5.10) are valid only in the context of the decomposition procedure of holding one side of the market stationary and examining the other.

The symmetry in the above description of buyers' and sellers' market behavior was clearly reflected in the differential equations describing the adjustment processes. In the following discussion, we shall extend the adjustment model by making the entry rates of new participants endogenous and examining equilibrium under such conditions.

5.4 Extensions

Introduction

The assumption of exogenous entry rates for new market participants is very restrictive. Making these entry rates endogenous provides the model with greater structure and thus enhances the range of comparative statics and short-run dynamics questions that can be asked. Specifying the entry rates of new participants is equivalent to making assumptions about individuals' participation decisions. Here, however, we will not be concerned with these decisions in the manner they were discussed in Chapters Three and Four. Instead we shall treat $\bar{x}$ and $\bar{v}$ as functions of proxies of the possibilities for buyers to make contacts with sellers and vice versa, and of a price variable.

For our purposes, the price variable to be chosen must convey information about the state of the market and must be observable by prospective buyers and sellers. Since transactions take place simultaneously at different prices, the average price of current transactions conveys information about which way the market is going. Also, informational requirements for its observability by prospective market participants are modest. As proxies for contact making possibilities, we shall use the intensities of the process of contacts for buyers and sellers.

In the remainder of this section, we shall first discuss the average price of current transactions. Then we shall examine its responsiveness to changes in market conditions. Finally, we shall formally specify the entry rates as functions of the average price of current transactions and of the intensities of the process of contacts.

The Average Price of Current Transactions

The traditional analysis of price equilibrium views the market equilibrium price as a guide variable that consumers and producers look to in making decisions. Here, we shall attempt to answer the question whether the average price of current transactions is a reasonable extension of the above notion for the type of market processes which are examined in this work.

Let $(X_1, \ldots, X_m)$ and $(V_1, \ldots, V_m)$ be the numbers of prospective buyers and sellers, respectively, in the market at some time. Also, let P_D^c and P_S^c be the average price of current transactions, looked at from the demand and supply sides of the market respectively.

It turns out that by employing the consistency condition (5.1) we can show that, as expected, $P_D^c = P_S^c$. This is proven in the following corollary.

<u>Corollary 5.1</u>. The average price of current transactions looked at from the demand side, P_D^c is equal to P_S^c, the average price of current transactions looked at from the supply side.

<u>Proof</u>. For P_D^c, computing the contribution to the average from p^i, $i = 1,m$, yields

$$P_D^c = \frac{L \sum_{i=1}^{m} g_i x_i \frac{1}{g_i} \sum_{k=1}^{i} \frac{p^k V_k}{V}}{L \sum_{i=1}^{m} g_i X_i} . \tag{5.11}$$

In like manner, by working from the supply side and remembering that transactions are carried out at the sellers' asking prices, we obtain:

$$P_D^c = \frac{\lambda \sum_{i=1}^{m} G_i V_i p^i}{\lambda \sum_{i=1}^{m} G_i V_i} . \tag{5.12}$$

The coefficient of p^i in (5.11) is equal to the coefficient of p^i in (5.12), if the consistency condition (5.1) is employed. Since this holds for all i's, $P_D^c = P_S^c = P^c$. Q.E.D.

Hence we can write

$$P^c = \frac{1}{\sum_{i=1}^{m} G_i V_i} \sum_{i=1}^{m} G_i V_i p^i . \tag{5.13}$$

We should note from (5.13) that P^c is a function only of the distributions of asking and reservation prices. That is, it does not depend explicitly upon the intensity of the process of contacts of buyers with sellers and vice versa, nor upon the absolute numbers of market participants.

In order to examine the responsiveness of this price variable to changes in market conditions, we shall investigate its dependence upon the numbers of buyers and sellers at different price levels, $(X_1, \dots, X_m)$ and $(V_1, \dots, V_m)$. We can differentiate P^c with respect to X_i and V_i, $i = 1, \dots, m$; concise formulas for the derivatives are summarized in the following lemma.

Lemma 5.2. Definition (5.13) by partial differentiation with respect to V_i, $i = 1, \dots, m$, and X_j, $j = 1, \dots, m$, yields:

$$\frac{\partial P^c}{\partial V_i} = \frac{G_i}{\sum_{i=1}^{m} G_i V_i} (p^i - P^c) \qquad i = 1, \dots, m; \tag{5.14}$$

$$\frac{\partial P^c}{\partial X_j} = \frac{1}{X} \frac{1}{\sum_{i=1}^{m} G_i V_i} \sum_{k=j}^{m} V_k (p^k - P^c), \quad j = 1, \dots, m . \tag{5.15}$$

Proof. (5.14) is immediate:

$$\frac{\partial P^c}{\partial V_i} = \frac{G_i p^i \sum_{i=1}^{m} G_i V_i - G_i \sum_{i=1}^{m} G_i V_i p^i}{\left(\sum_{i=1}^{m} G_i V_i\right)^2} = \frac{G_i}{\sum_{i=1}^{m} G_i V_i} (p^i - P^c) . \quad \text{Q.E.D.}$$

For the partial derivative of P^c with respect to X_j:

$$\frac{\partial P^c}{\partial X_j} = \frac{1}{\sum_{i=1}^{m} G_i V_i} \sum_{i=1}^{m} \frac{\partial G_i}{\partial X_j} V_i p^i - \frac{1}{\left(\sum_{i=1}^{m} G_i V_i\right)^2} \sum_{i=1}^{m} G_i V_i p^i \sum_{i=1}^{m} \frac{\partial G_i}{\partial X_j} V_i . \tag{5.16}$$

$$\frac{\partial G_i}{\partial X_j} = \frac{X \frac{\partial}{\partial X_j}(X_i + \cdots + X_m) - (X_i + \cdots + X_m)\frac{\partial X}{\partial X_j}}{X^2} ;$$

and

$$\frac{\partial G_i}{\partial X_j} = \frac{1 - G_i}{X} , \quad j \geq i ,$$

$$\frac{\partial G_i}{\partial X_j} = - \frac{G_i}{X} , \quad j < i .$$

Substituting the above for $\frac{\partial G_i}{\partial X_j}$ in (5.16) and performing the manipulations yields (5.15). Q.E.D.

We shall now use these results in examining the responsiveness of P^c to changes in market conditions. We shall first treat the supply side and then the demand side.

Supply side. Clearly, from (5.12), P^c is a convex combination of the $\{p^1, p^2, \ldots , p^m\}$. Hence $\frac{\partial P^c}{\partial V_i}$ will be negative for low i's and positive for high i's. We can draw a figure for $\frac{\partial P^c}{\partial V_i}$ as a function of i. Let us assume that $p^k = P^c$. The partial derivative

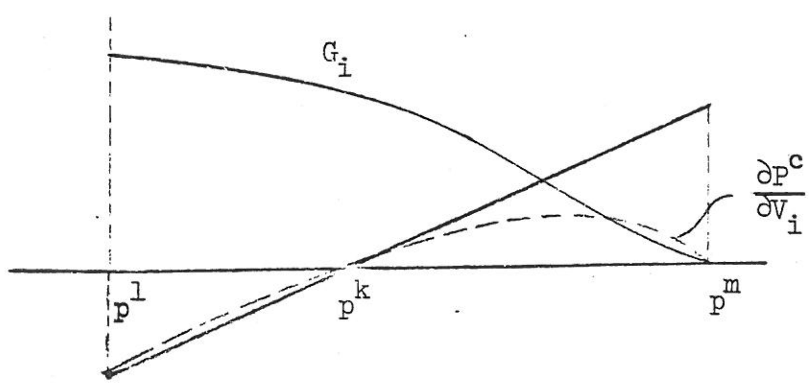

Fig. 5.2

of P^c with respect to V_i is negative for $i < k$, increases with i, becomes zero at $i = k$, remains positive for $k < i < m$, and becomes zero again for $i = m$.

Now, let us consider the effects on P^c of changes in the distribution of asking prices. Suppose that the number of sellers over the price range changes by ΔV_i, $i = 1, \ldots, m$. The change in P^c, in the absence of adjustment of the distribution of reservation prices, is given by:

$$\Delta P^c = \sum_{i=1}^{m} \frac{\partial P^c}{\partial V_i} \Delta V_i \ . \tag{5.17}$$

Suppose, for the sake of simplicity, that the distribution of asking prices changes so that values higher than p^k become more likely than values lower than p^k, where $p^k = P^c$. That is, $\Delta V_i \lesseqgtr 0$ for $i \lesseqgtr k$, ΔV_i is increasing for increasing i and $\sum_{i=1}^{m} \Delta V_i = 0$. (5.17) yields $\Delta P^c > 0$ and P^c increases; that is, a change in the distribution of asking prices corresponding to an increase in the number of vacancies with asking prices greater than P^c will cause an increase in P^c. Similarly, a change in the opposite direction will cause a decrease in P^c.

Let us examine the effects on P^c of a change in dispersion characteristics of the distribution of asking prices. We will assume that the concentration of vacancies around p^k increases, e.g., $\Delta V_i < 0$ for $i < q$, $\Delta V_q = 0$, $\Delta V_i > 0$ for $q < i < r$, $\Delta V_r = 0$, $\Delta V_i < 0$ for $i > r$ and $q < k < r < m$. The expression for $\frac{\partial P^c}{\partial V_i}$ suggests that P^c is more sensitive to changes in ΔV_i for lower i's rather than higher i's. Hence for the above change, it may happen that $\Delta P^c > 0$; namely that an increase in concentration will increase the average price of current transactions. This result, however, does not account for adjustment in the distribution of reservation prices that would be induced by the above change in the distribution of asking prices.

