Vacancies in Housing and Labor Markets[1]

Yannis M. Ioannides* and Jeffrey E. Zabel**

Department of Economics

Tufts University

Medford, MA 02155

Key Words: Housing Vacancies, Job Openings, Beveridge Curves, Housing and labor markets. JEL Codes: R23, J64

Abstract

The Great Recession of 2007–2009 has prompted a focus on the link between the housing and business cycles. We model the housing and labor markets by means of a DMP-type model that treats housing and labor supply as joint decisions and highlights the interdependence of vacancies in these markets. We estimate this at the MSA level using data on housing vacancies from the US Census Bureau's Housing Vacancy Survey (HVS) starting in 1986 and on job vacancies from the Conference Board's Help-Wanted Index starting in 1951. In particular, we estimate a Beveridge Curve for labor markets that includes spillovers from vacancies in the rental and homeownership housing markets, as well as novel Beveridge curves for owner and rental housing markets. We then estimate VAR models for housing and job vacancies. Results from impulse response functions show that shocks to rental and homeownership vacancies have negative and significant impacts on job vacancies.

[1]We are grateful for comments received during conversations with Allen Head, Rachel Ngai, Theodore Papageorgiou, and Chris Pissarides, and during presentations at Tufts, the 2014 Urban Economics Association meetings, Bethesda, MD, the ASSA/AREUEA meetings, Boston, January 3, 2015, HULM, St. Louis, April 10, 2015, Conference in Honour of Sir Christopher Pissarides, Sciences Po, July 1, 2015, Workshop on Public, Urban and Regional Economics, Cleveland Fed, February 26, 2016, Labour Workshop, CEP, LSE, May 3, 2016, and Asian Meeting of the Econometric Society, Hong Kong, June 5, 2017. We are especially grateful to Serena Rhee, Alan Manning, and Fudong Zhang for their careful and insightful comments. All errors are ours. **Version**: **June 27, 2017**

* 617-627-3294 Yannis.ioannides@tufts.edu ** 617-627-2318 jeff.zabel@tufts.edu

1 Introduction

The housing and business cycles are clearly tied together and this has become even more apparent since the Great Recession of 2007–2009. Figure 1 plots the national growth rate of real house prices, of real GDP, and the unemployment rate, 1976:1–2015:3. The house price index tracks quite well with both the real GDP growth rate and the unemployment rate; the correlation coefficients are 0.5 and -0.5, respectively. This hints that the national business and housing cycles are quite closely synchronized. In fact, Leamer (2007) has claimed that housing *is* the business cycle. He shows that, at the national level, residential investment is a much better predictor of recessions than aggregate business activity.[2] Bachmann and Cooper (2014) find using micro data, as we detail below, that housing turnover is pro-cyclical and leads the business cycle.

Starting with Oswald (1977a; b), the impact of homeownership on the labor market has received a lot of attention. He found that homeownership and unemployment rates are positively related given the relatively higher moving costs for homeowners.[3] However, as Beugnot *et al.* (2014) show theoretically, even if the unemployment and homeownership rates are positively correlated, individuals would be better off in economies where homeownership is promoted, and the costs from higher homeownership rates, if any, are principally associated with mobility costs, which are higher for homeowners. Karahan and Rhee (2014) link declines in house prices to geographical reallocation in the labor market by modeling the down payment requirement for purchasing a home as a financial friction. House price declines reduce homeowners' equity, impeding selling and moving when a local labor market is hit by a shock, thus generating a "house-lock effect" that may cause an increase in local unemployment and thus exacerbate further the local contraction. Their model accounts for 90% of the increase in dispersion of unemployment and the entire decline in net migration. Finally, Mian and Sufi (2014) and Mian, Sufi, and Trebbi (2014) have tied housing wealth to changes in the real economy.

In this paper, we provide a different link between the housing and labor markets by means

[2]Ghent and Owyang (2010) show, however, that this same relationship is not as strong at the MSA level.

[3]Also see Coulson and Fisher (2009) among others.

of a DMP model that treats housing and labor supply as joint decisions and highlights the interdependence of vacancies in these markets. Vacancy rates in both the housing and labor markets emerge naturally from search models. The link between them emanates from the use of the same utility function in analyzing bargaining between owners and renters in housing markets and between those same individuals and prospective employers.

