# A DMP MODEL OF INTERCITY TRADE

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**Abstract**

Job matching, a central feature of DMP theory, requires contacts between prospective employers and employees. This paper assumes that they may be either face-to-face and take place at city centers or via referrals from social contacts. The paper presents a model of an economy whose urban structure consists of cities of different types. All cities produce a non-tradeable final good using ranges of tradeable intermediate varieties. Each city has an internal spatial structure: individuals commute to the CBD in order to work, when employed, and to seek jobs, when unemployed. Hiring by each intermediate producing firm is subject to frictions, which are modeled in the Diamond–Mortensen–Pissarides fashion. City type is conferred by specialization in producing one of two types of intermediate varieties and there is intercity trade in intermediate varieties. The paper examines properties of equilibrium with intercity trade and its dependence on various parameters and their consequences for unemployment, output and welfare across the economy along a steady state. The model's use of international trade tools confers a central role to labor market tightness, akin to factor intensity. A natural dependence of unemployment on city size is generated. Equilibrium outcomes generically diverge from the planner's optimum: there exists mismatch generically. Socially optimal unemployment trades off the probability of employment to search costs of firms independently for each skill type and independently of city size. Socially optimal city sizes are independent of the matching model and therefore labor market tightness considerations but reflect both market size effects and the skill composition of the economy.

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# 1 Introduction

The Great Recession of 2007–2009 and its aftermath are amply demonstrating the importance of spatial variations in unemployment and economic activity. Not all parts of the US economy and not all economies of the world have been equally affected by it. The housing bubble has been most pronounced in some areas of the US, some of which experienced very dramatic declines in house prices. Some of those particular areas have experienced robust recoveries, while other areas are particularly slow in recovering. The variation of employment dynamics over the business cycle across federated states, regions and cities in large economies is particularly interesting. The present paper aims specifically at a better understanding of variations in economic activity and unemployment across an economy's cities while accounting for intercity trade. An urban perspective is made possible by a synthesis of new economic geography and urban economics, on the one hand, with the economics of markets with frictions, famously modeled by the Diamond-Mortensen-Pissarides (DMP) model, on the other.

The present paper proposes a coherent and firmly micro-founded theoretical model of urban unemployment in an economy of open, trading cities. The approach proposed here enhances the system-of-cities model by allowing for unemployment and fluctuations in economic activity that may differ systematically across cities in a large economy. It also enhances the DMP model of labor markets with frictions by introducing, in addition, spatial frictions of the sort that characterize urban economies. The model's use of international trade tools confers a central role to labor market tightness, akin to factor intensity. Equilibrium modeling of cities lends additional discipline to analyses of urban unemployment and provides a systematic way of exploiting Beveridge curves, as well, as an expository tool, as well. A specific application via job referrals by social contacts is another new feature. The model is cast in terms of certainty equivalents, which is in line with the original DMP literature. Equilibrium outcomes generically diverge from the planner's optimum: socially optimal unemployment trades off the likelihood of employment against search costs of firms, independently for each skill type and independently of city size. Socially optimal city sizes are independent of labor market tightness considerations but reflect both market size effects and the skill composition of the economy.

The present paper adopts a DMP approach, really following Pissarides (1985; 2000), embedded in a system-of-cities model, following Henderson (1974; 1987) as adapted by Fujita, Krugman and Venables (1999). It aims at accommodating the increasing mainly empirical literature on urban macroeconomics. Cities are specialized in terms of different ranges of differentiated intermediate varieties they produce. Only the intermediate varieties are tradeable, and are used by means of a Cobb–Douglas production function to produce in each city a final consumption good, which is neither storable nor tradeable. Each variety is produced by a single firm, which uses labor as its only input. There are two types of varieties, $\alpha-$ and $\beta-$varieties, with each being produced by a correspondingly different kind of labor in a monopolistic competition setting. The groups of firms producing the respective varieties make up the $\alpha-$ and $\beta-$ industries. Cities specialize by producing either type of varieties, provided they host the appropriate kind of labor.

Each intermediate producing firm hires in a frictional labor market, modeled in the standard Mortensen–Pissarides fashion. For simplicity I assume that jobs are destroyed at constant rates for each type of labor, and that forces individuals and firms back to the labor market. Matching of workers and vacancies is city-specific. Individuals maximize expected utility of consumption. Individuals need to travel to the central business district of the city where they reside in order to work, when employed, and to be matched, when unemployed. However, proximity is costly in terms of congestion, which in turn generates a land rent gradient. Locations nearer the city center are more attractive, but land rents adjust so as identical individuals be indifferent as to where they locate. The paper also examines matching via referrals from social contacts. The model of intercity trade in this paper in accommodating labor market frictions highlights the role of labor market tightness in intercity resource allocation that serves as a counterpart of factor intensities in conventional international trade models. A comparison between equilibrium allocations and those of a planner's problem concludes the paper.

## 1.1 Review of the Literature

Business cycle phenomena that have been addressed in urban contexts involve primarily measures of employment fluctuations. Taking cues from Helsley and Strange (1990), who have an explicit urban setting in mind, In broadly related research that emphasizes unemployment, Gan and Zhang (2006) link city size to matching efficiency. Simon (1988) argues that the more industrially diversified a city is the lower its frictional unemployment. He does not model the urban economy as such. Coulson (2006) points to a comparison of three MSAs, Los Angeles, CA, Detroit, MI, and San Jose, CA over the period 1956–2002 [*ibid.,* Figure 1]. While all three have employments that are trending upward over time during that period, the rate at which their employments do so certainly varies across cities and over time within cities.[1] High persistence in metro unemployment rates is documented by Kline and Moretti (2013), who develop a simple model of local labor markets and use it to study the effects of place-based policies in the form of local job creation programs.

Recent research throws additional light at employment and unemployment variations across US cities. Rappaport (2012) works with three possible explanations: one, skill mismatch of workers in high unemployment metro areas with the hiring needs of firms elsewhere; two, some metro areas offer intrinsic characteristics that make households and firms unwilling to move; three, high moving costs support long term divergence in metro unemployment areas. Rappaport's empirical analysis supports all three explanations. Metro workforce characteristics are able to account for the largest share of the variation in metro unemployment rates, when measured over complete business cycles. Characteristics more intrinsic to metro areas themselves account for much of this variation as well, though not as much as workforce characteristics. Estimated moving costs are sufficiently high to some households unwilling to move away from high-unemployment metros. Proulx (2013) finds that estimates for Okun's laws for U.S. MSAs show moderate to high cross-sectional dependence, a result which is robust to a number of different spatial proximity measures. In fact, the cross-sectional dependence appears to increase if instead of distance-based economic similarity-based measures are used. Decomposing the total effect of changes in the growth rate of real MSA GDP on the unemployment rate shows that the indirect effect of growth in GDP in neighboring cities dominates the direct effect of growth in local GDP.

Of relevance to the present paper is the international trade literature that emphasizes labor market frictions, such as Davidson, Martin, and Matusz (1999), who examine basic international trade issues in the light of frictional labor markets, and Costinot (2009) who rationalizes a positive association between unemployment and trade protection. Labor market flexibility as a source of comparative advantage is examined by Cuñat and Melitz (2012), Helpman and Itskhoki (2010), Helpman, Itskhoki, and Redding (2013) and Tang (2012) develop international trade models that emphasize frictional labor markets.[2] Dutt, Mitra, and Ranjan (2009) develop a simple model of international trade with search induced unemployment and show empirically that trade liberalization increases unemployment in the short run but reduces it in the long run, as economies adjust to a new steady state.

Anderson, Burgess and Lane (2007) report that thicker urban labor markets are associated with more assortative matching in terms of worker and firm quality and that production complementarity and assortative matching is an important source of the urban productivity premium. Bleakley and Lin (2012) confirm these findings with US Census microdata. Centralized matching of unemployed workers with vacancies may be considered as a proxy for social connections, in the close physical proximity afforded by urban living, in the functioning of urban labor markets.

Wasmer and Zenou (2006) and Zenou (2009a) provide an innovative bridge between urban economics and the DMP approach. The DMP approach to unemployment as well as all other aspects of changes in employment and labor force status has been particularly fruitful in understanding the spatial consequences of individuals' experiences through episodes of employment, underemployment, and out-of-the-labor force. The present paper emphasizes city specialization and intercity trade, while allowing for richness that is unmatched by other models, including how the prospect of unemployment affects urban structure under different settings for job matching. This paper is not the first to consider U.S. cities as subeconomies for the purpose of studying equilibrium unemployment. Some of the earlier contributions to the literature on macroeconomics with frictions address properties that the literature associates with effects of cities. As an example, Hall (1989) in discussing Blanchard and Diamond (1989), p. 61, wonders whether finding of constant returns to scale for the matching function would imply that active, dense labor markets "generate the same flow of matches, per given combination of unemployment and vacancies as do lower-density, smaller markets."[3] More recently, Shimer (2007) studies the process of labor market adjustment when numbers of workers and vacancies are typically mismatched by defining the markets for all well-defined occupations in each metropolitan area as distinct submarkets.[4]

## 2 A Model of Intercity Trade

I develop first a model of intercity trade.[5] In contrast to the model of Ioannides (2013), Section 7.8, which assumes that a tradeable final good is produced using raw labor and tradable intermediates, with only the labor market for raw labor being subject to frictions, here I follow Ziesemer (2003) and assume a dynamic monopolistic competition model for each of the industries producing the $\alpha-$ and $\beta-$varieties. Unlike Ziesemer's, the present model has two trading sectors and is applied to an intercity trade context.[6] In addition to centralized search at the CBD, the paper allows for decentralized search, where workers may get referrals to job openings from their social contacts.

Each city produces a final good, which is neither tradeable nor storable and is thus consumed locally. It is produced using two composite intermediate goods, each of which are produced by combining differentiated inputs in the form of $\alpha-$ and $\beta-$varieties, by means of CES production functions that exhibit constant returns to scale in the standard fashion of the new economic geography literature. The respective ranges are endogenous and denoted by $m_\alpha, m_\beta$, respectively. Cities may specialize in the production either of the $\alpha-$ or of the $\beta-$range of varieties. Let $n_\alpha, n_\beta$ denote the number of cities, respectively. This standard Dixit–Stiglitz–Krugman [ Dixit and Stiglitz (1977); Fujita *et al.* (1999) ] feature of the model gives rise to market size effects for each urban economy.

## 2.1 A Benchmark Production Model

To understand the significance of various features of the full model, I start with a stripped-down version without "bells and whistles." Specialized inputs of types $j = \alpha, \beta$, are used by both industries by means of Cobb-Douglas production functions $Y_\alpha = \zeta^{\phi}_{\alpha,\alpha}\zeta^{1-\phi}_{\beta,\alpha}$, $Y_\beta = \zeta^{\phi}_{\alpha,\beta}\zeta^{1-\phi}_{\beta,\beta}$, in each of $n_\alpha, n_\beta$ production sites (cities) of each type, with respective populations $N_\alpha, N\beta$. A type $j$ city is populated by labor that is necessary for the production of input $j$ : producing a unit of input $j$ requires $\varpi_j^{-1}$ units of labor of type $j$. The national supplies of each type of labor are $\bar{N}_j$, so that $\bar{N}_j = n_j N_j$. Both industries operate in each city type, but their outputs are neither tradeable nor storable and used for consumption locally. Thus cities of types $\alpha$ ($\beta$) produce and trade costlessly inputs of type $\alpha$ ($\beta$) and import inputs of type $\beta$ ($\alpha$) from cities of type $\beta$ ($\alpha$) at price $p_\alpha$ ($p_\beta$). Let $(\zeta_{\alpha,\alpha}, \zeta_{\alpha,\beta})$ be the quantities of $\alpha$ inputs used by $\alpha, \beta$ cities respectively, and $(\zeta_{\beta,\alpha}, \zeta_{\beta,\beta})$ be the quantities of $\beta$ inputs used by $\alpha, \beta$ cities respectively. They satisfy the following supply equations:

$$n_\alpha\zeta_{\alpha,\alpha} + n_\beta\zeta_{\alpha,\beta} = \varpi_\alpha^{-1} n_\alpha N_\alpha,\ n_\alpha\zeta_{\beta,\alpha} + n_\beta\zeta_{\beta,\beta} = \varpi_\beta^{-1} n_\beta N_\beta.$$

All $\beta$ cities spend a fraction $\phi$ of their aggregate income $p_\beta n_\beta N_\beta$ on $\alpha$ inputs, and all $\alpha$ cities spend a fraction $1-\phi$ of their aggregate income $p_\alpha n_\alpha N_\alpha$ on $\beta$ inputs. Equating those quantities for trade balance determines the terms of trade:

$$\frac{p_\alpha}{p_\beta} = \frac{\phi}{1-\phi}\frac{\varpi_\beta}{\varpi_\alpha}\frac{n_\beta N_\beta}{n_\alpha N_\alpha}. \tag{1}$$

This benchmark model determines the terms of trade based on aggregate factor supplies, productivities and factor shares, but it does not determine the numbers of cities nor city sizes, nor in the absence of shipping costs actual shipments of intermediate inputs. The aggregate quantities, however, of the inputs readily follow:

$$\zeta_{\alpha,\alpha} = \phi\varpi_\alpha\bar{N}_\alpha, \zeta_{\alpha,\beta} = (1-\phi)\frac{n_\alpha}{n_\beta}\bar{N}_\alpha; \zeta_{\beta,\alpha} = \phi\frac{n_\beta}{n_\alpha}\varpi_\beta\bar{N}_\beta, \zeta_{\beta,\beta} = (1-\phi)\varpi_\beta\bar{N}_\beta.$$