Demand side. Let us now examine the effect on P^c of changes in the distribution of reservation prices. For small changes in ΔX_i, ΔP^c is given by

$$\Delta P^c = \sum_{i=1}^{m} \frac{\partial P^c}{\partial X_i} \Delta X_i \ . \tag{5.18}$$

Contrary to the partial derivatives of P^c with respect to V_i, $i = 1, \ldots, m$, the sign of the partial derivatives of P^c with respect to X_j, $j = 1, \ldots, m$ cannot be easily predicted. From (5.15), the differences $p^k - P^c$ are weighed by the numbers of sellers with asking prices equal to p^k and then are summed up. However, let us consider an arbitrary $(V_1, \ldots, V_m)$ and draw $\frac{\partial P^c}{\partial X_j}$ as a function of j. (See Fig. 5.3.) Here, $\frac{\partial P^c}{\partial X_j}$ is positive for high

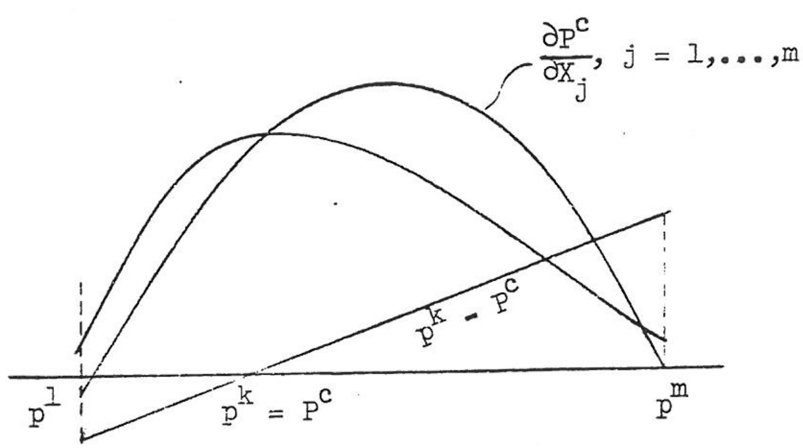

Fig. 5.3

j's and may become negative for very low j's. Conditions under which $\frac{\partial P^c}{\partial X_j}$ is equal to zero for $j = 1$ and positive for all other j's can be obtained. Unfortunately, however, they are not amenable to an economic interpretation, and will thus be deleted here.

An increase in the number of prospective buyers at different asking price levels over the upper part of the price range will always increase the average price of current transactions.

Regarding a change in dispersion characteristics of the distribution of reservation prices, we can work in a similar way; e.g., $\Delta X_i < 0$ for $i < q$, $\Delta X_q = 0$, $\Delta X_i > 0$ for $q < i < r$, $\Delta X_r = 0$, and $\Delta X_i < 0$ for $r < i$ and $q < k < r < m$. Suppose that originally $P^c \approx P^k$. Then for $j > k$, $\frac{\partial P^c}{\partial X_j} > 0$ and $\frac{\partial P^c}{\partial X_j}$ may become negative for low j's provided that the corresponding V_j's are sufficiently large. Whether an increase in dispersion of consumers according to reservation prices will have a downward effect upon P^c also depends upon the numbers of sellers with different asking prices.

In sum, we have defined the average price of current transactions and obtained an expression for it which involves only the distributions of reservation and asking prices. Neither the absolute number of market participants nor the intensities of the process of contacts enter this expression. Furthermore, we have examined the responsiveness of the average price of current transactions to changes in market conditions by studying properties of the derivatives of the average price of current transactions with respect to the numbers of market participants, prospective buyers and sellers, at different price levels. Finally, we used these derivatives to predict changes in the average price of current transactions caused by some specific cases of small changes in the price distributions. The following section completes the description of the new elements in the extended model.

Specification of the Entry Rates of New Market Participants

As we noted earlier, we shall make the entry rates of new market participants endogenous by specifying them as functions of the intensities of the process of contacts and of the average price of current transactions. We shall first discuss the demand side and then the supply side.

Demand side. The rate of entry of new prospective buyers, $\overline{x}$, will be assumed to be an increasing function of the expected intensity of the process of contacts, L, and a decreasing function[6] of the average price of current transactions, P^c. Analytically:

$$\overline{x} = \overline{x}(L,P^c), \quad \frac{\partial \overline{x}}{\partial L} > 0, \quad \frac{\partial \overline{x}}{\partial P^c} < 0 . \tag{5.19}$$

Furthermore, we shall assume differentiability up to the second order.

Supply side. Similarly to the treatment of the demand side, we shall simplify the seller's participation decision model by assuming that the rate of entry of new vacancies is an increasing function of the average price of current transactions and the intensity at which contacts are made. Analytically:

$$\overline{v} = \overline{v}(\lambda,P^c), \quad \frac{\partial \overline{v}}{\partial \lambda} > 0, \quad \frac{\partial \overline{v}}{\partial P^c} > 0 . \tag{5.20}$$

Again, we shall assume differentiability up to the second order.

Having extended our model by making the entry rates endogenous, we shall proceed in the following section to explore the existence and uniqueness of equilibrium in such a model and examine its properties.

6/ If units of different qualities are viewed as distinct, substitute commodities and several quality submarkets are considered $\overline{x}$ must be specified as a function of the vector P^c], the average price of current transactions in all submarkets. This possibility will not be pursued here.

Comparative statics and stability considerations are presented later in the chapter.

5.5 Existence and Uniqueness of Equilibrium for the Extended Model

Introduction

The section consists of three parts. In the first two parts, existence and uniqueness of equilibrium are discussed for the demand and supply sides of the market in the context of the decomposition procedure discussed earlier in this chapter. That is, by holding one side of the market at equilibrium the dynamics of adjustment and equilibrium in the other side are examined. The third part deals with the existence of equilibrium when both sides of the market are allowed to vary. It will be shown that existence of an overall equilibrium depends heavily upon properties of the informational structure of the market.

Existence and Uniqueness of Equilibrium: The Demand Side

Let us recall that the rate of entry of new buyers is assumed to be a continuous increasing function of L, the intensity of the process of contacts buyers make with sellers, and a decreasing function of P^c, the average price of current transactions. By assuming that the supply side is held at equilibrium, the set of differential equations (5.10a) describe the short-run dynamics of the adjustment of the numbers of prospective buyers at different price levels, X].

With the above specification of the rate of entry of new prospective buyers, $\bar{x}$, equ. (5.10a) is autonomous. Given initial conditions, they can be integrated[7/] over any finite interval of time. The following theorem shows the existence and uniqueness of a demand side equilibrium under these conditions.

Theorem 5.3. If the supply side is held at equilibrium and under the previously stated assumption, the adjustment process described by equ. (5.10a) has a unique equilibrium distribution of prospective buyers over the price range, $X^{o}]$.

Proof. Since equ. (5.10a) is autonomous, if an equilibrium $X]$ exists, it must satisfy the equation

$$MX] + \bar{x}(L,P^{c})\bar{E} = 0] \ .$$

Since M is non-singular, M^{-1} exists. Hence, the above equation becomes:

$$X] = \bar{x}(L,P^{c})\mu] \ , \qquad (5.21)$$

where $\mu]$ is as defined in (5.6). $\mu]$ is independent of $X]$.

If the function $\mu X] + \bar{x}(L,P^{c})\bar{E}$ has a zero point $X^{o}]$, from (5.21) $X^{o}]$ must belong to the cone in R^{m} defined by $b\mu]$--where b is a positive scalar. All points in this cone yield the same average price of current transactions. That is, since the asking price distribution is stationary and given and because the average price of current transactions depends only on the price distributions (and not on the absolute numbers of buyers and sellers and different

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7/ This can be easily shown; since M does not depend upon $X]$ and $\bar{x}$ is continuous, twice differentiable and bounded, the right-hand side of (5.10a) satisfies a Lipschitz condition. This is sufficient for the existence of the solution.

price levels), P^c is determined by $\mu]$ and f_V.

Therefore, since L is constant the equilibrium distribution of prospective buyers over the price range (i.e., the number of prospective buyers at different reservation price levels), $X^o]$,

$$X^o] = \overline{x}(L, P^c(\mu]))\mu] \ , \tag{5.22}$$

is unique by construction. The corresponding distribution of reservation prices is obtained by normalization. Q.E.D.

We should emphasize that the above theorem holds even if we relax the assumption, made in section 5.2, of the linearity of $L^*(V)$ with respect to V, as long as initiative in search is attributed to buyers.[8/] Finally we should note that the model for the demand side equilibrium adjustment implies a one-to-one correspondence between the distribution of asking prices, behavioral characteristics of buyers, and the intensity of the process of contacts on the one hand and the distribution of prospective buyers over the price range on the other. The latter is dependent upon supply side configurations by means of continuous functions.

Existence and Uniqueness of Equilibrium: The Supply Side

We shall now explore the existence and uniqueness of equilibrium implied by the short-run dynamics of adjustment of the distribution of asking prices under stationary conditions on the demand side. (That is, the total number of prospective buyers and their distribution over the

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8/ If we assume, as Theorem 5.3 requires, that the supply side is held at equilibrium, then V, the number of sellers in the market, is stationary and so is L.

price range is stationary.) Assuming perfect information and in view of the discussion in section 5.2, the intensity at which sellers make contacts with buyers remains constant. Therefore, the questions of existence and uniqueness of the supply side equilibrium are entirely symmetrical to those of the demand side discussed above. Hence a theorem, similar to 5.3, guaranteeing existence and uniqueness of a supply side equilibrium will be stated without proof.

Theorem 5.4. If the demand side is held at equilibrium and under the previously stated assumptions, the adjustment process described by equ. (5.10b) has a unique equilibrium distribution of sellers over the price range, $V^{o}]$,

$$V^{o}] = \bar{v}(\lambda, P^{c}(\nu]))\nu] \;, \tag{5.23}$$

and $\nu]$ is as defined in (5.9).