The vacancy rate is a long-established concept in the labor market and is central to the study of frictions in this market as established under the DMP framework. Once created, job openings may remain unfilled until suitable workers are found. Vacancies of dwelling units in the housing market are also rigorously grounded in a search model. Housing units may remain unoccupied until buyers or renters are found. See Wheaton (1990) and Ngai and Sheedy (2013) for the ownership market and an earlier literature by Arnott (1989) and Igarashi (1991) for the rental market. Studies of housing market adjustment through search have been long-standing (Ioannides, 1975; Genovese and Han, 2012).

An important feature of the housing market is the coexistence of tenure modes, renting and owning, which results from households' choices of renting versus owner-occupancy. Dwelling units in both the rental and ownership markets may be vacant. The housing and labor markets have not been studied jointly by means of consistent theoretical and empirical models that utilize vacancy data and that take both renting and owning into account.

An important contribution of the DMP framework is a rigorous foundation for the Beveridge Curve. Several recent papers have made the Beveridge Curvecentral to business cycle analyses of labor markets.[4] Our theoretical model develops the counterpart of the Beveridge Curvein the housing market. While vacant units in housing markets naturally correspond to job vacancies in the labor market, the concept of unemployment is difficult to translate in the housing market. Our proposed solution is motivated by two sources of information on the cyclical dependence of housing turnover, namely the work by Bachmann and Cooper (2014) and the evidence on the correlation between residential moves and job changes. We posit that frictions affecting renters generate an "unfulfilled" demand for owner occupied housing (just

[4]See Diamond and Şahin (2014) for a discussion of the significance of shifts in the Beveridge Curveand Ellsby et al. (2015) for the latest survey of the literature.

as unemployment is the unfulfilled demand for employment). That is, some renters would rather own, given fundamentals, but are rationed out because they cannot get a mortgage or for other reasons [Henderson and Ioannides (1986)]. A similar concept holds for owners who would rather rent. For them to rent, they have to deal with frictions associated with selling a home and moving. We find that the cyclical movement along this housing market Beveridge Curveis in the opposite direction as that in the labor market Beveridge curve. We believe that we are the first to develop a housing market counterpart of the Beveridge curve. Our model of housing and labor market vacancies originates in viewing housing and employment as joint decisions, and thus explicitly captures the interdependence of the two markets via the joint setting of tightness and wages, as we elaborate in detail further below. Empirically, vacancies in the housing market can shift the labor market Beveridge Curveand vice versa.[5]

Our estimates are based on data obtained from: the US Census Bureau's Housing Vacancy Survey (HVS),[6] the national version of the American Housing Survey (NAHS),[7] the Help-Wanted Index from the Conference Board,[8] and BLS's Job Opening and Labor Turnover Survey (JOLTS).[9] We estimate models at the national level as well as for 37 US Core-Based Statistical Areas (CBSAs). Annual national-level data on housing vacancies are available starting in 1950 and MSA-level data are measured as far back as 1986. Monthly data on job vacancies begin in 1951. Following Barnichon (2010), we combine the early print index with the recent online index to construct a consistent index of job openings for 1951-2014 at both the national and CBSA level.

At the national level, we show that housing market vacancies shift the labor market

[5]We recognize that according to Bachmann and Cooper, total housing turnover is positively but weakly correlated with and leads the rental vacancy rate, while it is positively but weakly correlated with and lags the owner vacancy rate. However, those calculations are based on HP-filter detrended data. Detrending is of course critical for understanding the cyclical patterns but interdependence in the raw data is of interest in its own right, especially when we allow for geographic detail in the data.

[6]`http://www.census.gov/housing/hvs/data/index.html`

[7]`http://www.census.gov/programs-surveys/ahs/data.html`

[8]`https://www.conference-board.org/data/`

[9]`http://www.bls.gov/jlt/data.htm`

Beveridge Curve. We estimate "unfulfilled" homeownership and rental rates using a housing tenure choice equation that is estimated with multiple waves of the NAHS. We then use these predictions to estimate the housing market versions of the Beveridge Curve. Our results are consistent with the theoretical prediction that the cyclical movement along this Curveis opposite to that of the labor market Beveridge curve.