## 2.2 A Full Dixit-Stiglitz-Krugman Model

I assume next that each city's final goods sector is made up of many competitive firms which combine the two composite intermediates to produce a final good according a Cobb-Douglas production function. That is, output of the final good per unit of time is:

$$Y = \left[\left(\sum_{n=1}^{n_\alpha}\sum_{m=1}^{m_\alpha} z_{\alpha_{nm}}^{1-\frac{1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}\right]^{\phi}\left[\left(\sum_{n=1}^{n_\beta}\sum_{m=1}^{m_\beta} z_{\beta_{nm}}^{1-\frac{1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}\right]^{1-\phi},\ 0<\phi<1,\ 1<\sigma, \tag{2}$$

where $z_{\alpha_{nm}}$ and $z_{\beta_{nm}}$ denote the demand by a firm in a city that produces the $\alpha-$composite and $\beta-$composite good, respectively, for an $\alpha-$variety from the range $m = 1, \dots, m_\alpha$, produced in city $n = 1, \dots, n_\alpha$, and for an $\beta-$variety from the range $m = 1, \dots, m_\beta$, produced in city $n = 1, \dots, n_\beta$. Since the marginal product of any variety tends to infinity as its quantity tends to zero, all available varieties in the economy will be used, $m_\alpha n_\alpha$ and $m_\beta n_\beta$, respectively. Using symmetry for each city that produces $m_\alpha$ $\alpha-$varieties, and respectively $m_\beta$ $\beta-$varieties, relative to all other cities in (2), yields the simplified expression:

$$Y_\alpha = \left[ \left( m_\alpha z_{\alpha,\alpha}^{1-\frac{1}{\sigma}} + m_\alpha (n_\alpha - 1) z_{\alpha,-\alpha}^{1-\frac{1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \right]^\phi (n_\beta m_\beta)^{(1-\phi)\frac{\sigma}{\sigma-1}} \left[ z_{\beta,\alpha} \right]^{1-\phi}, \tag{3}$$

$$Y_\beta = (n_\alpha m_\alpha)^{\phi \frac{\sigma}{\sigma-1}} \left[ z_{\alpha,\beta} \right]^\phi \left[ \left( m_\beta z_{\beta,\beta}^{1-\frac{1}{\sigma}} + (n_\beta - 1) m_\beta z_{\beta,-\beta}^{1-\frac{1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \right]^{1-\phi}, \tag{4}$$

where $z_{\alpha,\alpha}, z_{\beta,\alpha}$ are the quantities of an intermediate variety $\alpha, \beta$ demanded by, respectively, the $\alpha$ composite industry in a city of type $\alpha$, and correspondingly $z_{\alpha,\beta}, z_{\beta,\beta}$ in a city of type $\beta$, and $z_{\alpha,-\alpha}, z_{\beta,-\beta}$, the quantities of an intermediate variety $\alpha, \beta$, respectively, demanded by an $\alpha, \beta$ city from another city of its own type, of which there exist $n_\alpha - 1, n_\beta - 1$. Separate accounting for imported varieties is necessary in order to be able to account for shipping costs.

Firms in the differentiated goods industry $j$, $j = \alpha, \beta$, require workers with specific skills. To produce any $\alpha-$variety, the firm that owns the technology incurs a fixed cost $\kappa_\alpha$ and requires ${\varpi_\alpha}^{-1}$ units of skilled labor, all per unit of time. That is, to produce $z_\alpha$ units of an $\alpha$ variety, a firm demands an amount of labor given by

$$h_\alpha = \kappa_\alpha + \varpi_\alpha^{-1} z_\alpha, \tag{5}$$

its employment, and similarly for $\beta-$varieties.[7] The larger is $\varpi_\alpha$, the less labor the production of each variety requires, and the more productive the activity is. Let $W_\alpha$ denote the nominal wage rate earned by workers employed by $\alpha-$variety producing firms. All such firms in a given city set the same monopolistic price, at the symmetric equilibrium, for each of their varieties of $\alpha-$ products, $p_\alpha$. I develop the wage setting model below after I complete the description of frictions in the labor market.[8] The technology requires that both $\alpha-$ and $\beta-$ industry goods be available in the economy, though they do not need to operate necessarily in each city, since both types of varieties are tradeable.

Proposition 1. *The first-order conditions for the demand of $\alpha-$varieties by the typical firm in a city specializing in $\alpha-$varieties yield the following:*

$$z_{\alpha,\alpha} = \frac{\phi}{\tilde{n}_\alpha} z_\alpha, \quad z_{\alpha,\beta} = \tfrac{(1-\phi)\tau}{n_\beta} z_\alpha; \tag{6}$$

$$z_{\beta,\alpha} = \frac{\phi\tau}{n_\alpha} z_\beta, \quad z_{\beta,\beta} = \tfrac{(1-\phi)}{\tilde{n}_\beta} z_\beta. \tag{7}$$

Proof. The typical firm faces a price $P_\alpha$ for its output $Y_\alpha$ of the final consumption good. The price will be expressed in terms of the ideal price index, which is introduced in section 4.2.1 below. Optimizing[9] with respect to each input $z_{\alpha,\alpha}$ and $z_{\alpha,-\alpha}$, respectively, gives the usual result that the demand for each variety is isoelastic in the price:

$$\phi \frac{P_\alpha Y_\alpha}{\left[ m_\alpha z_{\alpha,\alpha}^{1-\frac{1}{\sigma}} + m_\alpha (n_\alpha - 1) z_{\alpha,-\alpha}^{1-\frac{1}{\sigma}} \right]} z_{\alpha,\alpha}^{-\frac{1}{\sigma}} = p_{\alpha,\alpha}. \tag{8}$$

Correspondingly, for each imported intermediate variety we have:

$$\phi \frac{P_\alpha Y_\alpha}{\left[ m_\alpha z_{\alpha,\alpha}^{1-\frac{1}{\sigma}} + (n_\alpha - 1) m_\alpha z_{\alpha,-\alpha}^{1-\frac{1}{\sigma}} \right]} z_{\alpha,-\alpha}^{-\frac{1}{\sigma}} = p_{\alpha,-\alpha}. \tag{9}$$

Due to iceberg shipping costs, whereby of a unit of a variety shipped only fraction $\tau$ survives, $0 < \tau < 1$, the effective price of an imported good is greater than that of a domestically produced one by a factor $\frac{1}{\tau}$, $p_{\alpha,-\alpha,t} = p_{\alpha,\alpha t} \frac{1}{\tau}$. From (8) and (9) we have that:

$$z_{\alpha,-\alpha} = \tau^\sigma z_{\alpha,\alpha}. \tag{10}$$

Similarly, the first order conditions for the demands for $\beta-$varieties by a city specializing in $\alpha-$varieties are given by:

$$(1-\phi) \frac{P_\alpha Y_\alpha}{\left[ n_\beta m_\beta z_{\beta,\alpha}^{1-\frac{1}{\sigma}} \right]} z_{\beta,\alpha}^{-\frac{1}{\sigma}} = p_\beta \frac{1}{\tau}, \tag{11}$$

where I use the assumption of iceberg costs to write $p_{\beta t} \frac{1}{\tau}$ for the price of $\beta-$varieties imported by an $\alpha-$ type city. The counterparts of equations (8), (9) and (11) for the cities specializing in $\beta$ varieties may be obtained in like manner. Therefore,

$$z_{\beta,-\beta} = \tau^\sigma z_{\beta,\beta}. \tag{12}$$

These necessary conditions yield:

$$\phi P_\alpha Y_\alpha = p_\alpha \tilde{n}_\alpha m_\alpha z_{\alpha,\alpha};\ (1-\phi) P_\alpha Y_\alpha = p_\beta \frac{1}{\tau} n_\beta m_\beta z_{\beta,\alpha}; \tag{13}$$

$$\phi P_\beta Y_\beta = p_\alpha \frac{1}{\tau} n_\alpha m_\alpha z_{\alpha,\beta};\ (1-\phi) P_\beta Y_\beta = p_\beta \tilde{n}_\beta m_\beta z_{\beta,\beta}, \tag{14}$$

where the auxiliary variables $\tilde{n}_j$, defined as

$$\tilde{n}_j = 1 + (n_j - 1)\tau^{\sigma-1},\ j = \alpha, \beta, \tag{15}$$

denote the number of cities where $j-$varieties are produced, adjusted to account for intercity shipping costs. The demands for intermediates can be solved for explicitly, after trade balance is introduced, that is spending by all $\beta-$ cities on $\alpha-$ varieties should be equal to spending by all $\alpha-$cities on $\beta-$varieties. If all varieties used are produced domestically, then

$$n_\beta \phi P_\beta Y_\beta = n_\alpha (1-\phi) P_\alpha Y_\alpha.$$

This and the first-order conditions (13–14) yield:

$$n_\beta \tau^{-1} p_\alpha n_\alpha m_\alpha z_{\alpha,\beta} = n_\alpha \tau^{-1} p_\beta n_\beta m_\beta z_{\beta,\alpha}, \tag{16}$$

which together with conditions for equilibrium in the market for each variety,

$$z_{\alpha,\alpha}\tilde{n}_\alpha + n_\beta \tau^{-1} z_{\alpha,\beta} = z_\alpha;\; n_\alpha \tau^{-1} z_{\beta,\alpha} + \tilde{n}_\beta z_{\beta,\beta} = z_\beta, \tag{17}$$

yield (6) above. Q.E.D.

## 2.3 Production of Varieties and Employment

Hiring by each of the firms producing intermediate varieties is subject to frictions. Following the standard DMP approach, for a firm producing an $j-$variety, let $h_{j,t}$ denote employment, $q(\theta_{j,t})$ the rate at which employment prospects arrive per vacancy, $\theta_{j,t}$ the ratio of vacancies to unemployment for each firm, $V_{j,t}$ the stock of vacancies posted by an $j-$firm, and $\delta_j$ the rate at which jobs break up.[10] I follow Ziesimer's (2003) extension of the Pissarides model to production with a range of intermediates, and thus have for expected employment:

$$\dot{h}_{j,t} = q(\theta_{j,t}) V_{j,t} - \delta_j h_{j,t}. \tag{18}$$

This can be expressed alternatively in terms of the quantity of each variety, $z_{j,t}$,

$$\dot{z}_{j,t} = \varpi_j q(\theta_{j,t}) V_{j,t} - \delta_j \left[\varpi_j \kappa_j + z_{j,t}\right]. \tag{19}$$

<u>Proposition 2.</u> *Each intermediate producing firm sets production $z_{j,t}$, price $p_{j,t}$, and vacancies $V_{j,t}$, so as to maximize expected discounted profit:*

$$\int_0^\infty e^{-\rho t} \left[p_{j,t} z_{j,t} - W_{j,t}\left[\kappa_j + {\varpi_j}^{-1} z_{j,t}\right] - p_{j,t}\gamma V_{j,t}\right] dt, \tag{20}$$

*where $\gamma$ denotes the cost per unit of time for each vacancy, denominated in units of the respective variety.*

Part A. *The first-order condition of the typical $z_\alpha-$ producing firm yields the the pricing equation:*

$$p_{j,t} = \frac{\sigma}{\sigma-1}\left[{\varpi_j}^{-1} W_{j,t} + (\delta_j+\rho)\frac{\gamma \varpi_j^{-1}}{q(\theta_{j,t})} p_{j,t}\right]. \tag{21}$$

Part B. *At the steady state, the equilibrium output and employment respectively are given by:*

$$z_j = \frac{(\sigma-1)\varpi_j - \frac{\rho\gamma\sigma}{q(\theta_j)}}{1+\frac{\rho\gamma\sigma}{q(\theta_j)}\varpi_j^{-1}}\kappa_j; \tag{22}$$

$$h_j = \frac{\sigma\kappa_j}{1+\frac{\rho\gamma\sigma}{q(\theta_j)}\varpi_j^{-1}}. \tag{23}$$

*The corresponding level of vacancies at the steady state equilibrium is* $V_j = \frac{\delta_j}{q(\theta_j)}\left[\varpi_j{}^{-1}z_j + \kappa_j\right]$, *which in view of (23) becomes:*

$$V_j = \frac{\delta_j}{q(\theta_j)}\frac{\sigma\kappa_j}{1+\frac{\rho\gamma\sigma}{q(\theta_j)}\varpi_j^{-1}}. \tag{24}$$

Proof. Working in the standard fashion, if $\lambda$ denotes the Lagrange multiplier adjoining the evolution of output equation (19), the first order condition with respect to $V_{j,t}$,[11] by using the current value Hamiltonian yields:

$$\lambda = \frac{\gamma p_{j,t}\varpi_j^{-1}}{q(\theta_{j,t})}.$$

The first-order condition[12] with respect to $z_{j,t}$, yields, after assuming a steady state for $\lambda$ and using for it the above value[13],

Along the optimum path, free entry by potential $j-$variety producing firms ensures that profits per unit of time are driven to zero at *every* point in time:

$$p_{j,c}z_j - W_j\left(\varpi_j^{-1}z_j + \kappa_j\right) - p_{j,c}\gamma V_j = 0. \tag{25}$$

This condition sets the integrand in (20) equal to zero and thus implies that expected discounted profit from (20) is also zero. Using the pricing formula (21) together with (19) and (25) we obtain the equilibrium output (22), employment (23), and the corresponding level of vacancies (24) above.

Q.E.D.

Both expressions for output and the labor requirements for each variety, Eq. (22) and (23) above, are functions of $\theta_j$, labor market tightness for each variety, which in turn imply a similar relationship for $V_j$, (24), as a function of $\theta_j$. We see shortly that the rate at which employment prospects arrive per vacancy, $q(\theta_j)$, decreases with labor market tightness; thus equilibrium output[14] and employment[15] for each variety decreases in labor market tightness. Other things being equal, this would imply a lower unemployment rate. An increase in productivity increases output and employment for each of the varieties produced. Below we examine the effect of such a change on the number of varieties produced at equilibrium.

The pricing condition (21) is the counterpart here of the *job creation condition* in the canonical model of Pissarides (2000), p. 12, Eq. (1.9), except for the fact that the monopolistic competition model introduces a markup, $\frac{\sigma}{\sigma-1}$ over unit cost in the standard fashion. The price covers the cost of labor per unit of time which includes the wage costs, ${\varpi_j}^{-1} W_{j,t}$, plus the expected capitalized value of the future stream of hiring costs foregone. The latter involves costs per vacancy, $\gamma \varpi_j^{-1} p_{j,c,t}$, adjusted for the expected length of vacancy, $\frac{1}{q(\theta_{j,t})}$, and the probability of job destruction and time preference, $\delta_j + \rho$.[16] Note also that the pricing condition by anchoring labor demand and output supply of each of the intermediates on the wage rate. This obviates the need to express the decisions of the all firms producing intermediates in terms of a symmetric equilibrium.

From now on, time subscripts will be used only when they are necessary for clarity. Otherwise, they will be dropped, since much of the analysis is conducted along steady states.

# 3 Behavior of Individuals

Below I follow Wasmer and Zenou (2002), as adapted by Ioannides (2013), Ch. 5, and develop a DMP model of job matching in cities. I start by deriving expressions for expected lifetime utility, that is income, in a continuous-time model at steady state, under the assumption that individuals may lose their jobs when employed, and search for new jobs only when unemployed, both at constant probabilities per unit of time, and may borrow and lend at a constant rate of interest $\rho$. As in section 2, a benchmark mode is helpful.