Again, the equilibrium distribution of asking prices can be obtained by normalizing $\bar{v}(\lambda, P^{c}(\nu]))\nu]$. The results we have obtained so far in this section are quite strong. We should emphasize that they depend to some extent upon our assumption that new prospective buyers enter the market at the lower end of the price range and sellers at the upper end. While this assumption seems quite restrictive at first glance, if we recall that the approach is intended as an averaging procedure, then the above can be partly justified.

The decomposition procedure is crucial to our approach. In the physical and social sciences, abstractions of this type commonly enable us to obtain insight into very complex processes. Although it is hard to justify the decomposition procedure by means of an economic process, we shall attempt to provide descriptions of economic adjustment processes which underlie the analytical formulations employed so far in this chapter. Finally, it is interesting to note here that the stationarity of the distributions, while a most important building block of our approach, turns out to be a crucial vehicle of the dynamic equilibrium adjustment.

In the remainder of this section we shall examine the existence of an overall equilibrium. Comparative statics and stability considerations are provided later in the chapter.

Overall Quality Submarket Equilibrium

Introduction. So far, we have considered the supply and demand sides of the market separately. In this section, we shall show that an overall equilibrium exists, by utilizing the earlier descriptions of the short-run dynamics of price and stock adjustment and making use of properties of the informational structure of the market. The existence[9] of equilibrium satisfies a fundamental consistency requirement for our model. Some of the economic processes associated with an overall equilibrium will be examined below. This should by no means be construed as giving prominence to the analytical over the economic content of our approach. We shall start with a comprehensive discussion of the informational structure of the market.

Informational structure of the market. As we mentioned in sections 3.6 and 4.4, dealing with market participants' use of informational

[9] Equilibrium existence theorems in economic theory and, to some extent, stability results serve the function of ensuring consistency and thus support claims of relevance to real world problems. Furthermore, from the point of view of economic theory in general and disequilibrium economics in particular, mathematical existence results are of dubious value unless they are supplemented by an economic process corresponding to behavioral rules that economic agents are supposed to follow.

intermediaries, we have restricted ourselves to describing market participants' purchase of information in order to improve upon their possibilities for making contacts with participants on the other side of the market.[10] Here, we shall retain the assumptions made in section 5.2 that buyers have the initiative to discover existing vacancies and contact sellers, while sellers are passive searchers who wait for prospective buyers to come in now and then. Buyers set their search policies on the basis of an anticipated intensity of the process of contacts that at equilibrium must be compatible with the demand side of the market.

Recall that the intensity of the process of contacts consumers make with sellers was assumed to depend only upon the number of sellers, in addition to the technology of information acquisition and dissemination. We can thus write:

$$L = L(\tau, V) \tag{5.24}$$

with $\frac{\partial L}{\partial \tau} > 0$ and $\frac{\partial L}{\partial V} > 0$, where τ stands for the technology of information and V for the total number of sellers (vacancies) in the market. Individuals' purchase of such information either on their own or through the use of informational intermediaries gives rise to the overall (market) technology[11] of information, proxied by τ in (5.24)

[10] We have not been concerned with purchase of information about the distribution of either reservation or asking prices. Our model, however, can handle the problem--although not explicitly treated--of an individual's purchasing information regarding "effective" contacts; that is, a prospective buyer can go to an agent and give him information about how much he is willing to pay and how long he would be willing to wait.

[11] Information may take the form of a collectively consumed good or of a purely private good. We have not identified who bears the costs to support the informational structure of the market, nor whether the

above. The relationship $L = L(\tau,V)$, i.e., the intensity of contacts as a function of the technology of information and the number of vacancies in the submarket is simply the function $L^*(V)$[12] we derived in section 4.4. At informational equilibrium, sellers expect the following intensity of the process of contacts to prevail:

$$z = \frac{L^*X}{V} = X\,\frac{L^*(\tau,V)}{V}\,. \tag{5.25}$$

Hence z is proportional to the number of searchers, X, and it would be independent of the number of vacancies if and only if L^* is a linear function of V. In general, z would be an increasing (decreasing) function of the number of vacancies, V, if the elasticity of L^* with respect to V is greater (less) than one, that is,

$$\frac{\partial z}{\partial V} = X\,\frac{\frac{\partial L^*}{\partial V}V - L^*}{V^2}\,,$$

and

$$\frac{\partial z}{\partial V} \lesseqgtr 0 \quad \text{iff} \quad \epsilon = \frac{V}{L^*}\frac{\partial L^*}{\partial V} \gtreqless 1\,. \tag{5.26}$$

In the following discussion, we shall abstract from a behavioral determination of the technology of information acquisition and dissemination and assume that it cannot indefinitely bring about an improvement

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participants' purchase of such information is at the social optimum level (an efficiency issue). Interesting questions that should be analyzed include whether such information should be provided publicly or purchased privately, and the equity question regarding the impact on market performance of the fact that individuals face differential search costs. Such issues will not be pursued here.

12/ If initiative in search were placed upon sellers, and if buyers were considered as passive searchers, the function $\lambda^*(X)$ would be the relevant relationship.

in the intensity at which contacts are made. As we shall see, this assertion is crucial for establishing the existence of a stationary equilibrium and is also necessary for the existence of a finite optimal L^* buyers choose.

In order to ensure that z will be finite, we should require that z, from (5.25) above, remains finite when $V \to 0$. It is sufficient to assume that

$$\frac{\partial L^*}{\partial V} < \infty, \quad \text{for} \quad V = 0, \quad \text{and} \quad L^*(\tau,o) = 0 \; . \tag{5.27}$$

Finally, we should note that our model depends heavily upon the decomposition procedure discussed in Chapter Two which makes the rate of contacts and the probability distribution functions of reservation and asking prices crucial information and thus dynamic features of the approach.

This completes our restatement of the informational structure of the market. We shall now proceed to a discussion of the main features of the overall equilibrium model.

Existence of an overall equilibrium. As we discussed in section 5.3, our adjustment models are based on the procedure of holding one side of the market at equilibrium and examining the other. We have not offered a model of an overall equilibrium adjustment.

The adjustment models we have discussed hinge on the assumption of perfect knowledge of price distributions and the intensities of contacts. Implications for market adjustment of relaxing some of these assumptions will be examined later in a heuristic fashion. The smoothing of the dynamic adjustment and the approximation of continuous distributions by discrete ones were carried out under the assumption that participants on both sides of the market act on the expectation of stationary conditions.

At a stationary equilibrium, when expectations are fulfilled and market variables remain constant, the derivation of the adjustment equations is still valid. Hence the question of the existence of an overall equilibrium becomes that of whether there exists a pair of vectors $X*]$ and $V*]$ such that the set of equations

$$MX*] + \overline{x}(L,P^c)\overline{E} = 0], \quad \text{and} \tag{5.28a}$$

$$NV*] + \overline{v}(\lambda,P^c)\underline{E} = 0] \tag{5.28b}$$

are satisfied.

The rate of entry of new consumers depends upon L, which is a function of τ, informational technology, and V, the total number of vacancies, which is obtained as the sum of the elements of $V*]$ and the average price of current transactions, P_c. The latter depends explicitly only upon the distributions of reservation and asking prices obtained by normalizing $X*]$ and $V*]$ respectively, and not upon the absolute number of market participants, nor upon the intensity of contacts.

The entry rate of new vacancies (sellers) depends on λ, which is a function of τ, X, the total number of prospective buyers, and the average price of current transactions.

We can invert the matrices M and N and obtain the set of equations

$$X*] = \overline{x}(L,P^c)\mu] , \tag{5.29a}$$

$$V*] = \overline{v}(\lambda,P^c)\nu] , \tag{5.29b}$$

which are equivalent to equs. (5.28a) and (5.28b). The components of $\mu]$ depend upon the distribution of asking prices, the intensity of contacts, L, and behavioral characteristics embodied into the stepwise function description of reservation price policies. The components of $\nu]$ depend upon the distribution of reservation prices, the intensity of contacts, λ, and behavioral characteristics embodied into the stepwise function description of asking price policies.

The following theorem guarantees the existence of a set of stationary solutions $X^*]$ and $V^*]$ but not uniqueness. The proof is based on the Brouwer fixed point theorem.

Theorem 5.5. Under the previously stated assumptions, there exists a solution $(X^*]; V^*])$ to the set of equ. (5.29); the solution yields a set of equilibrium distributions of reservation and asking prices and of equilibrium numbers of buyers and sellers.

Proof. We shall show that the mapping defined by the right-hand side of equ. (5.29) has a fixed point. This mapping is a continuous point-to-point mapping. We shall specify a closed and bounded interval, B, in $R^m \times R^m$ and show that the above mapping maps B into itself.

Let us take a point in B, $(X^o], V^o]) \in B$ and consider its image $(\bar{x}\mu]; \bar{v}\nu])$. For the components of the vector $\mu]$ from (5.6) we have:

$$\mu_i = \frac{1}{(L\frac{g_1}{\theta_1}+1)\cdots(L\frac{g_{i-1}}{\theta_{i-1}}+1)(Lg_i+\theta_i)} . \tag{5.30}$$

Hence: $\mu_i < \frac{1}{Lg_i + \theta_i}$. Since $Lg_i > 0$, $\mu_i < \frac{1}{\theta_i}$ by the definition of θ_i becomes:

$$\mu_i < T_i e^{Lg_i T_i} < T_i e^{LT_i} .$$

Since we have assumed a finite time horizon of search, T, the T_i's are bounded upwards by T. Furthermore, they are also bounded downwards because the derivative of an optimal reservation price curve with respect to time is positive and bounded upwards. Finally, let us recall that $L^*(\tau,V)$ was assumed to be bounded. Hence $T_i e^{LT_i}$ is bounded upwards and downwards. For the same reason, μ_i is bounded upwards. Furthermore, from (5.30) and because θ_i is bounded, μ_i is also bounded downwards--barring a trivial distribution of asking prices.