We complete the empirical analysis by using the data at the CBSA level to estimate a VAR model of housing and labor market vacancies. We use the results to calculate impulse response functions in order to study how shocks to either the housing or labor market will propagate themselves in the other market. The results show that shocks to the owner and rental vacancies have negative and significant impacts on job vacancies. This is consistent with the notion that during the Great Recession of 2007-2009 it was the downturn in the housing market that led to the subsequent decline in the labor market and the real economy.

The remainder of the paper is organized as follows. Section 2 provides a review of the recent literature that employs search models in the empirical study of housing markets. Section 3 discusses important aspects of the theoretical model, emphasizing those that capture the interdependence of the housing and labor markets with frictions. The full development of our model is relegated to Appendix A (section A). Section 4 describes the data, section 5 presents the results, and section 6 concludes.

2 Literature Review

A number of papers in the literature employ search models in the empirical study of housing markets, though very few among them examine both the housing and labor market by means of the full complement of ideas proposed here. Both Head and Lloyd-Ellis (2012; 2014) and Rupert and Wasmer (2012) develop models of joint housing-labor search, which are complementary to one another.

Rupert and Wasmer (2012) develop a theory of the relationship between unemployment and housing market frictions that focuses on the trade-off between commuting time and location decisions within a single labor market. Rupert and Wasmer show that a higher arrival rate of housing opportunities makes workers less choosy about jobs. In a variation of the model, workers receive demographic ("family") shocks, which change the valuation of the current dwelling and prompts them to sample from the existing stock of job vacancies, as opposed to just new vacancies when their jobs break up. Job separations now reflect the possibility that workers may not find an acceptable offer (vacancy), and the distribution of commuting distances occupied by workers is suitably adjusted [*ibid.,* Eq. (20)].[10] With job and housing vacancy searches being jointly indexed by commuting distance, the housing search process is subsumed into the job search. The housing market is not explicitly modeled, however, and the spatial distribution of new and existing vacancies plays the role of housing supply, but demand is not rationed by housing price. In a notable recent study Limnios (2014) explores whether frictions in the rental housing market can help explain frictions in the labor market.

Unlike Rupert and Wasmer, Head and Lloyd-Ellis (2012) focus on frictions in the housing market and the role of housing markets in generating frictions between labor markets. They do not, however, allow for frictions originating in the labor market, which they assume to be Walrasian. Head and Lloyd-Ellis do distinguish between homeownership and renting, with Bellman equations being defined separately for employed and unemployed renters and owners and are conditional on two different city types. The housing market is intermediated by real estate firms. Head and Lloyd-Ellis rely on the steady-state equilibrium values of the Bellman equations to establish that the rent differential across the two city types is determined by unemployed renters who are assumed to move costlessly between cities, even if they do not receive a job offer. In contrast, the differential in the value of houses is determined by the marginal (unemployed) home-owner who must first receive an outside offer and then incur

[10]In equilibrium, the distribution of workers' commuting distances is a linear combination of the distribution function of new vacancies, weighted by the rate at which new job opportunities arrive, of the distribution function of all vacancies, weighted by the rate at which demographic shocks arrive, and of the distribution of job offers over commuting distances, conditional on their coming from acceptable commuting distances, weighted by the rate of total separations. It is thus clear that labor turnover and frictions, including demographic shocks, have profound effects on individuals' location choices.

the (endogenous) liquidity cost of selling their house. This result suggests that anchoring the opportunity cost of homeownership calculations on rent differentials must account for basic characteristics of labor turnover across different city types. A key friction modelled by Head and Lloyd-Ellis pertains to the illiquidity of housing for homeownership. Because homeowners accept job offers from other cities at a lower rate than do renters, a link is generated between homeownership and unemployment both at the city level and in the aggregate. Their calibration of the model in order to match aggregate US statistics on mobility, housing, and labor flows predicts that the effect of homeownership on aggregate unemployment is small. When unemployment is high, however, changes in the rate of homeownership can have economically significant effects.