## 3.1 Frictional Labor Markets in the Benchmark Model

Suppose next that at each site, individuals are matched with firms according to the standard DMP machinery. Following the classic formulation of Pissarides (1985; 2000), we have the Bellman equations for the conditional value functions $(\Omega_{je}, \Omega_{ju})$:

$$\rho \Omega_{ju} = b_j p_j + \pi_j [\Omega_{je} - \Omega_{j,u}],$$

$$\rho \Omega_{je} = W_j + \delta_j [\Omega_{ju} - \Omega_{je}].$$

By substracting the first equation from the second, we may solve for $\Omega_{je} - \Omega_{ju}$ :

$$\Omega_{je} - \Omega_{ju} = \frac{W_j - b_j p_j}{\rho + \pi_j + \delta_j}.$$

This is the quantity that enters wage bargaining, and therefore determination of labor market tightness, as we see shortly.

On the firms' side, let $(V_{j,u}, V_{j,e})$ denote, respectively, the value of an unfulfilled, filled job vacancy. If $\gamma$ is the real cost of holding an unfulfilled vacancy and $q(\theta_j)$ the rate at which individuals arrive at firms and vacancy are filled, then the Bellman equations are:

$$\rho V_{j,u} = -\gamma p_j + q(\theta_j)[V_{j,e} - V_{j,u}], \; \rho V_{j,e} = \varpi_j p_j + \delta_j [V_{j,u} - V_{j,e}].$$

Solving for $V_{j,u} - V_{j,e}$ and using the free entry condition $V_{j,u} = 0$, we have two alternative expressions for $V_{j,e}$ :

$$V_{j,e} = \frac{\gamma p_j}{q(\theta_j)} = \frac{\varpi_j p_j - W_j}{\rho + \delta_j}.$$

Worker and firm Nash bargaining over the wage rate $W_j$ in order to allocate joint surplus $\Omega_{je} - \Omega_{ju} + V_{j,e}$, so as to maximize $[\Omega_{je} - \Omega_{ju}]^{\vartheta} V_{j,e}^{1-\vartheta}$ yields the job creation condition

$$W_j = \varpi_j p_j - (\rho + \delta_j) \frac{\gamma p_j}{q(\theta_j)}.$$

The corresponding nominal wage rate satisfies the wage curve:

$$W_j = (1 - \vartheta) b_j p_j + \vartheta p_j [\varpi_j + \gamma \theta_j].$$

Solving the job creation condition and the wage curve as a simultaneous system of equations determine uniquely the real wage and labor market tightness. The rate at which individuals find jobs follows:

$$\pi_j = \pi(\theta_j) = \theta_j q(\theta_j).$$

Rewriting (1) in terms of expected employment in each city type, $\frac{\pi_j}{\pi_j + \delta_j} N_j$ yields:

$$\frac{p_\alpha}{p_\beta} = \frac{\phi}{1 - \phi} \frac{\varpi_\beta}{\varpi_\alpha} \frac{n_\beta \frac{\pi_\beta}{\pi_\beta + \delta_\beta} N_\beta}{n_\alpha \frac{\pi_\alpha}{\pi_\alpha + \delta_\alpha} N_\alpha}. \tag{26}$$

Given factor supplies and numbers of cities, the labor market tightness variables correspond naturally to factor intensities [ Davidson, Martin and Matusz (1999) ]. Since the employment rate, $\frac{\pi_j}{\pi_j + \delta_j}$, is increasing in $\theta_j$, the higher is the employment rate for $j$, the lower the relative price of input $j$.

This benchmark model determines employment rates in each city type and the respective terms of trade based on aggregate factor supplies, productivities and factor shares, but it does not determine the numbers of cities nor city sizes. The model that the remainder of the paper puts together determines the numbers and sizes of cities. This is made possible by the combined effect of two key assumptions: one, the love of variety feature of the Dixit–Stiglitz–Krugman production function (2); and two, by closing the matching model by assuming that job matching occurs either in each city's CBD, or via social referrals. The former is the counterpart here of the ad hoc assumptions about urban agglomeration effects made by Behrens and Robert-Nicoud (2015). The latter is motivated by the social nature of urban living. Even when traveling to the CBD is not necessary for job matching, the fact that urban living is spatially confined facilitates social interactions and thus social job referrals.

### 3.2 A Wasmer–Zenou DMP Model of Job Matching in Cities

Let $j$ index individuals by skill type, $j = \alpha, \beta$, as indicated by the industry where they qualify for employment. The derivation proceeds by first solving for an individual's expected lifetime income, conditional on being unemployed, and employed, $\Omega_{ju}(\ell), \Omega_{je}(\ell)$, respectively, as functions of distance from the CBD. Let the commuting costs per unit of time for, respectively, an employed, and an unemployed person, be linear functions of the distance from the CBD, $\bar{a}_{je}\ell, \bar{a}_{ju}\ell$. The rate of unemployment compensation (or home production) per unit of non-commuting time is denominated in terms of the good produced of the industry of employment $j = \alpha, \beta$, $b_j = bp_j$, where $p_j$ is the price of good $j$. Jobs break up at constant rates $\delta_j$ in industry $j$, and the rate at which unemployed workers find jobs in industry $j$ is denoted by $\pi_j$. If unsubscripted, it denotes the number $\pi$. Finally, $R(\ell)$ is the land rental rate at location $\ell$, and, $\bar{R}$ : total land rentals per capita in a city, both per unit of time. The city is circular around its CBD.

The expected values of unemployment and employment $\Omega_{ju}, \Omega_{je}$ satisfy the following Bellman equations:

$$\rho\Omega_{ju}(\ell) = b_j p_j (1 - \bar{a}_{ju}\ell) + \pi_j(\ell)[\Omega_{je}(\ell) - \Omega_{j,u}(\ell)] + \bar{R} - R(\ell), \tag{27}$$

$$\rho\Omega_{je}(\ell) = W_j(1 - \bar{a}_{je}\ell) + \delta_j[\Omega_{ju}(\ell) - \Omega_{je}(\ell)] + \bar{R} - R(\ell). \tag{28}$$

The steady state rate of unemployment for an individual who is employed in industry $j$ must be such that flows into unemployment equal flows out of unemployment. That is: $\delta_j(1 - u_j) = u_j\pi_j$, which by solving for $u_i$ yields:

$$u_j = \frac{\delta_j}{\delta_j + \pi_j}. \tag{29}$$

The steady state unemployment rate decreases as the job finding rate $\pi_j$ increases.

<u>Proposition 3.</u> *The expected present value of lifetime income of an individual of type $j$ at location $\ell$ along a steady state of the search process is given by:*

$$\rho\omega_\alpha(\ell) = \bar{R} - R(\ell) + \bar{\mathcal{D}}_\alpha - \mathcal{D}_\alpha\ell,\ \rho\omega_\beta(\ell) = \bar{R} - R(\ell) + \bar{\mathcal{D}}_\beta - \mathcal{D}_\beta\ell \tag{30}$$

where the auxiliary functions $(\bar{\mathcal{D}}_\alpha, \bar{\mathcal{D}}_\beta; \mathcal{D}_\alpha, \mathcal{D}_\beta)$ are defined as follows, $j = \alpha, \beta$ :

$$\bar{\mathcal{D}}_j \equiv \frac{\delta_j}{\delta_j + \pi_j} b_j p_j + \frac{\pi_j}{\delta_j + \pi_j} W_j. \tag{31}$$

$$\mathcal{D}_j \equiv \frac{\delta_j}{\delta_j + \pi_j} \bar{a}_{ju} b_j p_j + \frac{\pi_j}{\delta_j + \pi_j} W_j \bar{a}_{je}. \tag{32}$$

Proof. This follows trivially from (27–29) and the definition of the expected value of the process along the steady state:

$$\omega_j(\ell) = (1 - u_j)\Omega_e + u_j \Omega_{iu}.$$

Q.E.D.

### 3.3 The Matching Model

Unlike the benchmark model above, I embed job matching in the urban model by following Wasmer and Zenou (2002). I specify the employment and unemployment probability in the classic DMP fashion. Let $U_j$ denote the stock of all unemployed type $j-$workers associated with a particular $j-$firm and $V_j$ the stock of vacancies for jobs for which $j-$types qualify. The rate of contacts between unemployed workers and vacancies per unit of time is specified via a *matching function* [Pissarides (1985; 2000)], as a function of $(U_j, V_j)$ :

$$M(U_j, V_j). \tag{33}$$

Following Pissarides, I assume that the matching function of vacancies with unemployed workers, $M(U_j, V_j)$, exhibits constant returns to scale with respect to both of its arguments, the effective stock of unemployed and the stock of vacancies, respectively.[17] The probability that an individual of type $j$ will have a contact during a small interval of time $(t, t + dt)$ is given by $\pi_j dt$, where $\pi_j$ is given by

$$\pi_j = \frac{M(U_j, V_j)}{U_j} dt = M\left(1, \theta_j\right). \tag{34}$$

This is rewritten, by using the linear homogeneity property of the matching function, as a function of the tightness of a city's labor market for $j-$types, $\theta_j$, $\theta_j = \frac{V_j}{U_j}$:

$$\pi_j = \theta_j q(\theta_j), \tag{35}$$

where $q(\theta_j) \equiv M\left(\frac{1}{\theta_j}, 1\right)$, $q' = -M_1 \frac{1}{\theta j^2} < 0$, denotes the rate at which unemployed workers arrive at each searching vacancy (firm). Therefore, the greater is $\theta_j$, the tighter the labor market is, and the lower the probability of contacts, $q$, for each firm. It follows from (35) by differentiation that an individual's job contact probability, on the other hand, is increasing and concave in labor market tightness.

Anticipating the discussion below, each industry is analyzed at a symmetric equilibrium among all intermediate varieties-producing firms of the same type, at which labor market tightness, and the respective stocks of unemployed individuals and vacant jobs are equalized. We will rely on the constant returns to scale assumption for the matching function and aggregate matching up to the level of the industry.

# 4 The Urban Structure

The urban structure in this paper is assumed to consist of many homogeneous cities of each type, $\alpha$ and $\beta$, that is, each city is populated by individuals of the same skill.[18] Locational equilibrium within each city of either type requires that otherwise identical individuals be indifferent as to where they locate. I thus impose that expected lifetime utility be equalized across all locations within each city. Since all individuals employ the same exogenous search intensity, the job matching probability is independent of specific location within the city. For spatial equilibrium, the land rental rate must vary with location so as to equalize expected lifetime income across all locations. Thus, the bid rental rate associated with a particular individual type $R(\ell)$ reflects the value of accessibility to the CBD.

## 4.1 Intracity Spatial Equilibrium

Working from (30), for either city type, we have that for $\omega_j(\ell)$ to be constant across all locations and under the assumptions that the opportunity cost of land at the city's edge $\bar{\ell}$, is equal to 0 and that employment probabilities are independent of $\ell$, then $R_\beta(\bar{\ell}) = 0$. Spatial equilibrium within a $j-$populated city implies a land rental function

$$R_j(\ell) = \mathcal{D}_j(\bar{\ell} - \ell),\ j = \alpha, \beta, \tag{36}$$

which is linear in $\ell$, and $\mathcal{D}_j$ is defined in (32) above.

Under the assumption that employment probabilities are independent of location $\ell$, the land rental functions from (36) may be readily integrated. Then, by expressing the equilibrium lifetime utilities net of the redistribution of total land rents, we have the following expressions, which are of course independent of $\ell$ :[19]

$$\omega_j^* = \bar{\mathcal{D}}_j - \frac{2}{3}\mathcal{D}_j\bar{\ell}_j,\ j = \alpha, \beta, \tag{37}$$

where $\bar{\mathcal{D}}_j$ is defined in (31) above. Since land is consumed at the unit level only and the city is circular, it follows that

$$\bar{\ell}_j = \left(\frac{N_j}{\pi}\right)^{\frac{1}{2}}, \ j = \alpha, \beta. \tag{38}$$

Thus, equilibrium utilities are written in terms of populations of different skill types. They decrease with city size, *cet. par.,* an effect due to congestion. However, as I show further below, the properties of matching in cities confers an advantage that maybe traded off against congestion.

## 4.2 Wage Setting and Labor Market Equilibrium

Following the Mortensen and Pissarides assumption that the typical intermediates-producing firm opens vacancies as long as their expected value is zero yields an expression for the value of a filled job: $\frac{\gamma p_j}{q(\theta_j)}$. From the pricing condition (21), the value of a filled job may be expressed as: $\frac{1}{\delta_j+\rho}\left[\frac{\sigma-1}{\sigma}\varpi_j p_j - W_j\right]$. It enters the objective function for the Nash bargaining wage setting problem, Eq. (39) below. That is, the typical intermediates-producing firm in an $j-$producing city and the typical worker jointly choose $W_j$, the nominal wage rate, so as to maximize[20]:

$$\left[\bar{\mathcal{D}}_j - \frac{2}{3}\mathcal{D}_j\bar{\ell}_j\right]^{\vartheta}\left[\frac{\sigma-1}{\sigma}\varpi_j p_{j,t} - W_{j,t}\right]^{1-\vartheta}, \tag{39}$$

where $\vartheta,\ 0 < \vartheta < 1$, is a parameter indicating the relative bargaining of an individual, with the firm's being denoted by $1-\vartheta$. This formulation presumes symmetry across all intermediates-producing firms. Stated formally the solution is given by Proposition 4. The proof is immediate.

Proposition 4. *The counterpart here of the* wage curve *in the DMP model, is given by the solution for the nominal wage rate to the bargaining problem of maximizing (39):*

$$W_j = \vartheta\frac{\sigma-1}{\sigma}\varpi_j p_j - (1-\vartheta)\frac{\delta_j}{\pi_j}\frac{1-\frac{2}{3}N_j^{\frac{1}{2}}\bar{a}_{j,u}}{1-\frac{2}{3}N_j^{\frac{1}{2}}\bar{a}_{j,e}}bp_j, \tag{40}$$

*where where* $a_{j,e} \equiv \bar{a}_{j,e}\pi^{-\frac{1}{2}}$, and $a_{j,u} \equiv \bar{a}_{j,u}\pi^{-\frac{1}{2}}$.