We now turn to the term $\bar{x}(L,P^c)$. The (scalar) function $\bar{x}$ was assumed to be bounded. Such an assumption can be justified because an entire housing market is assumed to consist of several hierarchies of finite size. At any rate, L is always bounded upwards and so is P^c by being a convex combination of the p^i's. Hence, since $\bar{x}$ cannot become zero--people always flow in--and because L and P^c are always bounded upwards, we can write:

$$\bar{x}_{min} < \bar{x} < \bar{x}_{max} .$$

Let μ_{max} and μ_{min} be the upper and lower bounds of the μ_i's, respectively. Also, let $\mu_{max}]$ and $\mu_{min}]$ be the vectors in R^m with all components equal to μ_{max} and μ_{min}, respectively. Finally, let us define the set B^X in R^m as follows: $B^X = \{X] \in R^m; \bar{x}_{min}\mu_{min}] \leq X] \leq \bar{x}_{max}\mu_{max}]\}$. B^X is convex.

A symmetrical procedure can be followed to show that the image of $V^o]$, $\bar{v}v]$, belongs to a bounded interval B^V, in R^m, which can be defined symmetrically to B^X. The only additional element used is the boundedness of $\frac{L^*(\tau,V)}{V}$. Let b be the upper bound of this expression; ν_{max} can then be defined as follows:

$$\nu_{max} = \bar{v}_{max} e^{m\mu_{max} \cdot b \cdot S} S .$$

Let us now define the set B as the cartesian product of B^X and B^V: $B = B^X \times B^V$. B, a subset of $R^m \times R^m$ is closed and bounded by definition and hence compact. The continuous point-to-point mapping $(\bar{x}\mu]; \bar{v}\nu])$ maps B into itself. Hence, according to the Brouwer fixed point theorem, this mapping has a fixed point. Furthermore, it is immediate from the above discussion that the zero vector in R^m is not a fixed point. Q.E.D.

Interpretations. We have now been able to show the existence of a stationary equilibrium for a quality submarket for market participants who are operating under conditions of uncertainty. Under the sometimes restrictive conditions of our model it is shown that the allocation process of consumers to houses has a stationary solution. Hence, a basic requirement for consistency of our model is satisfied.

The approach we have employed, although quite novel, is far from resolving all relevant issues. Let us first discuss some technical ones. We have dealt with discrete distributions, although the existence theorems do not depend upon m, the dimension of discretization. It thus seems likely that convergence results may be obtained so that existence results could be provided for continuous distributions--an elegant result that is by no means necessary, since m can be very large.

Economic models which have dealt with price dispersion have not managed to explain why it persists in terms of an equilibrium model. We have shown, by means of the theorem just proven, that price dispersion can persist, but we have not explained how such an equilibrium may be attained, nor have we considered any stability properties. Furthermore, it should be emphasized that if we start with a trivial distribution on one side of the market, our model, coupled with appropriate information flow, will not ultimately yield price dispersion. There are two basic reasons why price dispersion persists in our model; first, it takes time for market participants to make contacts with each other; and second, individuals search by means of finite time-horizons. All other possible causes of dispersion were held fixed.

Finally, two of the basic elements of our model merit attention: the aggregation procedure, implicit in the use of an "average" participant; and the set of assumptions defining his market performance. Regarding the former, any attempt towards true aggregation in economics is bound to encounter severe difficulties. Averaging may not be a serious handicap, provided that important characteristics are preserved. Regarding the latter, the most crucial of those assumptions is that prospective buyers enter the market at the lower part of the price range and sellers at the upper. Relaxing it by allowing individual market participants to enter the submarket with initial reservation or asking prices at any level within the price range will undoubtedly make the model less rigid. The mathematical analysis will be extremely tedious, however.

Summary and conclusions. In this section, we have discussed some important equilibrium properties of the model regarding the short-run dynamics of price and stock adjustment which were developed in section 5.3. Having extended the model by making the entry rates of new market participants endogenous, we explored its equilibrium properties by holding one side of the market at equilibrium and examining the other. Under these conditions, we showed that there exists an equilibrium and it is unique for both sides of the market.

Although we have not developed a model of overall adjustment, we were able to examine the existence of an overall equilibrium as follows: at stationary equilibrium, when expectations are fulfilled and market variables remain constant, the equilibrium relationships implied by the separate adjustment models, considered simultaneously, are valid. We showed, by means of the Brouwer fixed point theorem, that such an equilibrium exists, that it is characterized by price dispersion, and that it is not trivial from the point of view of our model.

Our approach suggests that the assumptions one makes about the nature of search significantly affect the form and properties of equilibrium. Contrary to various assertions assertions in the literature, search does not lead to an equilibrium characterized by a unique price. Our discussion of equilibrium is reminiscent of G. Stigler's theory[13/] of endogenous factors that tend to preserve price dispersion.[14/] In this model, investment in information about market conditions and about means of contacting market participants on the other side of the market becomes obsolete. This is one of the reasons that price dispersion persists.

[13/] Stigler (1961), op. cit., p. 219.

[14/] As J. Hirshleifer notes: "(M)obility of buyers and sellers in and out of the market (for example, in an intergenerational life cycle model) will provide a continuing demand for informational processes of search and advertising." Hirshleifer (1973), p. 37.

In the following sections of this chapter, we shall formula the comparative statics problem and examine some of its stability properties. These sections are intended as illustrations rather than exhaustive discussions of such properties.

5.6 Comparative Statics and Stability Properties of the Model

Introduction

Comparative statics is, in the classical sense, "the investigation of changes in a system from one position of equilibrium to another without regard to the transitional process involved in the adjustment."[15/] In the following discussion, we will be concerned with the adjustment process as well as the differences between equilibrium positions.

In this section, we shall first formulate the comparative statics problem for an overall equilibrium. Then we shall define the decomposition procedure employed earlier in terms of such a formulation. Next we shall present some local stability considerations, and finally we shall discuss, in a heuristic fashion, adjustment processes for some specific cases.

In view of Samuelson's correspondence principle, we should caution ourselves about the validity of comparative statics results when no information about the stability of the underlying dynamic system is available.[16/] However, the problem of stability will receive only rudimentary consideration in this work.

The Comparative Statics of an Overall Equilibrium

We showed in the previous section that an overall equilibrium

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15/ Samuelson (1965), p. 8.

16/ Ibid., pp. 257-260, 350-352.

exists. Such an equilibrium, described by numbers of prospective buyers and sellers at different price levels, is given by a solution $(X^*], V^*])$ to the set of equations

$$X^*] = \bar{x}\mu] \ , \tag{5.31a}$$

$$V^*] = \bar{v}\nu] \ . \tag{5.31b}$$

Having attributed initiative in search to buyers, the intensity of the process of contacts buyers make with sellers is a function of the number of sellers in the market. At equilibrium, the intensity of the process of contacts sellers make with buyers is a function of both the number of sellers and buyers in the market. Hence, $X]$ and $V]$ are the only endogenous variables in the model described by equ. (5.31).

For the sake of simplicity, let us consider a single exogenous variable β. The comparative statics problem is that of predicting the signs of the components of the vectors $\frac{dX^*]}{d\beta}$ and $\frac{dV^*]}{d\beta}$, where $X^*]$ and $V^*]$ are simultaneously determined by equ. (5.30). Differentiating (5.31) totally with respect to β yields:

$$dX] = \bar{x} \cdot \left[\nabla_X \mu]\right] \cdot \frac{dX]}{d\beta} \cdot d\beta + \bar{x} \cdot \left[\nabla_V \mu]\right] \cdot \frac{dV]}{d\beta} \cdot d\beta +$$

$$+ \bar{x} \cdot \frac{\partial \mu]}{\partial \beta} \cdot d\beta + \nabla_X \bar{x} \cdot \frac{dX]}{d\beta} \cdot \mu] d\beta + \nabla_V \bar{x} \cdot \frac{dV]}{d\beta} \cdot \mu] d\beta +$$

$$+ \mu] \cdot \frac{\partial \bar{x}}{\partial \beta} \cdot d\beta \ ,$$

and

$$dV] = \bar{v} \cdot \left[\nabla_X \nu]\right] \cdot \frac{dX]}{d\beta} \cdot d\beta + \bar{v} \cdot \left[\nabla_V \nu]\right] \circ \frac{dV]}{d\beta} \cdot d\beta +$$

$$+ \bar{v} \cdot \frac{\partial \nu]}{\partial \beta} \cdot d\beta + \nabla_X \bar{v} \cdot \frac{dX]}{d\beta} \cdot \nu] d\beta + \nabla_V \bar{v} \cdot \frac{dV]}{d\beta} \cdot \nu] d\beta +$$

$$+ \nu] \cdot \frac{\partial \bar{v}}{\partial \beta} \cdot d\beta \; .$$

All quantities above are evaluated at $(X^*]; V^*])$, the solution to equ. (5.31). Furthermore, for the sake of simplicity, the asterisks on $dX]$ and $dV]$ were omitted. The above equations can be written in compact matrix form as follows:

$$\left[\begin{array}{c|c} I-\{\nabla_X \mu]+\mu]\cdot\nabla_X \bar{x}\} & -\bar{x}\cdot\nabla_V \mu]-\mu]\cdot\nabla_V \bar{x} \\ \hline -\bar{v}\cdot\nabla_X \nu]-\nu]\cdot\nabla_X \bar{v} & I-\{\nabla_V \nu]+\nu]\cdot\nabla_V \bar{v}\} \end{array}\right] \cdot \left[\begin{array}{c} \frac{dX^*]}{d\beta} \\ \hline \frac{dV^*]}{d\beta} \end{array}\right] = \left[\begin{array}{c} \bar{x} \cdot \frac{\partial \mu]}{\partial \beta} + \frac{\partial \bar{x}}{\partial \beta} \cdot \mu] \\ \hline \bar{v} \cdot \frac{\partial \nu]}{\partial \beta} + \frac{\partial \bar{v}}{\partial \beta} \cdot \nu] \end{array}\right] . \qquad (5.32)$$

The above equations can be solved, in general, for $(\frac{dX^*]}{d\beta}, \frac{dV^*]}{d\beta})$ as a set of simultaneous linear equations.