In a sequence of papers, Ngai and Sheedy (2013; 2015) focus on the frictions associated with buying and selling homes. Ngai and Sheedy (2015) emphasize, in particular, the dynamic impact of the fact that the majority of housing purchase transactions involve households moving from one house to another, whereby they put their existing homes on the market and plan to buy new homes. This is motivated by households' desire to improve match quality, and consequently their decisions produce a cleansing effect on the quality distribution. Moving may be triggered by an event, like a demographic shock to a household that causes a reassessment of its housing demand. Ngai and Sheedy (2013) emphasize sellers' decisions, namely when to put a house up for sale and when to agree to a sale. They do not take a position on the interdependence between residential moves and job changes.

Particularly relevant for our paper are important facts reported by Bachmann and Cooper (2014), who use data from the 1969-2009 waves of the Panel Study of Income Dynamics (PSID). They report evidence on households' propensity to move and tenure choices and how such decisions correlate with aggregate economic activity.[11] For example, 15.3% of

[11]Pissarides (2013) wonders whether the recent housing market crash is an appealing explanation for the Great Recession of 2007–2009. Since homeowners are known to be less mobile than renters, the extraordinary expansion of homeownership in recent years might have contributed to the decline in residential mobility. He argues, however, that there is little evidence of a "house-lock effect", namely that falling house prices and the negative equity in many houses are factors behind the fall in mobility. Pissarides argues nonetheless that composition effects due to the shift to more homeownership could still be significant. He speculates that if

households move each year, with roughly 55% of these moves being by renters moving to new rental dwellings, 20% by owners changing homes, 10% by owners moving to rent and 15% by renters moving to own. Only a small fraction of these moves generated net additions to the stock of owners. Bachmann and Cooper also report that whereas total housing turnover is very weakly contemporaneously correlated with the unemployment rate, it is quite strongly correlated with the growth rate of GDP (detrended by means of an HP-filter). The correlation of the unemployment rate with the owner-to-owner moving rate is substantial and negative (-0.52), with the renter-to-renter moving rate is substantial and positive (0.51) and with the renter-to-owner moving rate is absolutely smaller and negative ($-.32$). So moving in order to own is negatively correlated with the unemployment rate and owner-to-owner and renter-to-owner moves are positively correlated with output growth, 0.44 and 0.59, respectively. When including leads and lags, owner-to-owner moves are contemporaneous with the business cycle, renter-to-owner and renter-to-renter moves lead it, and owner-to-renter moves are acyclical. Furthermore, turnover seems to lead house prices, especially renter-to-owner moves, which also lead aggregate economic activity. The authors speculate that households start buying houses because of good news about economic activity and about the housing sector.

Bachmann and Cooper report correlations between the specific categories of housing turnover with vacancy rates. Interestingly, the correlations with the rental vacancy rate are weak but the ones with the owner vacancy rate are much stronger. The data suggest that when there are many owner-to-owner moves, which correlates strongly with higher economic activity, the owner-occupied market has fewer vacancies. When there are many owner-to-renter moves, which correlates with lower economic activity, the owner-occupied market has a larger number of vacancies. Renter-to-owner moves are also negatively correlated with owner-occupied vacancies. Thus, the notion that lots of vacancies in the owner-occupancy market lead to more turnover activity terminating in that market is not supported by the data. Bachmann and Cooper argue that, instead, the data point to the importance of an

---

a secular decline in mobility, whatever its origin might be, is to persist, we should expect future recessions in the US to definitely impact the labor market Beveridge curve.

underlying factor, aggregate economic activity, that prompts households to move, and the resulting housing market adjustment is associated with higher house prices and lower vacancy rates. Because housing turnover is largely unrelated to vacancies in the rental market (*ibid.,* Table 4), which, in turn, are unrelated to economic activity (*ibid.,* Table 5), the data (plus correlations with a number of demographic characteristics that Bachmann and Cooper also report) suggest that in order to understand housing market dynamics better, deeper analyses of housing market adjustment are necessary, using both aggregate and demographic data. Bachmann and Cooper speculate that a larger number of vacancies do not seem to induce larger turnover, but rather, higher turnover activity leads to less vacancies. So overall, turnover in the U.S. housing market is procyclical and tends to lead the business cycle, as summarized by *ibid.,* Figure 7. This also corroborates Leamer's claim from a disaggregated perspective.