Note that the nominal wage rate depends explicitly on city size provided that the commuting costs depend on the employment state, $a_{j,u} \neq a_{j,e}$. The wage curve plays the role of labor supply: other things being equal, the wage rate increases in the respective labor market tightness. Here, this is effect is present because the employment rate is an increasing function of labor market tightness. An additional effect will be revealed when the equilibrium prices $(p_\alpha, p_\beta)$ are shown to be functions of labor market tightness in each type of city.

The associated expression for expected flow of nominal income per period in a type-$j$ city is:

$$\rho\, \omega_{j,\text{equ}} = \vartheta \frac{\pi_j}{\pi_j + \delta_j} \left[1 - \frac{2}{3} N_j^{\frac{1}{2}} a_{j,e}\right] \frac{\sigma - 1}{\sigma} \varpi_j p_j + \vartheta \frac{\delta_j}{\pi_j + \delta_j} \left[1 - \frac{2}{3} N_j^{\frac{1}{2}} a_{j,u}\right] b p_j, \; j = \alpha, \beta. \quad (41)$$

Real income follows by dividing expected nominal income by the respective ideal price index that is developed below.

### 4.2.1 City-specific Price Index

The ideal price index[21] accounts for the effective range of intermediate varieties used. In particular, a specialized city of type $\alpha$, imports all $\beta$ varieties, and there are $m_\beta n_\beta$ of them, incurring shipping costs, and correspondingly for cities of type $\beta$, that import all $\alpha$ varieties, of which there are $m_\alpha n_\alpha$. Therefore, the price indices (inclusive of shipping costs) are, respectively:

$$P_\alpha^* = p_\alpha^\phi p_\beta^{1-\phi} \left[m_\alpha \tilde{n}_\alpha\right]^{\frac{\phi}{1-\sigma}} \left[m_\beta n_\beta \tau^{\sigma-1}\right]^{\frac{1-\phi}{1-\sigma}}, P_\beta^* = p_\alpha^\phi p_\beta^{1-\phi} \left[m_\alpha n_\alpha \tau^{\sigma-1}\right]^{\frac{\phi}{1-\sigma}} \left[m_\beta \tilde{n}_\beta\right]^{\frac{1-\phi}{1-\sigma}}, \quad (42)$$

where the auxiliary variables $\tilde{n}_j,\; j = \alpha, \beta$, defined in (15) above, denote the number of cities adjusted for shipping costs. It is worth noting that the ideal price indices depend on the labor frictions aspects of the model.

### 4.2.2 Labor Market Equilibrium

Labor market equilibrium in our model is entirely defined in terms of labor market tightness. This is formally stated in Proposition 5.

Proposition 5. *The labor market tightness $\theta_j$ for the $j-$ variety producing firms satisfies:*

$$\frac{\gamma}{q(\theta_j)} = \frac{1 - \vartheta}{\delta_j + \rho} \left[\frac{\sigma - 1}{\sigma} \varpi_j + \frac{\delta_j}{\pi_j} \frac{1 - \frac{2}{3} N_j^{\frac{1}{2}} a_{j,u}}{1 - \frac{2}{3} N_j^{\frac{1}{2}} a_{j,e}} b_j\right], \; j = \alpha, \beta. \quad (43)$$

*Its properties are summarized by:*

$$\theta_j = \Theta(\underset{(+)}{N_j}\; ; \underset{(+)}{\varpi_j}\; , \underset{(+)}{b_j}\; , \underset{(?)}{\delta_j}). \quad (44)$$

Proof. The job creation condition (21) may be rewritten as $\frac{\gamma}{q(\theta_j)} = \frac{1}{\delta_j + \rho}\left[\frac{\sigma-1}{\sigma}\varpi_j - \frac{W_j}{p_j}\right]$. It yields a downwards sloping curve in $(\theta_j, W_j)$ space. Using the solution for $\frac{W_j}{p_j}$ from the bargaining outcome, (40), yields (43).

The left hand side is increasing in $\theta_j$ and the right hand side is decreasing in $\theta_j$ via its effect on $\pi_j$. Thus, (43) uniquely determines, in terms of exogenous variables and parameters, labor market tightness in a specialized city. Higher productivity $\varpi_j$ increases equilibrium labor market tightness, which in turn implies higher employment rate and lower unemployment rate. The right hand side increases (decreases) with $N_j^{\frac{1}{2}}$, provided that $a_{j,e} > \ (<) \ a_{j,u}$. It is reasonable to assume that $a_{j,e} > a_{j,u}$ — the unemployed do not have to commute to the CBD as frequently as the employed. Thus, the properties of the implicit solution of (43) for $\theta_j$ are summarized in (44).

Q.E.D.

A larger city size is associated with greater labor market tightness and therefore higher employment rate and lower unemployment rate, *cet. par.* The effect of the job destruction rate is positive (negative) if $\frac{\rho}{\pi_\alpha} \frac{1-\frac{2}{3}N_\alpha^{\frac{1}{2}} a_{\alpha,u}}{1-\frac{2}{3}N_\alpha^{\frac{1}{2}} a_{\alpha,e}} b_\alpha > \ (<) \ \frac{\sigma-1}{\sigma}\varpi_\alpha$. Unemployment is independent of city size only if the loss of time due to commuting is independent of employment status: $\bar{a}_{j,e} = a_{j,u} = a, j = \alpha, \beta$. In that case, employed and unemployed would differ only with respect to the rate of pay, wage rate versus unemployment compensation. Also, this margin provides the tradeoff for endogenizing search intensity. Greater intensity improves the likelihood of employment, but requires more frequent visits to the CBD. See Zenou (2009a, b).

In sum, under the reasonable assumption that commuting is more costly to those employed, the determination of labor market tightness according to (43) reflects that larger city size are associated with higher labor market tightness, and thus higher employment rates and lower unemployment rates. Thus, higher commuting costs are offset by better employment prospects. The effects of other parameters on labor market equilibrium, as summarized by (44), also agree with intuition.

## 4.3 Beveridge Curve

The *Beveridge curve*, an empirical tool that the study of labor markets with frictions aimed at predicting [Pissarides (1986); Pissarides (2000), p. 32], has been adopted by the U.S. Bureau of Labor Statistics a standard device in tracking labor market conditions.[22] Plotting the vacancy rate against the unemployment rate allows one to compare the impact of the business cycle on the labor market.

The Beveridge curve is an "accounting" relationship [see Diamond (2011); Mortensen (2011); Pissarides (2011)] in the sense it expresses "combinations of vacancies and unemployment that are consistent with equality between the entry into unemployment and the exit from it" [Pissarides (2011), p. 1095)]. I apply this concept to each intermediate-variety producing firm by defining the vacancy rate as the ratio of vacancies to vacancies plus employment. Using (23) and (24) yields:

$$\upsilon_\alpha = \frac{V_\alpha}{V_\alpha + h_\alpha} = \frac{\delta_\alpha}{\delta_\alpha + q(\theta_\alpha)}. \tag{45}$$

Thus, the vacancy rate increases in $\theta_\alpha$, labor market tightness and the steady state unemployment rate decreases. Therefore, changes in $\theta_\alpha$ trace the movement along a curve in $(u, \upsilon)$ space, a Beveridge curve for each intermediate variety producing firm, a downward sloping curve in unemployment-vacancy space.

Applying the above definition at the city level and for each city type at the symmetric equilibrium, we have:

$$\upsilon_{\alpha,N_\alpha} = \frac{m_\alpha V_\alpha}{m_\alpha V_\alpha + m_\alpha h_\alpha} = \frac{\delta_\alpha}{\delta_\alpha + q(\theta_\alpha)}. \tag{46}$$

Thus, city-level Beveridge curves coincide with the respective firm-specific ones in homogeneous cities: $\upsilon_{\alpha,N_\alpha} = \upsilon_\alpha$. We return below to the definition of aggregate, economy-wide Beveridge curves. Once we have defined aggregate equilibrium, we confirm this definition at equilibrium.

The impact of changes in productivity $\varpi_\alpha$ on $\theta_\alpha$ depends from (43) on city size. Larger cities are associated with Beveridge curves closer to the origin. With constant rates of job destruction, it is evident from the above definition that the properties of the Beveridge curve reflect the matching function. The Beveridge curve has become particularly useful in tracing the evolution of the business cycle. See, in particular, Diamond and Ṣahin (2014). It follows directly from the definitions that, for any given level of labor tightness, the more efficient is matching of job openings with workers, for any given level of labor tightness, the nearer is the Beveridge curve to the axes. Where we are on the Beveridge curve for an $\alpha-$variety producing firm depends on the value of labor market tightness. I note that the above analysis treats job destruction as independent of productivity. Allowing for job destruction to depend on shocks to productivity $\varpi_\alpha, \varpi_\beta$ requires suitably adapting the firms' search model [see Mortensen and Pissarides (1999)].

## 4.4 Aggregate Equilibrium in an Economy with Many Cities of both Types

With the number of cities hosting skilled labor of type $\alpha$ ($\beta$) given by $n_\alpha$ ($n_\beta$), and their respective populations being $N_\alpha$ ($N_\beta$), for national labor market equilibrium we have,

$$n_\alpha N_\alpha = \bar{N}_\alpha, \;\; n_\beta N_\beta = \bar{N}_\beta, \tag{47}$$

where $\bar{N}_\alpha$ and $\bar{N}_\beta$ denote the given national population of labor of the respective type.

Aggregate equilibrium involves the labor labor market tightness variables, which follow from Proposition 5, and are obtained implicitly as the solutions to (43) for each city type, $(\theta_\alpha, \theta_\beta)$. They are functions of $(N_\alpha, N_\beta)$, the respective city sizes only. The corresponding employment rates $(\pi_\alpha, \pi_\beta)$ readily follow and also depend on $(N_\alpha, N_\beta)$. The respective equilibrium quantities of each intermediates follow from Proposition 2, Part B, and given by (22). In the remainder of this section, I discuss the determination of ranges of intermediates $(m_\alpha, m_\beta)$, the terms of trade, $\frac{p_\alpha}{p_\beta}$, and real income for each individual skill type at equilibrium.

In an $j-$ type city, the demand for labor for the production of all $\alpha$ varieties, $m_\alpha h_\alpha$, is equal to expected labor supply, $(1-u_\alpha)\bar{H}_{e,\alpha}$, where $\bar{H}_{e,\alpha}$ is net labor available for production, which accounts for time spent on commuting when employed. That is $\int_0^{(N_j/\pi)^{\frac{1}{2}}}[1-\bar{a}_e\ell]2\pi\ell d\ell$:

$$\bar{H}_j(N_j) = N_j\left(1 - \frac{2}{3}a_{j,e}N_j^{\frac{1}{2}},\right),\ j=\alpha,\beta, \tag{48}$$

where $a_{j,e} \equiv \bar{a}_{j,e}\pi^{-\frac{1}{2}}$ was defined in Proposition 4.

The ranges of varieties $(m_\alpha, m_\beta)$, of varieties used at equilibrium are determined by imposing equilibrium in the labor market in each city type. The demand for labor for the production of all varieties in a city of type $\alpha$, respectively $\beta$, is simply equal to expected employment:

$$m_\alpha = \frac{1}{h_\alpha(\theta_\alpha)}\frac{\pi_\alpha}{\pi_\alpha+\delta_\alpha}\bar{H}_\alpha(N_\alpha),\quad m_\beta = \frac{1}{h_\beta(\theta_\beta)}\frac{\pi_\beta}{\pi_\beta+\delta_\beta}\bar{H}_\beta(N_\beta), \tag{49}$$

where each city's labor supply, $\bar{H}_{je},\ j=\alpha,\beta$, is given by (48) above, and functions $h(\theta_j)$ are given by Proposition 2, Part B above. Conditions (49) determine the range of varieties as function of labor market tightness and net labor in the respective city type: $m_\alpha = m_\alpha(\theta_\alpha, \bar{H}_{e,\alpha}),\ m_\beta = m_\beta(\theta_\beta, \bar{H}_{e,\beta})$, given all parameters describing the productivity characteristics of the technologies producing the differentiated varieties. Since employment for each variety decreases in labor market tightness and employment rate increases, it follows from (49) that the range of varieties increases with labor market tightness:

$$\frac{\partial m_j(\theta_j,\bar{H}_j)}{\partial\theta_j} > 0;\quad \frac{\partial m_j(\theta_j,\bar{H}_j)}{\partial\bar{H}_j} > 0,\ j=\alpha,\beta. \tag{50}$$

<u>Proposition 6.</u> *Given city sizes, $N_j, j=\alpha,\beta$, the aggregate equilibrium is defined as follows.*

*Part A. The ranges of intermediates are given by:*

$$m_j = \frac{1}{z_j(\theta_j)}\left[\frac{\delta_j+\rho\vartheta}{1-\vartheta}\frac{\gamma}{\frac{\delta_j}{\theta_j}+q(\theta_j)} - \frac{\delta_j}{\pi_j(\theta_j)+\delta_j}\frac{1-\frac{2}{3}N_j^{\frac{1}{2}}\bar{a}_{ju}}{1-\frac{2}{3}N_j^{\frac{1}{2}}a_{j,e}}b_j\right]\bar{H}_j,\ j=\alpha,\beta. \tag{51}$$

*provided that the r.h.s. be positive.*

*Part B. The equilibrium terms of trade for the tradeable intermediate varieties satisfy trade balance. By using (47) and (48) to rewrite (26), we have:*

$$\frac{p_\alpha}{p_\beta} = \frac{\phi}{1-\phi}\frac{\bar{N}_\beta}{\bar{N}_\alpha}\frac{\left[\frac{\delta_\beta+\rho\vartheta}{1-\vartheta}\frac{\gamma\theta_\beta}{\pi_\beta+\delta_\beta}\left(1-\frac{2}{3}N_\beta^{\frac{1}{2}}a_{\beta,e}b_\beta\right)-\frac{\delta_\beta}{\pi_\beta+\delta_\beta}\left(1-\frac{2}{3}N_\beta^{\frac{1}{2}}a_{\beta,u}\right)\right]}{\left[\frac{\delta_\alpha+\rho\vartheta}{1-\vartheta}\frac{\gamma\theta_\alpha}{\pi_\alpha+\delta_\alpha}\left(1-\frac{2}{3}N_\alpha^{\frac{1}{2}}a_{\alpha,e}b_\alpha\right)-\frac{\delta_\alpha}{\pi_\alpha+\delta_\alpha}\left(1-\frac{2}{3}N_\alpha^{\frac{1}{2}}a_{\alpha,u}\right)\right]}. \tag{52}$$