Let B be the $2m \times 2m$ matrix on the left-hand side of (5.32). Some of the signs of the elements of B may be available. However, even if all of them were known, that, in general, would not be sufficient for complete comparative statics information.

It is clear from this exposition that comparative statics analysis of our model is extremely complicated. An additional complicating factor inheres in our approach: Although in developing the adjustment model, we assumed that market participants follow optimal search policies, it is very difficulty analytically to express this fact through necessary conditions due to the averaging procedure we employed.

Typically, using such necessary conditions one may obtain an envelope-type theorem which would simplify the comparative statics analysis.

Because of these difficulties, we shall simplify the problem partly by making appropriate assumptions and partly by using some heuristic illustrations. Although such illustrations are of dubious predictive validity, in the context of the adjustment problem they do offer some insight into adjustment processes.

The Decomposition Procedure and Comparative Statics

We shall again employ the familiar decomposition procedure of holding one side of the market at equilibrium and examining the other. It is clear from the above formulation what this procedure involves. For example, assuming that the supply side is held at equilibrium is equivalent to reducing equ. (5.32) to

$$[I - \{\nabla_X \mu] + \mu] \cdot \nabla_X \bar{x}\}] \cdot \frac{dX^*]}{d\beta} = \bar{x}\,\frac{\partial \mu]}{\partial \beta} + \frac{\partial \bar{x}}{\partial \beta}\,\mu] \, , \tag{5.33}$$

which is what we would have obtained by totally differentiating equ. (5.21) with respect to β. Furthermore, since $\mu]$ does not explicitly depend upon $X]$, $\nabla_X \mu] = [o]$. Hence equ. (5.33) reduces to:

$$[I - \mu] \cdot \nabla_X \bar{x}] \cdot \frac{dX^*]}{d\beta} = \bar{x}\,\frac{\partial \mu]}{\partial \beta} + \frac{\partial \bar{x}}{\partial \beta}\,\mu] \, . \tag{5.34}$$

Let us write: $\nabla_X \bar{x} = (\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_m)$--an m-dimensional row vector. For the term $\bar{x}_j$, we have:

$$\bar{x}_j = \frac{\partial \bar{x}}{\partial X_j} = \frac{\partial \bar{x}}{\partial P^c}\,\frac{\partial P^c}{\partial X_j} \, ;$$

Since $\frac{\partial \bar{x}}{\partial P^c} < 0$, the sign of $\bar{x}_j$ depends upon the sign of $\frac{\partial P^c}{\partial X_j}$. Let us recall Lemma 5.2 and the discussion of the signs of the partial derivatives of P^c with respect to X_j: for high j's, $\frac{\partial P^c}{\partial X_j}$ will always be positive, but for low j's, it may be negative.

On the other hand, the components of the vector $\mu]$ are all known and positive. This is as far as we can go in predicting the signs of the elements of the matrix $[I-\mu] \cdot \nabla_X \bar{x}]$.

This completes our general discussion of the comparative statics of the demand side of the market. Evidently, the discussion of the supply side would be symmetrical to the one for the demand side.

In summary, having formulated the overall comparative statics problem, we were able precisely to define what the decomposition procedure involves in the context of comparative statics. We will now proceed to a discussion of the comparative statics of an exogenous shift in the rate of entry of new prospective buyers. The case of an exogenous shift in the rate of entry of sellers can be treated in a similar fashion.

An Example: An Exogenous Shift in the Rate of Entry of New Buyers

Let us consider the exogenous variable β as a shift parameter of the rate of entry of new buyers, $\bar{x}$. We shall consider the effects of a small increase in β. The increase in the exogenous variable β could reflect such factors as an influx of new population because of structural changes in the economy of the area, an increase in the rate of household formation, a decrease in interest rates which might induce more people to buy their own houses rather than continue renting, changes in tax laws that make owner-occupied housing more attractive as an investment opportunity.

Let us assume $\frac{\partial \bar{x}}{\partial \beta} > 0$. Equ. (5.34) then becomes:

$$\{I - \mu] \cdot \nabla_X \bar{x}]\}\frac{dX^*]}{d\beta} = \frac{\partial \bar{x}}{\partial \beta}\mu] \ . \tag{5.35}$$

Recall that $\mu] = -M^{-1}\bar{E}$. It can easily be shown from equ. (5.35) that the new solution vector is proportional to the original one. This is immediate from equ. (5.21), if we realize that the change in the rate of entry of new buyers does not affect $\mu]$.

In sum, if we restrict ourselves to examining changes under stationary conditions on the supply side, an exogenous upward shift in the entry rate of new buyers will cause the equilibrium numbers of prospective buyers at different price levels to increase uniformly, but the distribution of reservation prices will remain unaffected. This discussion does not, however, account for the impact of this change on the supply side.

The process of adjustment from the original equilibrium to the new one will not be described. It is similar to that involved in the case of an exogenous shift in the rate of entry of new vacancies, which will be discussed below. We shall now turn to stability considerations.

Stability Considerations

Here we shall explore local stability properties of the model which describes the short-run dynamics of adjustment under the decomposition procedure. We shall confine our attention to the supply side, as the demand side is completely symmetric. As noted above, the stability of the overall equilibrium will not be formally examined. Although we have not developed a model of overall adjustment, stability questions could be explored in terms of the separate adjustment models. If we confine our attention to local stability, the two models, taken together, can be considered as an approximation of the overall adjustment process.

Let us recall the differential equation (5.10a) describing the adjustment process of numbers of sellers with different asking prices. Let $V^o]$ be the stationary equilibrium vector from (5.23) and write: $\bar{v}_j = \frac{\partial \bar{v}}{\partial V_j}$. We shall examine small perturbations around the solution, $\Delta V] = V] - V^o]$. Equations (5.20a) and (5.23) yield:

$$\dot{\Delta V}] = N\Delta V] + \begin{bmatrix} & & 0 & & \\ \hline \bar{v}_1 & \cdots & \bar{v}_j & \cdots & \bar{v}_m \end{bmatrix} \Delta V] \ . \tag{5.36}$$

Recall that the rate of entry of new vacancies depends upon the intensity of the process of contacts for sellers with buyers, λ, and the average price of current transactions. Hence,

$$\bar{v}_j = \frac{\partial \bar{v}}{\partial P^c} \frac{\partial P^c}{\partial V_j} \ .$$

Also, we assumed that $\frac{\partial \bar{v}}{\partial P^c} > 0$; $\frac{\partial P^c}{\partial V_j}$ is given by (5.14). Note that the derivative of P^c with respect to V_j depends upon the distribution of reservation prices and the distribution of vacancies; it is positive (negative) for $j > j_o$ $(j < j_o)$ and equal to zero for $j = j_o$, where j_o is defined by $p^{j_o} = P^c$. Equation (5.36) can be written concisely as follows:

$$\dot{\Delta V}] = \bar{N}\Delta V] \ , \tag{5.37}$$

and $\bar{N}$ is given by:

$$\overline{N} = \begin{bmatrix} -\lambda G_1 - \gamma_1 & \gamma_2 & 0 & \cdots & 0 \\ 0 & -\lambda G_2 - \gamma_2 & \gamma_3 & & 0 \\ & & & & \vdots \\ & & & & 0 \\ & & & & \gamma_m \\ \overline{v}_1 & \overline{v}_2 & & & -\lambda G_m - \gamma_m + \overline{v}_m \end{bmatrix} .$$

The solution to equ. (5.37) is asymptotically stable at $V^o]$ if the eigenvalues of the matrix $\overline{N}$ all have negative real parts. The characteristic equation for $\overline{N}$ is: $\det[\overline{N} - \Lambda I] = 0$, where Λ (scalar) is an eigenvalue of $\overline{N}$. It is hard to predict the sign of the real part of the eigenvalues of the matrix $\overline{N}$, although we do know the signs and to some extent, the relative sizes of the terms $\overline{v}_j$. Also, it seems unlikely that we could examine in more detail the types of instability that the above adjustment process might exhibit, e.g., exponential instability (positive real eigenvalues) or cyclical instability (non-zero imaginary parts of eigenvalues). In general, the eigenvalues depend upon the technology of information acquisition and dissemination, the distribution of reservation prices, behavioral characteristics of sellers (discount rates and time-horizons of search) and the responsiveness of the participation decision to changes in the distribution of vacancies over the asking price range.