Housing search is often associated with, as well as prompted by, job change. Using data from the PSID for 1991-1993, Ioannides and Kan (1996) report that for 1974-1983, the proportion of moves combined with job changes was 6% for household heads, while the proportion of job changes was 15% per year, and that for residential moves was 15.6%. Thus, more than 40% of the movers also changed jobs, which implies a substantial correlation between moving and job change. Furthermore, nearly two-thirds of movers did so to rent, and one-third to own.[12]

A distinguishing feature of the housing market is the coexistence of tenure modes, renting and owning and the accordant household's choice of renting versus owner-occupancy, with rental and homeownership vacancy rates, and interesting dynamics, as just discussed. We propose a joint model of frictional labor and housing markets that allows for tenure modes and use it to motivate empirical analyses of both types of vacancy rates.

To the best of our knowledge, ours is the first paper to introduce a Beveridge Curvefor housing markets in a manner that is consistent with the original definition for the labor

[12]These facts agree with data from the CPS for 2004: Ioannides and Zanella (2008), Table 1, report that 17% of residential moves occur for work-related reasons and 52.7% for housing- and neighborhood- related reasons.

market.[13] We arrive at this result by extending Head and Lloyd-Ellis (2012) in order to account for frictional rental markets as well as frictional tenure choice. Furthermore, we examine the interdependence of labor and housing market vacancies by extending Head and Lloyd-Ellis (2012) to allow for frictional labor markets.

3 Model

In this section we develop a joint model of labor and housing markets with frictions. The full details of the development of a theoretical model are presented in Appendix A. Employed and unemployed owners and renters follow optimal plans, which we describe by means of Bellman equations for the respective conditional value functions. We model the supply of dwelling units, separately for the owner-occupied and rental segments of the housing market. This leads to the determination of the value of vacant housing, which reflects critically the illiquidity of housing. We solve the Bellman equations after we have established the relative numbers of agents in the four states, the probabilities that individuals may be found as employed and unemployed owners and renters. We develop fully the case of additional frictions in the labor market and and rental housing market (neither of which are allowed by Head and Lloyd-Ellis 2012) and show how to extend the model to allow for frictions leading to turnover in the homeownership market

The frictions in the rental market originate in down-payment constraints, bad credit ratings, and discrimination. A consequence of these frictions is the existence of unfulfilled renters who prefer to be owners but are unable to do so because they cannot get a mortgage. The frictions in the ownership market include moving costs and having an underwater mortgage. A consequence of these frictions is the existence of unfulfilled owners who pre-

[13] Peterson (2009) introduces a Beveridge Curvefor housing markets based on a relationship between the vacancy rate for housing and the rate of household formation, which he intends as a "long-run supply" relationship. Peterson argues that the rate of capital formation is decreasing in the housing vacancy rate because: one, the marginal cost of a new house is decreasing in the growth rate of the housing stock; and two, the probability of selling a new house is decreasing in the vacancy rate. Whereas the former assumption is counterintuitive, in view of urban congestion, the latter does agree with intuition.

fer to be renters. This owner and renter mismatch allows us to introduce a concept of "unemployment" in the housing market which we use to develop housing Beveridge curves.

A key feature of our model is the central friction that characterizes housing markets, namely renting versus owning. In a perfectly competitive economy and in the absence of uncertainty, where all assets earn the equilibrium rates of return, there should be no advantage of owning over renting for individuals with homogeneous preferences. With taste heterogeneity regarding family size and life styles, uncertain lengths of stay, heterogeneity of dwelling quality, the decentralized nature of the allocation of housing and a myriad other types of frictions (including the provision of local amenities like education), individuals must search for dwellings to rent or to own. Suppliers of rental housing services charge rents which compensate them for holding wealth in the form of rental housing stock. Suppliers of newly constructed homes for owner-occupancy (under the simplifying assumption that such dwellings comprise a market that is distinct from that for rental housing) must be compensated for their construction costs as well as for inventory holding costs incurred while waiting for prospective buyers. Because prospective tenants and owner occupants must search in order to find dwelling units, landlords and sellers of dwelling units anticipate that their units may stay vacant until rental agreements and purchase transactions may be completed.