*Part C. Equilibrium real income per unit of time along the steady state in each city type is given by (41) after dividing by the respective ideal price index (4.2.1). We have, for $\alpha-$ type cities:*

$$\frac{\rho\ \omega_{\alpha,\text{equ}}}{P_\alpha^*} = \left(\frac{p_\alpha}{p_\beta}\right)^{1-\phi}[m_\alpha\tilde{n}_\alpha]^{\frac{\phi}{\sigma-1}}\left[m_\beta n_\beta\tau^{\sigma-1}\right]^{\frac{1-\phi}{\sigma-1}}\vartheta$$

$$\left[\frac{\pi_\alpha}{\pi_\alpha+\delta_\alpha}\left[1-\frac{2}{3}N_\alpha^{\frac{1}{2}}a_{\alpha,e}\right]\frac{\sigma-1}{\sigma}\varpi_\alpha+\frac{\delta_\alpha}{\pi_\alpha+\delta_\alpha}\left[1-\frac{2}{3}N_\alpha^{\frac{1}{2}}a_{\alpha,u}\right]b_\alpha\right]; \tag{53}$$

*and for $\beta-$ type cities:*

$$\frac{\rho\ \omega_{\beta,\text{equ}}}{P_\beta^*} = \left(\frac{p_\beta}{p_\alpha}\right)^{\phi}\left[m_\alpha n_\alpha\tau^{\sigma-1}\right]^{\frac{\phi}{\sigma-1}}[m_\beta\tilde{n}_\beta]^{\frac{1-\phi}{\sigma-1}}\vartheta$$

$$\left[\frac{\pi_\beta}{\pi_\beta+\delta_\beta}\left[1-\frac{2}{3}N_\beta^{\frac{1}{2}}a_{\beta,e}\right]\frac{\sigma-1}{\sigma}\varpi_\beta+\frac{\delta_\beta}{\pi_\beta+\delta_\beta}\left[1-\frac{2}{3}N_\beta^{\frac{1}{2}}a_{\beta,u}\right]b_\beta\right]. \tag{54}$$

<u>Proof.</u>

Part A. By writing $m_\alpha z_\alpha = \frac{z_\alpha}{h_\alpha}m_\alpha h_\alpha$ we have after using the steady state solutions for intermediate output and employment, (22) and (23), the labor market equilibrium condition (49) and the job creation condition (43):

$$m_j z_j = \left[\frac{\sigma-1}{\sigma}\varpi_j\frac{\pi_j}{\pi_j+\delta_j}-\frac{\rho\gamma}{\frac{\delta_j}{\theta_j}+q(\theta_j)}\right]\bar{H}_j, \tag{55}$$

which after using (49) yields (51). A sufficient condition for the positivity of the r.h.s. of (51) is:

$$\theta_j > \frac{\delta_j(1-\vartheta)}{(\delta_j+\rho\vartheta)\gamma}, \tag{56}$$

which amounts to a lower bound on labor market tightness, given parameters $\delta_j$ and $\vartheta$.

Part B. With $m_\alpha = m_\alpha(\theta_\alpha,\bar{H}_\alpha)$, $m_\beta = m_\beta(\theta_\beta,\bar{H}_\beta)$ implicitly determined by (51), the equilibrium terms of trade, $\frac{p_\alpha}{p_\beta}$, from (13), (14) and (16) become:

$$\frac{p_\alpha}{p_\beta} = \frac{\phi}{1-\phi}\frac{n_\beta}{n_\alpha}\frac{m_\beta}{m_\alpha}\frac{z_\beta}{z_\alpha}. \tag{57}$$

By Walras' law, at the price ratio that satisfies this condition, and given the free entry (25) and the pricing (21) conditions, all markets clear. Presence of intercity shipping costs of the iceberg type does not affect this conclusion. By using (47), (48) and (51) in (57), we obtain (52).

Part C. By Walras' law, given city numbers and populations, $(n_\alpha, N_\alpha; n_\beta, N_\beta)$, the equilibrium terms of trade, defined by Eq. (52), together with equilibrium labor market tightness in each city type, given by (43), ensure that individuals' demand for output in each city must equal its supply. That is, individuals spend the flow of their permanent income, given by (41), on their city's output,[23]

$$P_j^* Y_j = \rho\, \omega_{j,\text{equ}} N_j,\ j = \alpha, \beta. \tag{58}$$

Real income at equilibrium is given by (41), divided by the ideal price index for $\alpha-$type cities, $P_\alpha^*$, adjusted to account for intercity shipping costs associated with importing all $\beta$ varieties, that is (42), and respectively for $\beta-$type cities. Therefore, given the equilibrium value of the terms of trade from (52), real nominal income is fully determined by the model. That is, the expression for nominal permanent income in (41) is homogeneous of degree one in $p_\alpha$, so that dividing it by the ideal price index yields an expression for real permanent income that is a function of the terms of trade $\left(\frac{p_\alpha}{p_\beta}\right)$, and, in addition, of the ranges of varieties and number of cities. This yields (53) and (54) above.

Q.E.D.

Regarding condition (56), the larger the weight of individuals in wage bargaining, the lower the labor market tightness can be, and therefore higher unemployment at the steady state.

For brevity, the exposition suppresses intermediate results that show that given number of cities and city sizes, a higher labor market tightness in $\alpha-$type cities reduces its own terms of trade while it increases employment and reduces unemployment in $\alpha-$type cities. This is due to the productivity enhancing role of the range of varieties. Even though the production of each variety decreases, when labor market tightness increases, the increase in the range and in the employment rate more than make up. An increase in the productivity of $\alpha-$varieties implies higher labor market tightness, which in turn induces increase in production which in turn leads to fall in their relative price. Note that the terms of trade equation above (52) is reminiscent of (26) of the benchmark model in section 3.1 above. The complications have to do with the presence of intermediates in the Dixit-Stiglitz-Krugman production model of section 2.2 above.

In sum, given the number of cities and city populations of each type, job creation conditions for either city type determine labor market tightness in each city, $\theta_\alpha, \theta_\beta$. Labor market tightness in turn determines employment at free entry equilibrium for each variety, and from the conditions for labor market equilibrium in each city, (49), the ranges of intermediates $m_\alpha, m_\beta$ are determined. The equilibrium terms of trade follow from (52), which establishes an one-to-one correspondence between labor market tightness in each city type and terms of trade. Here, labor market tightness variables act a bit like factor intensities in international trade models, which is reminiscent of Davidson *et al.* (1999).

What does the intercity trade equilibrium condition (52) tells us about the impact of city sizes? In view of (56), a larger city type improves its own terms of trade. That is, the total effect of a larger value of $N_\alpha$ (alternatively $N_\beta$) operates via the impact on equilibrium labor market tightness, from (43), and on congestion. Labor market tightness increases with city size (from (43)), which in turn increases the total input of respective intermediates (from (51)). Given fixed total population of each skill type, the number of the respective cities decreases, which reduces the variety of intermediates and yields a total effect of decreasing spending on them.

In sum, given $(N_\alpha, N_\beta)$, (47) determine the number of each city type, $(n_\alpha, n_\beta)$. Conditions (43) determine labor market tightness for each city type, $(\theta_\alpha, \theta_\beta)$, as functions of the respective city sizes. The respective employment rates follow from their definitions, $\pi_j = \theta_j q(\theta_j),\ j = \alpha, \beta$. The inputs of intermediates follow from (22), and the ranges of intermediates $(m_\alpha, m_\beta)$ follow from (51). The equilibrium is summarized by Proposition 6: The equilibrium terms of trade are given by (52), and real income for each individual skill type follows from 53–54).

## 4.5 Equilibrium City Size

Given the terms of trade, ranges of intermediates and numbers of cities of each type — with all relevant variables entering the first line of the expressions for real permanent income in (53–54) above — individuals of either skills seek to locate so as to maximize the permanent income. Conveniently, changes in city populations affect equilibrium real income in each city directly through city geography and through equilibrium labor market tightness, these being variables that enter the bracketed term in second lines of the expressions for real income in (53–54) above, respectively for $\alpha-$ and $\beta-$ type cities. If a particular city offers greater utility than other cities in each type within a system of cities, it would experience in-migration; if it offers less, it would experience out-migration. In this fashion, we may define equilibrium city size as the value of $N_j,\ j=\alpha,\beta$, for which the quantity

$$\frac{\pi_j\frac{\sigma-1}{\sigma}\varpi_j+\delta_j b_j}{\pi_j+\delta_j}-\frac{2}{3}N_j^{\frac{1}{2}}\left[\frac{\pi_j\frac{\sigma-1}{\sigma}\varpi_j a_{je}+\delta_j a_{ju}b_j}{\pi_j+\delta_j}\right], \tag{59}$$

is maximized for each city type, given the terms of trade, ranges of intermediates and numbers of cities of each type.

The first-order condition yields an explicit quadratic equation in $N_j^{\frac{1}{2}}$, with coefficients that depend on $N_j$ however. [24] Therefore, in general, a stable solution for the equilibrium city size for each city type exists, given labor market equilibrium in each city. Given city sizes, the conditions for labor market equilibrium (43) determine labor market tightness and employment in each city. The equilibrium number of cities thus follows. At the competitive equilibrium populations of agents with different skills distribute themselves across cities.

# 5 Comparative Dynamics

With the bulk of the analysis having been carried out at a steady state, it is interesting to perform comparative dynamics on output and individuals' welfare of changes in the productivity parameters, $(\varpi_j,\kappa_j), j=\alpha,\beta$, the matching mechanism, $M(.,.)$, and the rate of job destruction, $\delta_j, j=\alpha,\beta$.

## 5.1 The Effects of Labor Productivity

I consider first the impact on the steady state outcomes of a lower value for the marginal labor requirement, or equivalently, a higher value of $\varpi_j$, an improvement in productivity, which may be also seen as an improvement in TFP. A larger $\varpi_\alpha$ would affect production of all firms producing $\alpha-$ varieties in a single $\alpha-$type city, or all in all $\alpha$ cities. The impact in a particular city is complicated to analyze, because of free entry. It is easier to assume that it affects all firms producing $\alpha-$ varieties in all $\alpha$ cities. From (22) and (23), such a change has direct positive effects on the equilibrium quantities of output and employment for each variety. Via the job creation and wage setting conditions (43), such a change also leads to an increase of labor market tightness, which is in turn associated with higher employment and lower unemployment rates. However, this makes it harder for firms to hire and therefore at the steady state, from Equ. (22) and (23), this implies lower employment and output for each variety, *cet. par.*. By using (43) in (22), the resulting expression for $z_\alpha$ is an increasing function of $\varpi_\alpha$ and of $\theta_\alpha$.[25] It turns out that the net effect on output and employment for each variety is positive. From (49) such a change leads to a greater range of varieties.

Therefore, for given terms of trade, the effect on permanent income in an $-\alpha$ type of a positive productivity shock is positive.

Such a change may be traced along the Beveridge curve for the particular city. Recall section 4.3, where a changing labor market tightness produces an downward-sloping curve in $(u, v)$ space. An increase in $\varpi_\alpha$, *cet. par.*, from (4.3) increases the vacancy rate and reduces the unemployment rate, thus shifting the Beveridge curve for each variety-producing firm by making it "more vertical." The Beveridge curve is identical across all $\alpha-$variety producing firms as well as for the specialized city. This follows readily because the unemployment rate is the same across all $\alpha-$variety producing firms; and, in defining the vacancy rate in (4.3), the number of firms $m_\alpha$ multiplies both numerator and denominator in (4.3).

Considering the Beveridge curve for all cities specializing in $\alpha-$varieties, one may think of it as a spectrum of curves, or as an "average" of the respective curves. Alternatively, one may obtain the aggregate unemployment and express it in terms of the respective labor forces, on one hand, and vacancies as a share of vacancies and employment across all cities, on the other. At a first level of approximation, since the $\alpha-$variety producing cities have the same sizes, their labor forces are the same. So, the aggregate unemployment rate is the average unemployment rate across the respective cities. However, labor market tightness, and thus unemployment and vacancies in each city is determined by the realization of $\varpi_\alpha$.

The effects of productivity changes on the terms of trade is readily obtained from (52) through the effects on equilibrium labor market tightness. E.g., a productivity improvement in the production of $\alpha$ varieties in the form of an increase in $\varpi_\alpha$ leads to an increase in the term of trade of type$-\alpha$ cities (and, similarly, for $\beta-$cities). The net effect of symmetrical productivity improvements on the terms of trade depends on the sizes of the the two types of cities.

## 5.2 The Productivity of Matching

Consider first the impact of a larger total factor productivity (TFP) of the matching function. We write

$$M(U, V) \equiv \bar{M}\mu(U, V),$$

where $\bar{M}$ denotes total factor productivity in matching. From (43) by total differentiation we have that [26]:

$$\frac{d\theta_\alpha}{d\bar{M}} > \ (<)\ 0, \ \text{ iff: } \theta_\alpha > \ (<)\ \frac{(1-\vartheta)\delta_\alpha}{(\delta_\alpha+\rho)\gamma}.$$

In other words, a higher value of the TFP of the matching process has a positive effect if labor market tightness exceeds a certain threshold.

It is interesting to relax another aspect of the matching process in our context, that is, to assume that unemployed workers find out about jobs via referrals from their social contacts and do not need to travel to the CBD in order to search for jobs. See section 6 below. In that case, returning to Equ. (43), we note that provided that unit commuting costs are greater for employed than for unemployed individuals the labor market would be tighter in larger cities and the unemployment rate would be lower. A decrease in the unit commuting costs that is greater for unemployed individuals, as by their not being required to commute as frequently, than for employed ones brings about an increase in labor market tightness.

### 5.3 The Rate of Job Destruction

The rate of job destruction, an exogenous parameter in this paper, is different across industries. The effect of higher value of the rate of job destruction readily follows from (43). We find [27] that

$$\frac{d\theta_\alpha}{d\delta_\alpha} < \ (>)\ 0, \ \text{ iff } \ \theta_\alpha > \ (<)\ \frac{1-\vartheta}{\gamma}.$$

That is, when the cost of vacancies is sufficiently high relative to the weight of firms in bargaining, then firms find it disadvantageous to keep up with opening up vacancies and labor market tightness is lower, the employment rate is lower and the unemployment rate is higher. Cities specializing in industries that are more prone to job destruction, say manufacturing, would experience higher steady state unemployment than those specializing in services.