An alternative approach to stability analysis is the use of Lyapunov's second method. The Lyapunov stability theorem ensures stability if there exists a symmetric positive definite matrix W such that $-W\overline{N}^T - \overline{N}W$ is positive definite. We shall try the diagonal $m \times m$ matrix with elements $\frac{1}{\lambda G_j + \gamma_j}$, $j = 1, \ldots, m$,

$$W = \begin{bmatrix} \dfrac{1}{\lambda G_1 + \gamma_1} & & 0 \\ & \ddots & \\ 0 & & \dfrac{1}{\lambda G_m + \gamma_m} \end{bmatrix} .$$

Let us write $-W\bar{N}^T - \bar{N}W = [w]$. The matrix $[w]$ is as follows:

$$[w] = \begin{bmatrix} 2 & -\dfrac{\gamma_2}{\lambda G_2 + \gamma_2} & 0 & \cdots & -\dfrac{\bar{v}_1}{\lambda G_1 + \gamma_1} \\ -\dfrac{\gamma_2}{\lambda G_2 + \gamma_2} & 2 & & & -\dfrac{\bar{v}_2}{\lambda G_2 + \gamma_2} \\ 0 & & 2 & \cdots & \vdots \\ \vdots & & & & \\ 0 & & & & -\dfrac{\bar{v}_{m-1}}{\lambda G_m + \gamma_m} \\ -\dfrac{\bar{v}_1}{\lambda G_1 + \gamma_1} & -\dfrac{\bar{v}_2}{\lambda G_2 + \gamma_2} & & \cdots & 2 - 2\dfrac{\bar{v}_m}{\lambda G_m + \gamma_m} \end{bmatrix} .$$

The system (5.37) is stable, if $[w]$ is positive definite. Hence it is sufficient for the stability of $\bar{N}$ if all principal minors of $[w]$ are positive. It can easily be shown by expansion that the $(m-1)$ principal minors of $[w]$ are positive definite. With respect to the sign of $\det[w]$, no definite answer can be given because the relative sizes of the terms $\dfrac{\bar{v}_j}{\lambda G_j + \gamma_j}$ are involved.

If the responsiveness of the rate of entry of new vacancies is much smaller compared to the corresponding adjustment coefficient, then local asymptotic stability is guaranteed. In case $\dfrac{\partial \bar{v}}{\partial P^c} = 0$ (i.e., the rate of entry of new vacancies does not depend upon the average price of current rransactions), then the supply side short-run dynamics are stable.[17/]

Our inability to guarantee local stability may reveal properties of a much wider class of dynamic processes, namely those which are essentially based upon behavioral assumptions of perfect foresight. Models based on such assumptions cannot adequately deal with situations of unfulfilled expectations. The rigidness thus introduced is an apparent cause of instability.

In view of Samuelson's correspondence principle, assuming stability of the underlying system may facilitate comparative statics analysis. However, we will not pursue this in this work. We should, however, mention that the lack of symmetry of the characteristic matrix precludes us from using the theory of symmetric stable matrices. In the remainder of this section we shall examine equilibrium adjustment processes in a heuristic fashion.

A Heuristic Illustration of Adjustment Processes

In the beginning of this section, we presented a formulation of the comparative statics problem. We can use our adjustment model to describe the process of price and stock adjustment in a particular case for which the comparative statics analysis is conclusive. We shall be concerned with the case in which an original supply side equilibrium is disturbed by an exogenous factor, while the demand side remains at equilibrium. This is of course intended as a mere illustration rather than an exhaustive analysis of the subject.

17/ This is immediate because for $\bar{v}_j = 0$, $j = 1, \ldots, m$, then $\bar{N} = N$ and the eigenvalues of N are equal to $-\lambda G_j - \gamma_j$, $j = 1, \ldots, m$.

We shall examine the effects upon the equilibrium on the supply side of the market of an exogenous (to the specific quality submarket under consideration) shift in the rate of entry of new vacancies. Such a shift can be caused by a variety of factors, such as a structural change in the normal operation of the filtering process, higher construction rates because of increased subsidies, lower construction costs, etc. We shall assume that the exogenous factor that shifts upwards the rate of entry of new sellers does not affect the rate of entry of new prospective buyers. We can use the formulation of the comparative statics problem to predict the effect of such a change on the supply side, which is a uniform increase in the numbers of sellers at different asking price levels while the distribution of asking prices remains unaffected.

We can easily illustrate the process of adjustment set in motion by such a change by using the adjustment equations (5.10b). The submarket is at equilibrium when the exogenous small shift in $\overline{v}$ takes place. For the new rate of entry of vacancies $\overline{v}'$, $\overline{v}' > \overline{v}$ for the same λ and P^c. The new sellers start their search by setting their optimal asking price policies. The inflow of new vacancies with asking prices at the upper part of the price range will increase the stock of sellers there. As a result, more transactions will take place at higher prices and since $\frac{\partial P^c}{\partial V_j}$ for high j's is positive, the average price of current transactions will increase. The latter will cause a further increase in the rate of entry of new vacancies, which in turn reinforces the above tendency. Some of the new sellers, who either make contacts with buyers with reservation price below their current asking prices and thus do not sell, or are not contacted by any buyers, adjust their asking prices downwards. Eventually asking prices of new sellers will decrease below the original level of P^c. The resulting increase in the stock of vacancies there will cause a downward pressure on P^c. Since this effect is exacerbated, the further asking prices of sellers decrease and P^c will be pulled downwards towards its original level. In such a way the distribution of vacancies may reach its equilibrium position, provided that the process is stable. At any rate, the average price of current transactions may fluctuate around its equilibrium value. At the new equilibrium, the distribution of asking prices is the same, while the numbers of sellers at different price levels have increased uniformly. Thus the average price of current transactions is again seen to be the vehicle of equilibrium adjustment.

An interesting result of our model of adjustment is that infusion of new vacancies will initially cause an increase in the average price of current transactions, contrary to what one might expect on the basis of traditional microeconomic theory and the "excess demand creates upward pressure on price" postulate of tâtonnement adjustment. It should be noted, however, that our assumption that new sellers enter the submarket at the upper part of the price range is partly responsible for this result.

At the new equilibrium, the total number of vacancies in the market has increased. This will, of course, affect conditions on the demand side of the market. In particular, if buyers are well informed, the increase in the number of vacancies will produce an increased intensity of the buyers' process of contacts. According to our assumption regarding the information production function, the intensity of the process of contacts for sellers will not be affected, although supply side conditions will ultimately be affected. The increase in L, the intensity of the process of contacts for buyers, will affect the distribution of reservation prices and buyers over the reservation price range. Finally, the distribution of asking prices will be affected, provided that sellers perceive the change in demand side conditions.

The above heuristic exposition underscores the difficulty one encounters in attempting to examine such questions.[18]

Despite its lack of rigor, such an exposition does provide insight into the adjustment process. A large number of questions can be dealt with in such a fashion. In addition to treating each side of the market separately, we can, for small changes, treat the overall adjustment problem by considering the two sets of adjustment equations and using them in an iterative fashion. However, such issues will not be further pursued in this work.

Summary

In this section, we have dealt with comparative statics and stability properties of the model. We formulated the comparative statics problem for an overall equilibrium, and used this precisely to define the decomposition procedure of holding one side of the

[18] Such a model may lend itself to studying market operation through simulation. Work devoted to the study of residential segregation, Freeman & Sunshine (1970), through computer simulation on the basis of behavioral models similar to the ones employed here came to my attention at a time when most of this work has been completed. Their behavioral models are very poorly developed. Computer simulation seems to be one of the tools for fruitful research along directions suggested by our work.

market at equilibrium and examining the other. We then examined the comparative statics of an exogenous upward shift in the entry rate of new buyers. It was shown that if the supply side is held at equilibrium, such a shift leaves the distribution of reservation prices unaffected and increases the number of prospective buyers in the market.

Finally, we discussed stability and adjustment processes for the supply side of the market, stating sufficient conditions for stability. We discussed in a heuristic fashion the adjustment process involved in the case of an equilibrium which is disturbed by an exogenous shift in the rate of entry of new vacancies.

5.7 Summary and Interpretations

This chapter was devoted to the development of models of price and stock adjustment. In terms of the short-run dynamics of adjustment, we introduced and characterized an equilibrium concept for the submarket. Let us first describe the conceptual scheme which underlies our equilibrium adjustment models and then briefly summarize some of the results we obtained in this chapter.

Market participants search on expectations of stationary market conditions. For the purpose of equilibrium analysis, optimal asking and reservation price policies that are obtained from our behavioral models are discretized. Equilibrium adjustment of the distributions of asking and reservation prices was described by modelling the market participants' "flowing through" the discrete steps of a price range characterizing the quality submarket. Some make contacts and others do not, and some of the contacts made do not lead to transactions. Buyers and sellers set finite horizons for their search. If, at the end of the time-horizon, a seller has not sold his house or a buyer has not bought a house, they leave the market. We offered a description of the short-run dynamics of adjustment by means of a system of ordinary differential equations.

Individuals enter the submarket at rates which depend upon market conditions. We showed that if one side of the market is held at equilibrium the adjustment process on the other side is characterized by a unique equilibrium price distribution and number of market participants. In the general case of an overall equilibrium, we showed the existence of a configuration of distributions of asking and reservation prices and of numbers of market participants.

Next we formulated the general comparative statics problem for an overall equilibrium. For this purpose, the decomposition procedure that we have employed in this work was defined precisely. We examined the comparative statics of a demand side equilibrium when it is disturbed by an upward shift in the rate of entry of new buyers. It was shown that such a change leaves the reservation price distribution unaffected but increases the number of prospective buyers in the market. Finally, we examined the stability of a supply side equilibrium and provided a heuristic exposition of the equilibrium adjustment process when such an equilibrium is disturbed by an exogenous shift in the rate of entry of new sellers. We were unable, however, to prove stability in the general case of an endogenous entry rate of new sellers, but we stated sufficient conditions for stability.

We have been concerned with equilibrium in a single quality submarket of a housing quality hierarchy. We have not attempted explicitly to incorporate interactions between different levels of a quality hierarchy. In general, this is too big a problem because of conditions of uncertainty and search. An analytical treatment would have to resort to averages and aggregates, i.e., reduce the problem to its deterministic dimensions.