Whereas the rental market may well be approximated as a Walrasian one, the homeownership market typically involves a prospective buyer interacting with a potential seller over the terms of exchanging a substantial asset (the typical household's most important asset). Thus, the housing market with frictions lends itself conveniently to modeling by means of the tools known as the Diamond-Mortensen-Pissarides model, DMP for short. The dynamic model of the housing market with frictions, due to Head and Lloyd-Ellis (2012), incorporates many of these desirable features. It models frictions in the DMP tradition, expressed in terms of housing vacancies in the homeownership market, which reflect the fact that renters must search before they may become homeowners, and that sellers of dwelling units have to hold their properties vacant until transactions take place.

We extend the Head and Lloyd-Ellis model in two directions: one is to account for a frictional labor market. We do so by using the same conditional value functions for individuals as the ones defined for housing decisions to structure labor market bargaining. It is this feature of our model that leads naturally to the interdependence between frictional housing and labor markets. A second extension is to allow for frictions in the rental housing market. The purpose of the extended model is to characterize job vacancy rates, on the one hand, and rental and home ownership vacancy rates, on the other, in housing and labor markets at equilibrium. Our model of frictional housing and labor markets in the DMP tradition allows us to solve for the states in which individuals may be found in the economy, that is employed and unemployed owners and renters. This, in turn, allows us to solve for wage rates for renters and owners in terms of job, rental housing and home ownership vacancy rates, and finally to characterize the steady state equilibrium. The result is spillovers between housing and labor markets; housing vacancy rates are affected by job vacancy rates and vice versa. This in turn justifies our empirical investigation of vacancy rates in the housing and labor markets.

3.1 Definitions and Notation

We introduce notation and follow it with the basic ingredients of the model:

- The population consists of identical individuals and is given, $N : N(t) = N(0)e^{\nu t}$, where ν is the population growth rate.

- The housing stocks are endogenous, with total and per person owner-occupancy stock, H and $h = \frac{H}{N}$, and total and per person rental stock, R and $r = \frac{R}{N}$.

- The numbers of individuals found in the four different states are denoted by N^{WR}, N^{UR}, N^{WH}, and N^{UH}, where superscripts W and U denote employed and unemployed, and R and H denote renter and homeowner; they may be alternatively expressed as the probabilities that agents may be found in the respective states, n^{WR}, n^{UR}, n^{WH}, and n^{UH}, where

$$n^{WR} + n^{UR} + n^{WH} + n^{UH} = 1. \tag{3.1}$$

- The stocks of vacant units in the owner and rental segments of the housing market, denoted by υ^H and υ^R, are endogenous. The homeownership vacancy rate is

$$\text{vown} = \frac{\upsilon^H}{H} = \frac{H - N^{WH} - N^{UH}}{H} = 1 - \frac{1}{h}\left(n^{WH} + n^{UH}\right). \tag{3.2}$$

The rental vacancy rate is

$$\text{vrent} = \frac{\upsilon^R}{R} = \frac{R - N^{WR} - N^{UR}}{R} = 1 - \frac{1}{r}(n^{WR} + n^{UR}). \tag{3.3}$$

- The unemployment rate, u, is endogenous; the stock of unemployed is uN.

- The employment rate, μ, is also endogenous; the stock of employed is μN.

- The job vacancy rate, υ, is endogenous; the stock of job vacancies is υN.[14]

- Labor market tightness, θ, the ratio of the job vacancy rate to the unemployment rate, defined as $\theta = \frac{\upsilon}{u}$.

- Homeownership market tightness,ϕ^H, is defined as the ratio of the number of prospective homeowners (which is initially assumed to be all renters), to the number of vacant units in the homeownership market per person, $\phi^H = \frac{n^{WR}+n^{UR}}{h-n^{WH}-n^{UH}}$

- $\bar{\lambda}^H$, the exogenous arrival rate of vacant dwelling units per prospective buyer.

- $\bar{\lambda}^R$, the exogenous arrival rate of vacant dwelling units per prospective renter.

- γ^H, the contact rate, the rate per unit of time at which prospective buyers arrive per vacant homeownership unit, is defined as the product $\bar{\lambda}^H$ and ϕ^H :

$$\gamma^H = \bar{\lambda}^H \phi^H = \bar{\lambda}^H \frac{n^{WR} + n^{UR}}{h - n^{WH} - n^{UH}}. \tag{3.4}$$

Unlike in the canonical DMP model, the matching rate $\bar{\lambda}^H$ is exogenous, but the contact rate for units with prospective buyers is endogenous because homeownership market tightness is endogenous.