## 6 Aspatial Job Matching via Referrals

Formal and informal methods of search coexist even in our modern economy. How do such options impact urban equilibrium and urban spatial structure? The paper examines next first the case of individuals' receiving referrals[28] from others who are dispersed around the urban area and then the case of individuals' receiving tips from their social contacts. In the former case, individuals incur cost to visit others and interact with them; in the latter, individuals benefit by merely being associated with others.

So far all job matching takes place at the CBD and requires individuals to travel to the CBD in order to work and to be matched. It is interesting to examine how the implications of the model change, if individuals may match to jobs via referrals through individuals' social contacts.

I consider an unemployed individual who is in direct contact with $k$ other individuals. The probability that any of her contacts is employed and hears of a vacancy is $(1-u)\frac{V}{U}u =$

$(1-u)\theta u$. The probability that the individual finds a job thanks to one of her direct contacts[29] is $(1-u)\theta u \frac{1-(1-u)^k}{u}\frac{1}{k}$. The probability of finding a job through through referral by her social contacts is thus:

$$\text{Prob}_{\text{ref}}(u,\theta;k) = 1 - \left[1-(1-u)\theta\frac{1-(1-u)^k}{k}\right]^k. \tag{60}$$

If $k$ is large, the RHS in (60) is approximated[30] by

$$\text{Prob}_{\text{ref}} = \theta u(1-u)k. \tag{61}$$

The probability of finding a job though a referral from a friend is multiplicative in the total number of people contacted and in the labor market tightness, and in a quadratic function of the unemployment rate. Since labor market tightness is defined as $\theta = \frac{v}{u}$, the approximation in (61) implies that high unemployment rate implies a lower probability that social contacts are employed and thus less likely to pass on a job opening tip.

The assumption of referral matching from social contacts requires adapting the above Bellman equations by

$$\rho\Omega_{ju}(\ell) = b_j p_j + u_j(1-u_j)\theta E[k]\left[\Omega_{je}(\ell) - \Omega_{iu}(\ell)\right] + \bar{R} - R(\ell). \tag{62}$$

In view of the referral matching probability, for large $k$, from (61), the steady state unemployment rate is given by:

$$u_j = \frac{\delta_j}{\theta_j u_j(1-u_j)E[k] + \delta_j}. \tag{63}$$

Noting that Eq. (63) is cubic in $u_j$, allows me to solve explicitly[31] for the unemployment rate:

$$u_{j,\text{ref}} = \left(\frac{\delta_j}{\theta_j E[k]}\right)^{\frac{1}{2}}, \ j = \alpha, \beta. \tag{64}$$

It is the counterpart here of (29). It is decreasing in labor market tightness, just as in the original case of Eq. (29), and in the expected number of contacts each individual has: more social contacts, more referrals. Obviously, this solution is acceptable provided that

$$\frac{\delta_j}{E[k]} < \theta_j, \ j = \alpha, \beta. \tag{65}$$

When the average number of contacts is too low, relative to the rate of job destruction, vacancies must be sufficiently high to allow matching.

I invoke the logic of the definition of the matching function, section 33, to define the rate at which vacancies make contacts with workers as:

$$q_{\text{ref}}(\theta_j) = \frac{1}{\theta_j}\text{Prob}_{\text{ref}} = \left(\frac{\delta_j}{\theta_j E[k]}\right)^{\frac{1}{2}}\left[1-\left(\frac{\delta_\alpha}{\theta_\alpha E[k]}\right)^{\frac{1}{2}}\right]E[k], j = \alpha, \beta. \tag{66}$$

Under the feasibility condition (65), $q_{\text{ref}}(\theta_\alpha)$ decreases with $\theta_\alpha$, which agrees with the respective property of the matching model. It is social contacts that hear of vacancies and pass on the word to individuals.

Expected utility in the case of referrals from social contacts thus becomes:

$$\rho\omega_{j,\text{ref}} = \frac{\theta_j u_j(1-u_j)E[k]}{\theta_j u_j(1-u_j)E[k]+\delta_j} W_{j,\text{ref}}(1-\bar{a}_{j,e}\ell) + \frac{\delta_j}{\theta_j u_j(1-u_j)E[k]+\delta_j} bp_j + \bar{R} - R(\ell). \quad (67)$$

I exploit the aspatial nature of referral matching by assuming[32] that parameter values are such that make it dominate CBD matching.

Allowing for the fact that unemployment pay in the definition of expected utility (32) for those relying on referrals from social contacts does not depend on distance from the CBD yields:

$$\mathcal{D}_j \equiv \frac{\theta_j u_j(1-u_j)E[k]}{\theta_j u_j(1-u_j)E[k]+\delta_j} W_j \bar{a}_{e,j},$$

the wage setting model in an $j-$type specialized city yields:

$$W_{j,\text{ref}} = \vartheta\frac{\sigma-1}{\sigma}\varpi_j p_j - (1-\vartheta)\frac{\delta_j}{\theta_j u_j(1-u_j)E[k]} \frac{1}{1-\frac{2}{3}N_j^{\frac{1}{2}}a_{j,e}} bp_j, \; j=\alpha,\beta. \quad (68)$$

The associated expression for expected nominal income per period in a type-$j$ city is:

$$\vartheta\frac{\theta_j u_j(1-u_j)E[k]}{\theta_j u_j(1-u_j)E[k]+\delta_j}\left[1-\frac{2}{3}N_j^{\frac{1}{2}}a_{j,e}\right]\frac{\sigma-1}{\sigma}\varpi_j p_j + \vartheta\frac{\delta_j}{\theta_j u_j(1-u_j)E[k]+\delta_j} bp_j, \; j=\alpha,\beta. \quad (69)$$

Proposition 7. *The labor market tightness $\theta_j$ for the $j-$ variety producing firms in the case of aspatial matching via referrals satisfies:*

$$\frac{\gamma}{q_{\text{ref}}(\theta_j)} = \frac{1}{\delta_j+\rho}(1-\vartheta)\left[\frac{\sigma-1}{\sigma}\varpi_j + \frac{\left(\frac{\delta_j}{\theta_j E[k]}\right)^{\frac{1}{2}}}{1-\left(\frac{\delta_j}{\theta_j E[k]}\right)^{\frac{1}{2}}}\frac{1}{1-\frac{2}{3}N_j^{\frac{1}{2}}a_{j,e}} b_j\right], \quad (70)$$

*where $q_{\text{ref}}$, the rate at which firms generate hires per vacancy is now defined by (66) above, a decreasing function of labor market tightness.*

*Its properties are summarized by:*

$$\theta_j = \frac{1}{E[k]}\Theta_{\text{ref}}(\underset{(+)}{N_j}\; ; \;\underset{(+)}{\varpi_j}\; , \;\underset{(+)}{b_j}\; , \;\underset{(?)}{\delta_j}). \quad (71)$$

Proof. Condition (70) is the counterpart here of the job creation condition (43). The derivation follows from the explicit solution for the unemployment rate (64). Equilibrium labor market tightness and unemployment rate satisfies the system of (63)–(70). The left hand side of (70) is increasing, and its right hand side is decreasing, in $\theta_j$. Thus, in general, a unique solution exists, which shares the basic properties with model with CBD matching above. The solution for $\theta_j$ is in fact obtained via the solution for $\theta_j E[k]$, in terms of which it enters both sides of (70). The properties of the solution may be summarized in terms of $\Theta_{\text{ref}}$, the solution of (70) in terms of $\theta_j E[k]$, given in (71) above. In addition to the determinants of $\Theta(\cdot)$ in (44), $\Theta_{\text{ref}}(\cdot)$, (71) also includes the expected number of contacts per person, $E[k]$.

Q.E.D.

It is noteworthy that the effects of social connections work via the labor market tightness, and thus so do any differences across city types. Also, in describing social connections so far, I have assumed that individuals of the same type have an equal (average) number of connections with others. Suppose next that individuals' connectedness with others varies in a deterministic fashion across the population. For example, we could assume that social connectedness is described by means of a symmetric socio-matrix, or an adjacency matrix. Each row of this matrix gives the pattern of connectedness with others. That is, if $a_{i',i''} = 1$, individuals $i'$ and $i''$ are interconnected. How would different individuals locate? Would they segregate across different cities of the same type, or would they all locate in each city type? These are interesting questions may be pursued further.

Our extension of the model using referrals from social contacts instead of centralized matching has an interesting consequence. Steady state unemployment may differ across cities of the same type because of differences in social structure, as represented by the expected number of social contacts, which enters (64), the expression for $u_{j,\text{ref}}$. It may also differ because of heterogeneity in city geography. In any case, any such differences across city types are consistent with intercity spatial equilibrium.

# 7 Mismatch and the Planner's Optimum

I define next the social planner's problem of allocating the economy's resources, [33] defined as maximizing expected discounted income per person while respecting the informational structure of the economy. The planner's optimum defines the socially optimal level of unemployment and thus allows us to assess the extent of mismatch associated with the operation of a market economy in this setting. Mismatch is assessed in terms of comparing arbitrary allocations to socially optimal ones.

Specifically, given a total population of each type of skilled labor, $\left(\bar{N}_\alpha, \bar{N}_\beta\right)$, the planner's problem is to choose the number of cities of each types, $(n_\alpha, n_\beta)$ and their respective populations, $(N_\alpha, N_\beta)$, subject to (47), the range of intermediates produced in each city type, $(m_\alpha, m_\beta)$, their respective quantities net of search costs (see below), $(z_{\alpha,n}, z_{\beta,n})$, and the quantities of each to be used in each city type, $(z_{\alpha,\alpha}, z_{\alpha,\beta}; z_{\beta,\alpha}, z_{\beta,\beta})$, and the labor market tightness in each city, $(\theta_\alpha, \theta_\beta)$, so as to maximize total discounted average real per person in the entire economy at the steady state. Such an aggregation is appropriate because total income is defined in terms of the final good that is locally consumed.

Writing expected income per person per unit of time in each city type as output per person in each city type, Exp $\text{Inc}_j = \frac{Y_j}{N_j}$, we have:

$$\frac{n_\alpha N_\alpha}{\bar{N}} \times \frac{Y_\alpha}{N_\alpha} + \frac{n_\beta N_\beta}{\bar{N}} \times \frac{Y_\beta}{N_\beta} = \frac{1}{\bar{N}} \left[ n_\alpha Y_\alpha + n_\beta Y_\beta \right], \tag{72}$$

where $(Y_\alpha, Y_\beta)$ are given by (3), (4), respectively. Decomposing the problem of maximizing the present value of this quantity, subject to appropriate resource constraints, applied at the steady state, we note that the use of intermediates, $(z_{\alpha,\alpha}, z_{\beta,\alpha}; z_{\alpha,\beta}, z_{\beta,\beta})$, is a static problem. The properties of the planner's optimum are formally stated as follows.

Proposition 8.

*Part A. The maximization of total income per unit of time with respect to use of intermediates, subject to iceberg shipping costs, satisfy the resource constraints as per conditions (17) above, reproduced here in adapted notation:*

$$\tilde{n}_\alpha z_{\alpha,\alpha} + n_\beta \frac{1}{\tau} z_{\alpha,\beta} = z_{\alpha,n},\ n_\alpha \frac{1}{\tau} z_{\beta,\alpha} + \tilde{n}_\beta z_{\beta,\beta} = z_{\beta,n},$$

*where $z_{j,n}$ denotes the net quantity of intermediate of type $j$, that is the quantity produced, $z_{j,p}$, net of its producer's search costs,*[34] *$z_{j,n} = z_{j,p} - \gamma V_j$, and yields:*

$$z_{\alpha,\alpha} = \frac{1}{\tilde{n}_\alpha + \tau^{\sigma-1} n_\beta} z_{\alpha,n},\ z_{\alpha,\beta} = \frac{\tau^\sigma}{\tilde{n}_\alpha + \tau^{\sigma-1} n_\beta} z_{\alpha,n};$$

$$z_{\beta,\alpha} = \frac{\tau^\sigma}{n_\alpha \tau^{\sigma-1} + \tilde{n}_\beta} z_{\beta,n},\ z_{\beta,\beta} = \frac{1}{n_\alpha \tau^{\sigma-1} + \tilde{n}_\beta} z_{\beta,n}. \tag{73}$$

Part B. *The range of intermediates is given by:*

$$m_j = \frac{1}{h_{j,p}} \frac{\pi_j}{\pi_j + \delta_j} \bar{H}_{je}. \tag{74}$$

Part C. *The counterparts of (19) and (23) for the planner, $(z_{j,p}, h_{j,p})$ are given by:*

$$z_{j,p} = \varpi_j \kappa_j \frac{\sigma - 1 - \sigma \frac{\gamma\rho}{\varpi_j q_j(\theta_j)} + \frac{\gamma(\rho+\delta_j)}{\varpi_j q_j(\theta_j)}}{1 + \sigma \frac{\gamma\rho}{\varpi_j q_j(\theta_j)} - \frac{\gamma(\rho+\delta_j)}{\varpi_j q_j(\theta_j)}},\ j = \alpha, \beta; \tag{75}$$

$$h_{j,p} = \frac{\sigma\kappa_j}{1+\sigma\frac{\rho\gamma}{q(\theta_j)\varpi_j} - \frac{\gamma(\rho+\delta_j)}{\varpi_j q_j(\theta_j)}}, j=\alpha,\beta. \tag{76}$$

*The quantities of intermediates net of search costs, $z_{j,p} - \gamma V_j$ are given by:*

$$z_{j,n} = z_{j,p} - \gamma V_j = (\sigma-1)\kappa_j\varpi_j \frac{1-\frac{\gamma(\rho+\delta_j)}{\varpi_j q_j}}{1+\sigma\frac{\rho\gamma}{q(\theta_j)\varpi_j} - \frac{\gamma(\rho+\delta_j)}{\varpi_j q_j}}, j=\alpha,\beta. \tag{77}$$

Part D. *The planner's maximand may be rewritten as follows:*

$$\left[\frac{\left(1-\frac{\gamma(\rho+\delta_\alpha)}{\varpi_\alpha q_\alpha}\right)\left(\frac{\pi_\alpha}{\pi_\alpha+\delta_\alpha}\right)^{\frac{\sigma}{\sigma-1}}}{\left(1+\frac{\gamma\rho}{\varpi_\alpha q_\alpha}[\sigma-1-\delta_\alpha]\right)^{-\frac{1}{\sigma-1}}}\right]^{\phi}\left[\frac{\left(1-\frac{\gamma(\rho+\delta_\beta)}{\varpi_\beta q_\beta}\right)\left(\frac{\pi_\beta}{\pi_\beta+\delta_\beta}\right)^{\frac{\sigma}{\sigma-1}}}{\left(1+\frac{\gamma\rho}{\varpi_\beta q_\beta}[\sigma-1-\delta_\beta]\right)^{-\frac{1}{\sigma-1}}}\right]^{1-\phi}\mathcal{N} \tag{78}$$