There are many issues we could examine with our approach. We chose to examine a few, some easy and some much more difficult to deal with. Strong results were arrived at in a reasonably large number of cases, given the difficulty of extending economic theory into a virtually unexplored area.

Chapter Six

SUMMARY AND SUGGESTIONS FOR FUTURE RESEARCH

Although studies of housing markets have noted the importance of market participants' imperfect knowledge of market conditions, the role of the vacancy rate as a market variable, and the incidence of price dispersion, there has been no comprehensive theoretical analysis of these issues. This dissertation encompasses such a theoretical study of market structure and operation under conditions of uncertainty. The search behavior of housing market participants is modelled and utilized to derive an equilibrium theory for a quality submarket. Although we have drawn extensively upon an analysis of housing markets in putting forth theoretical arguments, both the analytical tools employed and the equilibrium concepts developed apply beyond the confines of housing markets. We have attempted to summarize our results as the exposition proceeded. Instead of repeating these results we will summarize the most important in the context of a discussion of the basic elements of our approach and will offer a critical review of some results in conjunction with suggestions for future research.

At any point in time a quality submarket is characterized by the probability distribution functions of sellers' asking prices and buyers' reservation prices and the number of prospective buyers and sellers in the market. Buyers' (sellers') search of the market was modelled as sampling, at random time, from the distribution of asking (reservation) prices. The process of contacts was described by a point process whose intensity is a function of market conditions and information acquisition and dissemination technology. The informational structure of the market was rigorously founded upon behavioral models of buyers' and sellers' use of informational intermediaries. The sensitivity of the optimal search policy of buyers and sellers to variation in the discount rate, time-horizon of search, number of market participants, information technology and characteristics of asking- and reservation price distributions was examined.

Under the conditions of finite time-horizon of search and exponential discount rates, the behavioral models for buyers and sellers were integrated into a theory of incremental price and stock adjustment. The mathematical model describing the short-run dynamics of adjustment for each side of the market was obtained under the assumption that the other side is held stationary. The two separate models describing the short-run dynamics of adjustment were considered simultaneously for the purpose of defining an overall equilibrium for a quality submarket. Such an equilibrium was defined as a configuration of stationary probability distribution functions of reservation and asking prices and stationary numbers of housing market participants. It was shown that such an overall equilibrium exists and is characterized by price dispersion, vacancies and unsatisfied demand. That is, the market does not "clear" and no unique equilibrium price prevails. Furthermore, the adjustment process can be classified as a non-tâtonnement process.

By holding each side of the market at equilibrium, we examined properties of equilibrium on the other side. We showed that such an equilibrium exists and is unique and stated sufficient conditions for local stability properties. Also, we formulated the general comparative statics problem for an overall equilibrium. Finally, we examined the comparative statics of shifts in the (endogenous) entry rate of new market participants and described the equilibrium adjustment process which would follow such disturbances.

These are the basic elements of our approach. The models we obtained are general enough to facilitate rigorous analysis of a large class of non-tâtonnement adjustment processes. We shall now proceed to a critical review of some aspects of this work.

The analytical formulation of behavioral models for market participants, buyers and sellers, exhibits a general time-dependence that has been only partly explored in this work. The dynamic programming formulation adopted is amenable to generalization in several directions, such as consideration of several alternative courses of action open to market participants that may or may not be statistically independent, or participants' behavior under a variety of disequilibrium situations. Hence, the dynamics of the participation decision can be further explored at the behavioral level. The determinants of the (aggregate) entry rates of new participants should be rigorously obtained from the participation decision. For the purpose of empirical testing, in particular, the effects of employment opportunities within the geographical area of a particular housing quality hierarchy, of income changes or housing subsidies, of construction lags and of the availability of finance capital upon the households' participation decisions should be considered.

The model can account for intramarket turnover, once a particular classification of new participants in terms of their origin is introduced (i.e., whether they enter a hierarchy for first time or previously occupied dwelling units within the same hierarchy). This should, of course, be reflected in the participation decision and would bear upon market adjustment.

Future theoretical research will benefit from a comparative analysis of unit and price competition. Such an analysis may be of use in providing appropriate limiting conditions for a model which will account for game-like interactions between buyers and sellers--a hitherto unexplored aspect of housing markets.

The informational structure of the market, as specified in this work, can be explored further so that the model will attain greater flexibility. First, we may introduce a seller's asking price as a determinant of the intensity of the process of contacts he makes with buyers. Furthermore, we may delineate a market for informational services. From such a viewpoint, the demand side of such a market was examined in sections 3.6 and 4.4, which describe the use of informational intermediaries by sellers and buyers. The effects of information pooling on individuals' behavior and market adjustment can be examined with the tools suggested in this dissertation.

A host of other issues can also be analyzed. Participants familiar with a particular housing market may enjoy informational advantages in comparison with newcomers; the opportunity cost of resources, expended by participants to ensure for themselves a specific intensity of the process of contacts or information technology may differ among participants. As a result, different individuals might make contacts at different rates. By using such notions, it is possible to incorporate another element of intramarket turnover and to analyze the market performance of poorly informed individuals.

Another area of practical as well as theoretical significance, that has been considered by some studies of housing markets, is the process of discrimination in housing and its effects. The advantages of our model in examining this problem are two-fold: differential transaction and psychic search costs[1] can be easily handled; and the interaction between buyers and sellers is at the center of our model, so that sellers' discrimination can be modelled by adjusting the intensity of the process of contacts. Furthermore, costs, or benefits, to buyers and sellers from discrimination can be estimated. The disequilibrium structure of the model would facilitate an examination of the incidence of discrimination as a function of market conditions. If nonwhites are discriminated against, their number relative to whites participating in the market would influence the intensity of the process of contacts. Similarly, transient housing market phenomena, such as neighborhood tipping, can also be investigated.

As we noted earlier, equilibrium for a quality submarket was investigated in terms of the dynamics of adjustment rather than as a specific case of general equilibrium under uncertainty. Our inability to obtain stability results hampered our examination of overall convergence conditions. Such conditions would justify the decomposition procedure adopted in this work, i.e., examining equilibrium adjustment in one side of the market while the other is held at stationary equilibrium.

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[1] "There is a great deal of qualitative evidence that nonwhites have difficulty in obtaining housing outside the ghetto (see James Hecht, Kain (1969), McEntire, and Karl and Alma Tauber). Persistence, a thick skin, and a willingness to spend enormous amounts of time house-hunting are minimum requirements for nonwhites who wish to move into white neighborhoods. These psychic and transaction costs may be far more significant than out-of-pocket costs to Negroes considering a move out of the ghetto." Kain and Quigley (1972), p. 264.

In future research the informational equilibrium, "desired equals actual," should be generalized, and adaptive behavior, in the sense of incorporating information acquired in the course of search, should be allowed. Finally and most importantly, because we dealt with expected values of market variables, our model did not require a knowledge of probabilistic laws that might characterize equilibrium adjustment. Investigation of such laws could be pursued as a much better alternative to an ad hoc modelling of equilibrium adjustment that typically lacks behavioral foundations.

In this work we assumed that the information sellers disseminate to the market about their units is sufficient for buyers to classify them in terms of quality; in addition, prospective buyers can readily assess the quality of a housing unit they visit. As a natural extension of our model, the following situation could be examined: buyers use prices of housing units to infer quality before they actually search out a housing unit in order to ascertain its quality. Sellers, on the other hand, may be assumed to know the quality of their units. In contrast to the model we examined, such a view of a housing market may be labelled as a case of informational asymmetry.

Preliminary investigation of behavioral models for the case of informational asymmetry has shown that the approach we have taken in this work can be applied to that case with only a few minor modifications. First, we need not restrict ourselves to a particular quality submarket--an entire housing quality hierarchy can be treated. Second, from the viewpoint of buyers, the market is segmented in terms of price, since buyers cannot directly observe quality and hence perceive price as a signal. Third, a better specification of the informational structure of the market may be adopted. That is, as suggested earlier in this chapter, the probability that during any interval of time a seller will make a contact with a buyer should depend upon the seller's asking price.

Although the case of informational asymmetry has been investigated[2] in other areas of economic literature, it is interesting to examine it in the context of housing markets for two reasons. First, it comprises a reasonable view of ownership and rental housing markets; and second, an examination of equilibrium in such an environment may enhance the understanding of market signalling and informational externalities, and thus contribute to the understanding of the theoretical underpinnings of market structure and price formation, as well as of the persistence of price dispersion.

As was mentioned in Chapter One, one of the goals of past research in disequilibrium economics was a theoretical derivation of the Phillips curve. These efforts have been only partially successful. In housing market terms, the issue becomes whether a stable relationship exists between housing price changes and vacancy rates. Housing market adjustment towards equilibrium, under any common notion of the term, takes place through changes in price as well as changes in the number of vacancies and consumer searchers. A relationship between price changes and vacancy rates--or whatever the measure of idle resources might be during the equilibrium adjustment process--that could be derived from behavioral models would be a tool of theoretical as well as practical significance.

[2] See Ackerlof (1970) and Spence (1972).

However, we have refrained from pursuing such a task here for two reasons: first, such a relationship would be rather meaningless unless an entire housing market is considered; and second, our discussion of equilibrium, carried out in Chapter Five, suggests that results may vary greatly depending upon the properties of the technology of information acquisition and dissemination.

Finally, no public policy issues were explicitly taken up in this dissertation. Government policies designed at improving the consumption of housing services by disadvantaged socioeconomic groups may be more efficient if implemented in conjunction with housing market information systems. It is expected that further development of the housing market model discussed in this work will lead to a framework for evaluating such policies.