- δ, the exogenous rate at which jobs break up.

[14]For simplicity, the number of jobs is defined as equal to the number of individuals.

- The unit value of vacant rental housing is equal to the unit rental housing supply cost, which is assumed to increase with rental housing per person.

- The unit value of the vacant homeownership housing stock is equal to the unit owner-occupied housing supply cost, which is assumed to increase with the owner-occupied housing stock per person.

- The job matching function in the labor market is defined as $M = \mathcal{M}(uN, \upsilon N)$, and assumed to be homogeneous of degree 1 in its arguments.

Expressing the flows of individuals across the four states yields three equations, which together with Eq. (3.1) comprise the flow equations that may be used to solve for the four state probabilities, n^{WR}, n^{UR}, n^{WH}, and n^{UH} in terms of μ, the employment rate, an endogenous variable that is derived to be as an increasing function of labor market tightness, θ, $\mu(\theta), \mu' > 0$, and the job matching function in the labor market. That is: $\mu(\theta) = \mathcal{M}(uN, \upsilon N)/uN$. The state probabilities also depend on γ^H, the contact rate between vacant dwelling units and prospective buyers, δ, the rate at which jobs break up, and ν, the population growth rate. The associated homeownership rate at the steady state is equal to $\frac{\lambda^H}{\lambda^H+\nu}$, and thus the homeownership market tightness, ϕ^H, is equal to $\frac{\nu}{(\lambda^H+\nu)h-\lambda^H}$, a decreasing function of the owner-occupied housing stock per person. The associated unemployment rate is equal to $\frac{\delta+\nu}{\delta+\nu+\mu}$. Derivations for these expressions are given in Appendix A.

In Appendix A, we present the standard DMP machinery, that is, the Bellman equations for renters and owners, and use their solutions to characterize the determination of wages and job vacancy rates via suitably defined bargaining models. The model predicts different wage rates for owners and renters. When individuals transition from renting to owning, their bargaining position in the labor market changes. This originates in the logic of firms' bargaining with workers over the division of the surplus, where both parties are aware that renters' lifetime utility reflects the prospect that they may become owners. Similarly, our extension of the Head and Lloyd-Ellis model that accounts for owner-to-renter transitions makes owners' lifetime utility reflect the prospect that they may, at some point, become renters.

If after becoming owners individuals remain owners forever, the *wage curve for owners*, that is the DMP counterpart of the labor supply curve for owners, expresses the wage rate for owners as a convex combination of unemployment compensation and the benefit to the firm from making a hire. The latter benefit is an increasing function of the labor market tightness for owners, and is equal to the marginal revenue plus the savings in hiring costs to the firm.

The *wage curve for renters*, that is the DMP counterpart to the labor supply curve for renters, also expresses the wage rate for renters as a convex combination of unemployment compensation and the benefit to the firm from making a hire, plus a second but negative component that adjusts for the spillover effect due to the fact that renting is associated with the prospect of becoming a homeowner. It is for this reason that the wage curve for renters also depends on the labor market tightness for owners. That is, other things being equal, the prospect from becoming an owner confers an indirect benefit from renting. It also depends on the housing market tightness in the homeownership market, as this determines the matching rate of renters with prospective sellers and therefore the prospect of the transition from renting to owning.

The demand by firms for labor is expressed through the *job creation condition*, defined for the average worker, who may be either a renter or an owner, and who are perfect substitutes in production. This is the DMP model counterpart for the demand for labor. The job creation condition equates the expected wage rate plus the capitalized value of the firm's hiring costs, which are foregone once a person is hired, to the marginal revenue of an additional worker.