*where $\mathcal{N} \equiv \mathcal{N}(n_\alpha, n_\beta; N_\alpha, N_\alpha)$, a function of the numbers of city types and their sizes, is defined as*

$$\mathcal{N} \equiv \varpi^* \frac{n_\alpha \tau^{\sigma(1-\phi)}\tilde{n}_\alpha^{\phi\frac{\sigma}{\sigma-1}} n_\beta^{(1-\phi)\frac{\sigma}{\sigma-1}} + \tau^{\sigma\phi} n_\beta (n_\alpha)^{\phi\frac{\sigma}{\sigma-1}}\tilde{n}_\beta^{(1-\phi)\frac{\sigma}{\sigma-1}}}{(\tilde{n}_\alpha + \tau^{\sigma-1}n_\beta)^{\phi\frac{\sigma}{\sigma-1}}(n_\alpha\tau^{\sigma-1}+n_\beta)^{(1-\phi)\frac{\sigma}{\sigma-1}}} \bar{H}_\alpha^{\phi\frac{\sigma}{\sigma-1}}\bar{H}_\beta^{(1-\phi)\frac{\sigma}{\sigma-1}}, \tag{79}$$

*where $\bar{H}_j, j=\alpha,\beta$, are defined in (47). Note that the number of cities adjusted for shipping costs, $(\tilde{n}_\alpha, \tilde{n}_\beta)$, are given in (15), and $\varpi^*$ is defined as a function of preference, aggregate production function and production of intermediates parameters:*

$$\varpi^* \equiv (\sigma-1)\sigma^{-\frac{\sigma}{\sigma-1}}\varpi_\alpha^\phi \kappa_\alpha^{-\frac{\phi}{\sigma-1}}\varpi_\beta^{1-\phi}\kappa_\beta^{-\frac{1-\phi}{\sigma-1}},$$

*where $\varpi^*$ aggregates the two types of productivity parameters (shocks), $\varpi_\alpha$, and $\varpi_\beta$.*

Proof.

Part A.

Part B. The range of intermediates is obtained by (49), and is equal to the respective expected city labor supply divided by expected employment in producing each intermediate. Thus (74) follows.

Part C. From the current value Hamiltonian, by expressing the first-order conditions and solving at the steady state, the counterparts of (19) and (23) for the planner, $(z_{j,p}, h_{j,p})$: are given by (75).

By using the expression for the stock of vacancies at the steady state, $V_j = \frac{\delta_j h_j}{q_j}$, we obtain a simplified expression for the quantities of intermediates net of search costs, $z_{j,p} - \gamma V_j$, given by (77).

Part D. By dividing expected employment in each city, $\frac{\pi_j}{\pi_j+\delta_j}H_{je}$, $j=\alpha,\beta$, where $\bar{H}_{je}$, are given by (48), by employment required for each intermediate variety, (76), we obtain expressions for the range of intermediates. Thus, we may rewrite the planner's maximand as in (78).

Q.E.D.

We note that the planner's allocations are different from the equilibrium ones, given by Proposition 1 and (6) above in particular, on several accounts. First, the quantities to be allocated are net of producers' search costs; second, the planner internalizes the resource costs associated with intercity shipping costs; and third, the aggregation in terms of incomes in the two city types according to (72) makes the shares $(\phi, 1-\phi)$ irrelevant at this level of analysis, though not for the entire planner's solution, as we see shortly below.

By comparing with the expressions for equilibrium output and employment, given by (22) and (23), it follows that social optimum requires greater quantities of both, for the same value of labor market tightness. That reflects the fact that the planner internalizes the search externality affecting producers of intermediates.

The tradeoffs addressed by the planner's optimum handle the several types of inefficiencies in this paper. One is the pecuniary one associated with the monopolistic competition model: a greater variety of intermediates improves welfare, but requires larger cities; larger cities involve greater congestion. A third is the potential inefficiency of search. My formulation of the planner's problem respects (as it ought to) the informational structure of the economy, and thus obviates the problem of multiplicity that is inherent in the essential decentralization of urban production (due to the lack of ability of agents to coordinate and locate in a particular city), and lends itself to formulations that where individual city characteristics differ. Therefore, the planner's optimum provides the best possible outcome given any set of values of underlying parameters.

The planner's choice of labor market tightness $(\theta_\alpha, \theta_\beta)$, follow from the maximization of the first (second) ratio in square brackets in (78) and is thus independent of city size, unlike the case at equilibrium, Eq. (43). Setting labor market tightness optimally involves trading off higher probability of employment — the greater the tightness the greater the probability of employment and the greater variety of intermediates — against greater search costs incurred by firms. That is, the planner's optimum level of employment for each intermediate, given by (76), decreases with labor market tightness, which means more intermediates can be produced, but the quantities of intermediates available for production decrease with labor market tightness. In our formulation of the planner's problem, these tradeoffs determine labor market tightness separably for each type of intermediate. This is also true in the equilibrium case, which of course reflects the counterpart of the job creation condition in the DMP model.

The tradeoffs associated with the numbers and sizes of cities are characterized by the properties of the auxiliary function $\mathcal{N}(n_\alpha, n_\beta; N_\alpha, N_\alpha)$, defined by (79) above, and are thus separable from the determination of labor market tightness and thus socially optimum unemployment. Here, intercity shipping costs are crucial in determining the tradeoff. Very small cities have little congestion, allow for greater variety of intermediates but impose greater shipping costs. Suppose that there are no intercity shipping costs, that is, $\tau = 1$. The variable part in the right hand side of (79) becomes:

$$\frac{N_\alpha^{\frac{1}{\sigma-1}} N_\beta^{\frac{1}{\sigma-1}}}{(\bar{N}_\alpha N_\beta + \bar{N}_\beta N_\alpha)^{\frac{1}{\sigma-1}}} \left(1 - \frac{2}{3} a_{\alpha,e} N_\alpha^{\frac{1}{2}}\right)^{\phi \frac{\sigma}{\sigma-1}} \left(1 - \frac{2}{3} a_{\beta,e} N_\beta^{\frac{1}{2}}\right)^{(1-\phi)\frac{\sigma}{\sigma-1}}.$$

Maximizing this quantity with respect to $(N_\alpha, N_\beta)$ gives the socially optimal city sizes and thus the numbers of cities, as well. The first-order conditions have the form of a system of algebraic equations in $(N_\alpha, N_\beta)$ whose solutions may be characterized easily and depend on parameters $\sigma, \phi, a_{\alpha,e}, a_{\beta,e}$ and the total number of skilled labor of the two types, $\bar{N}_\alpha, \bar{N}_\beta$. It can be shown that the system of equations admits two sets of solutions, one of which is stable and the other unstable. The presence of shipping costs complicates this tradeoff, but it can shown that the planner's optimum city sizes exist. Specifically, the planner's optimum city sizes are independent of labor market tightness and therefore of the frictions in the labor allocation mechanism. Thus, in general, it would differ from equilibrium city sizes, defined in section 4.5 above as the city sizes that maximize the quantity in (59). Mismatch arises generically in this model: The planner's optimum and equilibrium sizes coincide only by chance. Socially optimal city sizes involve tradeoffs between love of variety (or, what is the same, market size effects) and geography, both local geography due to congestion and national geography due to shipping costs.

The solution to the planner's problem is affected by geography via shipping costs. The productivity parameters $(\varpi_\alpha, \varpi_\beta)$ affect labor market tightness separably. Alternative sets of values allow us to trace the path of the vacancy and unemployment rates, the Beveridge curves for the two kinds of cities for the planner's optimum. We could perform additional comparative dynamics exercises, e.g., by allowing for different commuting costs (reflecting internal geography) across different sites, or for different shipping costs between the $\alpha$ and $\beta$ varieties (where one can be services and the other manufactured goods, or different kinds of specialized services produced by different occupations). Such extensions could easily be addressed by future research.

# 8 Conclusions

The variation of unemployment and the variability of economic activity across cities has been established but is still poorly understood phenomenon. The present paper proposes a coherent and firmly micro-founded theoretical model of urban unemployment in an economy of open, trading cities. The approach proposed here enhances the system-of-cities model by allowing for unemployment and fluctuations in economic activity that may differ systematically across cities in a large economy. It also enhances the DMP model of labor markets with frictions by introducing, in addition, spatial frictions of the sort that characterize urban economies. The model's use of international trade tools confers a central role to labor market tightness, akin to factor intensity. Equilibrium modeling of cities lends additional discipline to analyses of urban unemployment and provides a systematic way of exploiting Beveridge curves, as well, as an expository tool, as well. The specific applications whereby location decisions within urban areas are influenced by choice between job referrals by social contacts and centralized job matching are another new feature. The model is cast in terms of certainty equivalents, which is in line with the original DMP literature.

Equilibrium outcomes generically diverge from the planner's optimum: socially optimal unemployment trades off the likelihood of employment against search costs of firms, independently for each skill type and independently of city size. Socially optimal city sizes are independent of labor market tightness considerations but reflect both market size effects and the skill composition of the economy.

Different cities interact via intercity trade and migration, and the macroeconomic dynamics of urban business cycles reflect both those forces. Specialized cities, exemplified by the proverbial company town, may react differently to economy-wide shocks than diversified ones, exemplified by the large cities that accommodate many industries and activities. Cities that provide services to the international economy are affected differently by the national and international business cycle than large diversified regional centers. It is for such reasons that diversified cities deserve attention in future research.

Future research with models of the type used in this paper should be cast explicitly in stochastic terms and allow for a full dynamic stochastic analysis rather than a steady state analysis, now that progress has been made with the development of stochastic versions of the DMP model: see Petrosky-Nadeau and Wasmer (2015). In particular, it would be interesting to allow for intercity moves that follow city-specific shocks, thus allowing linkages between the housing and labor markets, and also introduce financial frictions.

# Notes

[1] A recent compilation using BEA data for GDP by MSA shows that even as growth in real U.S. GDP by metropolitan area slowed from 2.0 percent in 2007 to 0.8 percent in the first quarter of 2008, the distribution of growth in real U.S. GDP by metropolitan area varies from impressive growth in some areas, as in Grand Junction, CO at 12.3 per cent, to sharp decline, as in Kokomo, IN at 10 per cent.

[2] A key feature of Cuñat and Melitz (2012) is modeling flexibility versus rigidity in in the context of international trade. Helpman and Itskhoski (2010) address questions broadly related to those of Cuñat and Melitz (2012), except that their models also allow for unemployment. That is, one of the two sectors in their model produces a final good by means of a range of differentiated products, which are themselves produced with raw labor and whose production is subject to market frictions, as modeled in the DMP fashion. Trade integration benefits both countries but may raise their rates of unemployment.

[3] Hall argues that such a finding should be taken to imply a much higher level of specialization in the large dense markets, or else there would be no efficiency benefits from large dense markets. If specialization were not omitted then a given combination of unemployment and vacancies would generate "moderate matching rates for highly specialized workers rather then very rapid matching for the less specialized workers and jobs. Hall's remark suggests the possibility that accounting for job market matching can be used to help distinguish between the different roles of city size in facilitating MAR externalities, matching and labor pooling.

[4]In Shimer's calculation, with 134 million workers in 2000 in the US who are allocated in 362 different metropolitan areas (regions with at least one urbanized area with with population 50,000 or more) and 560 micropolitan areas (regions with an urban area with population 10,000 to 50,000) and 800 different occupations, there were $922 \times 800 \approx 740,000$ "markets." Shimer, *ibid.,* Proposition 4, *ibid.,* p. 1082, determines uniquely the number of workers, $M$, and jobs per market, $N$; given the national unemployment and vacancy rates at 5.4% and 2.3%, respectively, those numbers are $M = 244.2$ and $N = 236.3$, respectively. Therefore, this estimate implies 134 million / $244.2 \approx 550,000$ labor markets, which is indeed in the same range as the 740,000 number. A particularly interesting result, reported by Shimer, is that the theoretically predicted job finding rate varies with the vacancy–unemployment ratio in ways which imply a matching function that is Cobb–Douglas in vacancies and unemployment.

[5] The model borrows ideas from Kraay and Ventura (2007)'s model of the international economy, in order to describe intercity trade in the style of Anas and Xiong (2003) as adapted by Ioannides (2013), Chapter 7, Section 7.8, and where hiring is subject to frictions. Kraay and Ventura (2007) allow for a rich set of possible shocks, including monetary shocks, for which urban/regional aspects are particularly interesting but which are not adopted here. Francis, Owyang and Sekhposyan (2009) and Owyang, Rapach and Wall (2009) link a city's business cycle with its industrial structure.

[6] Broadly related approaches by Lentz and Mortensen (2012), which assumes a perfectly competitive model with product varieties and hiring with frictions by heterogeneous firms and by vom Berge (2011) do not model intercity trade.

[7]An apparent difference in this specification from vom Berge (2011), Eq. (12), is due to the fact that he assumes that search costs, as proportional to the stock of vacancies, constitute a component of labor demand by each variety-producing firms. The counterpart of this feature in my model is in Eq. (20), where I assume that a search cost in terms of the intermediate good itself. I thank Philipp von Berge for clarifying

correspondence.

[8]In a stochastic environment, it would be interesting to allow, like Kraay and Ventura, *op. cit.,* for the $\beta-$industry technology to be operated by $\alpha-$skill types as well. Kraay and Ventura's assumptions allow naturally for $\alpha-$ industry skills to be compatible with $\beta-$ industry employment. Firms in the $\beta-$ industry face infinitely elastic demand curves and behave competitively setting price equal to marginal cost $p_\beta = w_\beta \varpi^{-1}$. Such an asymmetry in the price elasticity of product demand across the $\alpha-$ and $\beta-$ industries adds richness to their model.

[9] The derivative is taken of each of the terms involving $z_{\alpha_{nm}}$ in the original description of the rate of output in (2), of which there exist $m_\alpha$, and then and then use symmetry to write the firm order conditions.

[10] Employment and production are of course random, but I work throughout with expected quantities.