This work has elaborated on a market structure characterized by what T. Koopmans referred to as "secondary" uncertainty.[3] Although we focused upon some hitherto neglected but nevertheless important aspects of housing markets, the non-tâtonnement equilibrium adjustment models we developed have a much wider applicability and will hopefully constitute a contribution to the literature of models of equilibrium adjustment.

[3] See Koopmans (1957), p. 163. Koopmans argues that "... the secondary uncertainty arising from lack of communication,...,is quantitatively at least as important as the primary uncertainty arising from random acts of nature and unpredictable changes in consumers' preferences."

BIBLIOGRAPHY

Ackerlof, G. A., "The Market for 'Lemons': Qualitative Uncertainty and the Market Mechanism," Quarterly Journal of Economics, Vol. 84, August, 1970.

Arrow, K. J., "Toward a Theory of Price Adjustment," in M. Abramovitz, The Allocation of Economic Resources, Stanford: Stanford University Press, 1959.

________, and M. Kurz, Public Investment, The Rate of Return, and Optimal Fiscal Policy, Baltimore: The Johns Hopkins Press, 1970.

Becker, G. S., Economic Theory, New York: Alfred A. Knopf, 1971.

Blackwell, D., "Discounted Dynamic Programming," Annals of Mathematical Statistics, Vol. 36, 1965.

Carman, J. M., "Vacancy and Turnover in Alameda County--Characteristics of Rental Housing and Population Mobility 1967-68," Technical Report No. 3, The Center for Real Estate and Urban Economics, University of California, Berkeley, 1969.

Cooper, C., "The House as Symbol of Self," Institute of Urban and Regional Development, University of California, Berkeley, Working Paper No. 120, May 1971.

DeGroot, M. H., Optimal Statistical Decisions, New York: McGraw-Hill Book Company, 1970.

DeLeeuw, F. and F. Ekanem, "The Supply of Rental Housing," American Economic Review, Vol. LXI, No. 5, December 1971.

Diamond, P. A., "A Model of Price Adjustment," Journal of Economic Theory, No. 3, 1971.

Edel, M., "Filtering in a Private Housing Market," in M. Edel and J. Rothenberg, Readings in Urban Economics, New York: The McMillan Company, 1972.

Elfving, G., "A Persistency Problem Connected with a Point Process," Technical Report No. 16, 1966, Department of Statistics, Stanford University.

Fisher, F. M., "Quasi-competitive Price Adjustment by Individual Firms: A Preliminary Paper," Journal of Economic Theory, No. 2, 1970.

Freeman, L. C. and M. H. Sunshine, Patterns of Residential Segregation, Cambridge, Mass: Schenkwan Publishing Company, 1970.

Gastwirth, J. L., "On Probabilistic Models of Consumer Search for Information," (Unpublished). Department of Statistics, Harvard University, Technical Report No. 134, June 1971.

________, and F. Mehran, "On Probability Models of Information in the Labor Market," (Unpublished). Harvard University, Department of Economics, May 1971.

Grigsby, W. G., Housing Markets and Public Policy, Philadelphia: University of Pennsylvania Press, 1963.

Gronau, R., "Information and Frictional Unemployment," American Economic Review, Vol. LXI, No. 3, June 1961.

Hahn, F. and T. Negishi, "A Theorem on Non-Tatonnement Stability," Econometrica, Vol. 30, No. 3, 1962.

Hirshleifer, J., "Where Are We in the Theory of Information," American Economic Review, Papers and Proceedings, Vol. LXIII, No. 2, May 1973.

Holt, C. C., (1970a) "Job Search, Phillips' Wage Relation, and Union Influence: Theory and Evidence," in E. Phelps et. al. (1970).

________, (1970b) "How Can the Phillips Curve be Moved to Reduce Both Inflation and Unemployment?" in E. Phelps et. al. (1970).

________, and M. David, "The Concept of Job Vacancies in a Dynamic Theory of the Labor Market," in The Measurement and Interpretation of Job Vacancies, New York: National Bureau of Economic Research, 1966.

Kain, J. F. and J. M. Quigley, "Measuring the Value of Housing Quality," Journal of the American Statistical Association, June 1970.

________ and ________, "Housing Market Discrimination, Homeownership, and Savings Behavior," American Economic Review, Vol. LXII, No. 3, June 1972.

Karlin, S., "Stochastic Models and Opitmal Policy for Selling an Asset," in K. J. Arrow et al. Studies in Applied Probability and Management Science, Stanford: Stanford University Press, 1960.

Koopmans, T. C., Three Essays on the State of Economic Science, New York: McGraw-Hill Book Company, 1957.

Lancaster, K., "A New Approach to Consumer Theory," Journal of Political Economy, Vol. 74, No. 2, 1966.

________, "Changes and Innovations in the Technology of Consumption," American Economic Review, Papers and Proceedings, Vol. L, May 1966.

Lansing, J. B., C. W. Ceifton and J. N. Morgan, New Homes and Poor People, Ann Arbor: University of Michigan, 1969.

Leland, H. E., "Theory of the Firm Facing Uncertain Demand," American Economic Review, Vol. LXII, No. 3, June 1972.

Luenberger, D. G., Optimization by Vector Space Methods, New York: John Wiley and Sons, 1969.

McCall, J. J., "The Economics of Information and Optimal Stopping Rules," Journal of Business, Vol. 38, 1965.

_______, "Economics of Information and Job Search," Quarterly Journal of Economics, Vol. 84, February 1970.

_______, "Probabilistic Microeconomics," Bell Journal of Economics and Management Science, Vol. 2, No. 2, Autumn 1971.

McNulty, P. J., "Economy Theory and the Meaning of Competition," Quarterly Journal of Economics, Vol. 82, November 1968.

Meilijson, I., "A Bargaining Problem," (Unpublished) Ph.D. Dissertation, Department of Statistics, University of California, Berkeley, 1969.

Mittelbach, F. G., et al., Optimization Problems in Housing Investment," preliminary paper presented to the Western Economic Association, August 1969.

Mortensen, D. T., (1970a) "A Theory of Wage and Employment Dynamics," in E. Phelps (ed.) 1970.

_______, (1970b) "Job Search, the Duration of Unemployment, and the Phillips Curve," American Economic Review, Vol. LX, No. 5, December 1970.

Morgenstern, O., "Thirteen Critical Points in Contemporary Economic Theory: An Interpretation," Journal of Economic Literature, Vol. X, No. 4, December 1972.

Muth, R. F., Cities and Housing, Chicago: The University of Chicago Press, 1969.

Muehlen, P. von zur, "Optimal Price Adjustment: A Test of a Price Equation in U. S. Manufacturing," Proceedings of IEEE Conference on Decision and Control, December 1972.

Negishi, T., "On the Formation of Prices," International Economic Review, Vol. 2, 1961.

Parzen, E., Stochastic Processes, San Francisco: Holden Day, 1967.

Phelps, E., et al., Microeconomic Foundations of Employment and Inflation Theory, New York: Norton and Co., 1970.

Quirk, J. and R. Saposnik, Introduction to General Equilibrium Theory and Welfare Economics, New York: McGraw-Hill Book Company, 1968.

Radner, R., "Equilibrium under Uncertainty," paper delivered at December 1972 meetings of the American Economic Association December 1972.

Rapkin, C. L. Winnick and D. M. Blank, Housing Market Analysis; A Study of Theory and Methods, Washington D.C.: U. S. Housing and Home Finance Agency, 1953.

Rees, A., "Information Networks in Labor Markets," American Economic Review, Vol. 56, No. 2, June 1966.

Ross, S. M., Applied Probability Models with Optimization Applications, San Francisco: Holden Day, 1970.

Rossi, P., Why Families Move--A Study of the Social Psychology of Residential Mobility, Glencoe, Illinois: The Free Press, 1955.

Rothenberg, J., Economic Evaluation of Urban Renewal, Washington, D.C.: The Brookings Institution, 1967.

Rothschild, M., "Models of Market Organization under Imperfect Information: A Survey," Harvard Institute of Economic Research, Discussion Paper No. 224, December 1971.

Samuelson, P. A., Foundations of Economic Analysis, New York: Atheneum 1965.

Schelling, T. C., "Neighborhood Tipping," Harvard Institute for Economic Research, Discussion Paper No. 100, 1970.

_______, "Models of Segregation," American Economic Review, Papers and Proceedings, Vol. LIX, No. 2, May 1969.

Smith, W. F., Housing--Social and Economic Elements, Berkeley, California: University of California Press, 1970.

Spence, A. M., "Market Signalling: The Informational Structure of Job Markets and Related Phenomena," Discussion Paper No. 4, Public Policy Program, Kennedy School of Government, Harvard University, February 1972.

Stigler, G. J., "The Economics of Information," Journal of Political Economy, Vol. LXIX, No. 3, June 1961.

________, "Information in the Labor Market," Journal of Political Economy, Vol. LXX, No. 5, 1962.

Sweeney, J. L., "A Dynamic Theory of the Housing Market," (Unpublished) Ph.D. Dissertation, Department of Engineering-Economic Systems, Stanford University, June 1971.

________, "Quality, Commodity Hierarchies, and Housing Markets," Econometrica (forthcoming).

________, "Housing Unit Maintenance and the Mode of Tenure," Journal of Economic Theory (forthcoming).

________, "Filtering in Housing and the Indirect Effects of New Construction," Unpublished paper, Department of Engineering-Economic Systems, Stanford University, June 1973.

Telser, L. G., "Searching for the Lowest Price," American Economic Review, Papers and Proceedings, Vol. LXIII, No. 2, May 1973.

Thurow, L. C. and R. E. B. Lucas, "The American Distribution of Income: A Structural Problem," U. S. Congress, Joint Economic Committee, 92nd Congress, 2nd Session, Washington: U. S. Government Printing Office, March 1972.