[11] $\frac{\partial}{\partial V_{j,t}}\mathrm{HAM} = 0$, where HAM denotes the current value Hamiltonian,

$$\mathrm{HAM} = p_{j,t}z_{j,t} - W_{j,t}\left[\varpi_j\kappa_j + z_{j,t}\right] - p_{j,t}\gamma V_{j,t} + \lambda\left[\varpi_j q(\theta_{j,t})V_{j,t} - \delta_j\left[\varpi_j\kappa_j + z_{j,t}\right]\right].$$

[12] $-\frac{\partial}{\partial z_{j,t}}\mathrm{HAM} = \dot{\lambda} - \rho\lambda.$

[13] $-\left[p_{j,t} + \frac{\partial p_{j,t}}{\partial z_{j,t}}z_{j,t} - \varpi_j{}^{-1}W_{j,t} - \delta_j\frac{\gamma p_{j,t}\varpi_j^{-1}}{q(\theta_{j,t})}\right] + \rho\lambda = 0.$

[14]

$$\frac{\partial z_j}{\partial\theta_j} = \frac{\frac{\rho\gamma q'(\theta_j)\kappa_j}{q^2(\theta_j)}}{\left(\frac{1}{\sigma} + \frac{\rho\gamma}{q(\theta_j)}\varpi_j\right)^2} < 0.$$

[15]

$$\frac{\partial h_j}{\partial\theta_j} = \frac{\frac{\rho\gamma q'(\theta_j)\varpi_j\kappa_j}{q^2(\theta_j)}}{\left(\frac{1}{\sigma} + \frac{\rho\gamma}{q(\theta_j)}\varpi_j\right)^2} < 0, \quad \frac{\partial h_j}{\partial\varpi_j} = \frac{\frac{\rho\gamma\varpi_j\kappa_j}{q(\theta_j)}}{\left(\frac{1}{\sigma} + \frac{\rho\gamma}{q(\theta_j)}\varpi_j\right)^2} > 0.$$

[16]Rearranging it allows one to express the "real wage" $\frac{W_j}{p_j}$ as a decreasing function of labor market tightness. "[A]t a lower wage rate, jobs are more profitable and more vacancies are created" [Pissarides (2011), p. 1095]. However, unlike in the original Pissarides framework where the job creation condition follows from free entry of firms in creating vacancies, here it follows from the pricing behavior of firms. Free entry is also imposed here in a dynamic sense (25), which along with the job creation condition leads to an expression for the value of vacancies. See (24) above.

[17] Helsley and Strange (1990) obtain a source of agglomeration economies rooted in job matching, in that larger labor markets may provide better matches between jobs and workers. Stevens (2007) endogenizes the Pissarides matching function. Petrongolo and Pissarides (2001) provide an excellent overview of the literature that the Pissarides matching function has given rise to. Mortensen (2009) also endogenizes the matching function, along the lines of Shimer (2007)'s formulation and shows that the flow of matches is an increasing and concave function of the number of jobs and workers to be matched, holding the other constant, but exhibits increasing returns to scale (except in the limiting case of large numbers of jobs and workers relative to the number of submarkets, "islands"). The Mortensen solution is particularly interesting because it rests in random matching, with the number of matches in each isolated island being the minimum of the realized number of available jobs and workers that search. The sums of the residuals are the numbers of unemployed workers and vacant jobs. The aggregate matching function that arises is the resulting statistical relationship between average meeting rates per island

and the aggregate numbers of unmatched workers and jobs per island. The 2010 Nobel Price citation, `http://static.nobelprize.org/nobel_prizes/economics/laureates/2010/ecoadv10.pdf` and the Prize Lectures by the laureates themselves are the best summary of this literature as of the time of writing. See Diamond (2011), Mortensen (2011) and Pissarides (2011).

[18]It is not a trivial task to determine city types in the sense of the paper empirically, because even the largest relative employment shares for industrial sectors in US cities are rather small in absolute terms. See Ioannides (2013), 7.2, 294–297.

[19] For a homogeneous city, say a type $\alpha-$city, $R_{\text{tot}} = \int_0^{\bar{\ell}} R_\alpha(\ell) 2\pi \ell d\ell = \frac{1}{3}\pi \mathcal{D}_\alpha \bar{\ell}^3$, for which $\bar{R} = \frac{1}{3}\pi\mathcal{D}_\alpha \bar{\ell}$.

[20] This deviates from Wasmer and Zenou, *op. cit.* and from the the standard structure of search models, as Chris Pissarides reminds me. My intention is to render the bargaining solution for the wage rate independent of location. In the present model, at the spatial equilibrium it is expected lifetime utility that is equalized across locations within and across cities. Associating the bargaining outcome with the increase of the expected value of employment over unemployment, $\Omega_e - \Omega_u$, would make it dependent on location. That is, the solution for the bargained wage is:

$$W_j = W_j(\ell) = \vartheta \frac{\sigma - 1}{\sigma} \varpi_j p_j \left[1 - \bar{a}_e \ell\right] + (1 - \vartheta) b p_j \left[1 - \bar{a}_u \ell\right].$$

Substituting back into the expression for utility renders expected equilibrium utility quadratic in $\ell$. For spatial equilibrium, $R(\ell)$ varies quadratically with $\ell$. This in turn produces an expression for total rents per person that is also quadratic in $\ell$ and yields in turn an expected utility net of redistributed rents that is quadratic in city size. This makes derivations very unwieldy.

I thank Frédéric Robert-Nicoud for directing my attention to the Stole–Zwiebel bargaining solution [ Stole and Zwiebel (1996) ], which has been adopted (after refinements) for multi-person bargaining settings by some recent applications of the DMP model. See Helpman, Otshkoki and Redding (2013). I think that the Pissarides-based large firm model, as adapted to the urban setting by Wasmer and Zenou (2002; 2006) is more suitable to my model.

[21] This follows from expression the unit cost function that corresponds to (2), the production function for the final good in each city.

$$B(p_\alpha(z), p_\beta(z)) = \left[\int_0^{m_\alpha} p_\alpha(z)^{1-\sigma} dz\right]^{\frac{\phi}{1-\sigma}} \left[\int_0^1 p_\beta(z)^{1-\sigma} dz\right]^{\frac{1-\phi}{1-\sigma}}, 0 < \phi < 1, \sigma > 1.$$

To this cost function ideal price indices may be defined for the $\alpha-$ and $\beta-$ industries:

$$P_\alpha = \left[\int_0^{m_\alpha} p_\alpha(z)^{1-\sigma} dz\right]^{\frac{1}{1-\sigma}}, \text{and } P_\beta = \left[\int_0^1 p_\beta(z)^{1-\sigma} dz\right]^{\frac{1}{1-\sigma}}.$$

[22]See Job Openings and Labor Turnover Survey, p. 5 `http://www.bls.gov/web/jolts/jlt_labstatgraphs.pdf` .

[23]It is easy to incorporate economy-wide trade in the final good, an extension not undertaken here.

[24] The quadratic equation is:

$$N_j \frac{2}{3} \frac{\sigma - 1}{\sigma} \delta_j (\varpi_j a_{j,e} - a_{j,u} b_j) \frac{\partial \pi_j}{\partial N_j} - N_j^{\frac{1}{2}} \frac{\sigma - 1}{\sigma} (\varpi_j - b_j) \frac{\partial \pi_j}{\partial N_j} + \frac{1}{3} \left[\pi_j \frac{\sigma - 1}{\sigma} \varpi_j a_{je} + \delta_j a_{ju} b_j\right] (\pi_j + \delta_j) = 0,$$

with two positive roots defined by functions of $N_j$, provided that the productivities $\varpi_j$ are sufficiently higher than the compensation of leisure, $b_j$. The expressions for the roots define alternative maximizing values as fixed fixed points. Note that $\frac{\partial \pi_j}{\partial N_j}$ is obtained from (43). Similarly to many other models in urban systems, the smaller root is associated with an unstable equilibrium, and the larger one with a stable. The existence of feasible values depends on sufficient concavity of the $\pi_j$ with respect to $N_j$.

[25]By using the definition of the vacancy rate to substitute for $\frac{\gamma}{q(\theta_\alpha)}$ in (22), we have:

$$z_\alpha = \frac{\frac{\sigma-1}{\sigma}\varpi_\alpha(\delta_\alpha + \rho\vartheta) - \rho(1-\vartheta)\frac{\delta_\alpha}{\pi_\alpha}}{\frac{1}{\sigma}\left[\delta_\alpha + \rho + \rho(1-\vartheta)(\sigma-1)\right] + \rho(1-\vartheta)\varpi_\alpha\frac{\delta_\alpha}{\pi_\alpha}}\kappa_\alpha.$$

Working similarly with (23), we have:

$$h_\alpha = \frac{1}{\frac{1}{\sigma}\left[\delta_\alpha + \rho + \rho(1-\vartheta)(\sigma-1)\right] + \rho(1-\vartheta)\varpi_\alpha\frac{\delta_\alpha}{\pi_\alpha}}\kappa_\alpha.$$

[26]

$$\frac{d\theta_\alpha}{d\bar{M}} = \frac{\frac{1}{\bar{M}^2\mu(U,V)}\left[\gamma\theta_\alpha - \frac{1-\vartheta}{\delta_\alpha+\rho}\delta_\alpha\right]}{\frac{\gamma m_1}{\bar{M}\mu^2(\theta_\alpha^{-1},1)\theta_\alpha^2} + \frac{(1-\vartheta)\delta_\alpha\mu_2}{(\delta_\alpha+\rho})\bar{M}\mu^2(1,\theta_\alpha}.$$

[27]

$$\frac{d\theta_\alpha}{d\delta_\alpha} = \frac{\frac{1-\vartheta-\theta_\alpha\gamma}{(\delta_\alpha+\rho)\pi_\alpha}}{-\frac{\gamma q'}{q^2} + \frac{1-\vartheta}{\delta_\alpha+\rho}\frac{\delta_\alpha}{\pi^2}\pi'}.$$

[28]Galenianos (2014) proposes a model of referral hiring. An equilibrium search model of the labor market is combined with a rudimentary social network. The key features are that the workers' network transmits information about jobs and that wages and firm entry are determined endogenously. The model has no spatial features.

[29]This is derived in detail in Calvó-Armengol and Zenou (2006) and Ioannides and Soetevent (2006). It is consistent with the identifying assumptions of Topa (2001).

[30]First, note that

$$1 - (1-u)^k \approx ku.$$

This yields in turn:

$$\text{Prob}_{\text{ref}}(u,\theta;k) \approx 1 - [1-\theta(1-u)u]^k \approx \theta u(1-u)k.$$

This calculation may also be seen as a simplification of matching model in Galeotti and Merlino (2014), Eq. (3).

An analysis by Calvó-Armengol and Zenou shows that as $k$ varies, the exact probability of $\text{Prob}_{\text{ref}}(u,\theta;k)$ increases initially until it reaches a unique maximum and decreases thereafter. The economic intuition for this finding is that increasing network size makes coordination failures more likely. Although unemployed workers receive on average more job openings through their social network as social network size increases, information about vacancies may be wasted as it becomes more likely that an unemployed worker receives multiple notifications of the same vacancy.

[31]Of the three roots, one is equal to 1, and of the other two, (64) is a feasible solution for unemployment.

[32] When this assumption is not appropriate, coexistence of CBD matching and referral matching may be handled by means of a heterogeneous (mixed) city model, as in section 5.7.3.1, 239–240, Ioannides (2013).

[33]I note that by choosing labor market tightness in each city type as decision variables, the planner'a problem respects the informational structure of the economy. This formulation is in agreement with that by Şahin et al. (2014).

[34] In expressing the current value Hamiltonian, we recognize that it is the quantities of intermediates net of search costs that are available for production. The current value Hamiltonian may be written as:

$$\text{HAM} = \left( n_\alpha (\tilde{n}_\alpha)^{\phi \frac{\sigma}{\sigma-1}} n_\beta^{(1-\phi)\frac{\sigma}{\sigma-1}} + n_\beta (n_\alpha)^{\phi \frac{\sigma}{\sigma-1}} \tilde{n}_\beta^{(1-\phi)\frac{\sigma}{\sigma-1}} \right) (z_\alpha - \gamma V_\alpha)^\phi (z_\beta - \gamma V_\beta)^{1-\phi}$$

$$\times [\kappa\alpha + \varpi_\alpha^{-1} z_\alpha]^{-\phi \frac{\sigma}{\sigma-1}} [\kappa_\beta + \varpi_\beta^{-1} z_\beta]^{-(1-\phi)\frac{\sigma}{\sigma-1}} \left[ \frac{\pi_\alpha}{\pi_\alpha + \delta_\alpha} H_{e,\alpha} \right]^{\phi \frac{\sigma}{\sigma-1}} \left[ \frac{\pi_\beta}{\pi_\beta + \delta_\beta} H_{e,\beta} \right]^{1-\phi}.$$

The production constraints, according to (19), are adjoined by:

$$+\lambda_\alpha \left[ \varpi_\alpha q(\theta_\alpha) V_\alpha - \delta_\alpha \left( \varpi_\alpha \kappa_\alpha + z_\alpha \right) \right] + \lambda_\beta \left[ \varpi_\beta q(\theta_\beta) V_\beta - \delta_\beta \left[ \varpi_\beta \kappa_\beta + z_\beta \right] \right].$$

The first order conditions with respect to $V_\alpha, V_\beta$, yield:

$$\lambda_\alpha \varpi_\alpha q(\theta_\alpha) = \gamma\phi \frac{\text{HAM}}{z_\alpha - \gamma V_\alpha}, \lambda_\beta \varpi_\beta q(\theta_\beta) = \gamma\phi \frac{\text{HAM}}{z_\beta - \gamma V_\beta}.$$

The first order conditions with respect to $\lambda_\alpha, \lambda_\beta$, yield:

$$-\phi \frac{\text{HAM}}{z_\alpha - \gamma V_\alpha} + \phi \frac{\sigma}{\sigma - 1} \frac{\text{HAM}}{\varpi_\alpha \kappa_\alpha + z_\alpha} + \delta_\alpha \lambda_\alpha = \dot{\lambda_\alpha} - \rho \lambda_\alpha;$$

$$-\phi \frac{\text{HAM}}{z_\beta - \gamma V_\beta} + \phi \frac{\sigma}{\sigma - 1} \frac{\text{HAM}}{\varpi_\beta \kappa_\beta + z_\beta} + \delta_\beta \lambda_\beta = \dot{\lambda_\beta} - \rho \lambda_\beta.$$

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