Working with the job creation condition and the wage curves as a system of simultaneous equations allows us to express wage rates and labor market tightness for owners and renters, as functions of the housing market tightness conditions. These solutions imply job vacancy and unemployment rates for owners and renters, which also depend on housing market tightness. The equilibrium rent readily follows from the rental housing supply equation, because the proportion of renters is equal to $\frac{\nu}{\lambda+\nu}$, and in the Head and LLoyd-Ellis model, the rental housing market is not frictional, and thus the number of rental vacancies is equal to zero. Once wage rates and labor market tightness have been solved for renters and owners, they are expressed as functions of housing market tightness. So are the conditional value functions, and thus via them the value of home owner vacancies. Therefore, the supply equation for owner-occupied housing determines the equilibrium value of the stock of owner occupied units. In other words, equilibrium is determined from asset equilibrium. The solution for the homeownership vacancy rate follows once the owner-occupied housing stock is determined. Thus, rental and owner vacancy rates are jointly determined with job vacancy and unemployment rates. The solutions establish the presence of spillovers from the housing market to the labor market and vice versa. Such spillovers are not present in the original Head and Lloyd-Ellis model. They emanate in our model from the assumption of a frictional labor market.

3.2 Two Extensions of the Head and Lloyd-Ellis Model

We go beyond Head and Lloyd-Ellis in two additional directions. First, we introduce symmetry to the two segments of the housing market by adding frictions in the rental market. Newly produced rental housing stock is assumed to enter the market as vacant, and its value must earn the equilibrium rate of return. Consistent with the homeownership market, its supply price depends on the total rental housing stock as a proportion of the population. The demand-side is expressed via the value of vacant rental housing which is in turn determined by Bellman equations for occupied and vacant rental housing units. These equations depend on the tightness in the rental and homeownership markets. Rental housing units vacated by renters who become owners must be matched with new prospective renters. Therefore, equilibrium conditions in the two segments of the housing market become interdependent, although the dwelling units are distinct and individuals are flowing through them as their mode of housing tenure changes. The equilibrium quantities of the two types of housing stock are jointly determined and so are their respective vacancy rates. Thus, given an exogenous population growth rate, the housing market adjusts to accommodate individual housing needs, while both segments of the housing market and the labor market are frictional and characterized by non-zero rental, owner, and job vacancy rates and unemployment rates.

A second direction in which we extend the model of frictions in the housing market is by allowing for agents' being unable to fulfill their desired choices. That is, owners may be rationed: given their circumstances, some owners would rather be renters, but are rationed and remain owners. Similarly for renters: given their circumstances, some renters would rather be owners but are rationed and remain renters. Both types of rationing express the joint impact of financial and mobility frictions.[15]

Let the numbers of mismatched individuals be $N_{u,rent}$ and $N_{u,own}$, rationed (unfulfilled) renters and owners, respectively. Let the respective shares of renters who would rather own and are rationed and remain renters, and of owners who would rather rent but are rationed and remain owners, be denoted by msm^R and msm^H, respectively:

$$\text{msm}^R = \frac{N_{u,rent}}{N^{WR} + N^{UR}}, \ \text{msm}^H = \frac{N_{u,own}}{N^{WH} + N^{UH}}. \tag{3.5}$$

The introduction of rationing of renters, $0 < msm^R < 1$, but not of owners, $msm^H = 0$, does not affect the flow equations, which continue to hold with the modification that instead of $\bar{\lambda}^H$, the rate at which prospective buyers find dwelling units, we now have

$$\lambda^H = \bar{\lambda}^H(1 - \text{msm}^R).$$

The introduction of owner rationing, $0 < msm^H < 1$, does affect the flow equations quite extensively, as we detail in Appendix A, sections A.2.4 and A.2.5. The rate at which prospective owners find rental dwelling units may now be written as

$$\lambda^R = \bar{\lambda}^R(1 - \text{msm}^H).$$

Allowing for owner rationing in the form of unfulfilled renters, msm^H, constitutes a more significant modification of the model than allowing for renter rationing because renting is no longer a transient state, with renters seeking to become owners at the first opportunity. There are now transitions of owners, both unemployed and employed, into renters, just as there are transition of renters, both unemployed and employed, into owners. While the model continues to be tractable, the homeownership rate in the long run is now less than one, if population growth is zero, which removes an awkward feature of our version of the Head and Lloyd-Ellis model. In the original Head and Lloyd-Ellis model owning is a terminal state.

[15]See Henderson and Ioannides (1986) for an estimation of tenure choice under this type of rationing.

3.3 Labor and Housing Market Beveridge Curves

The Beveridge Curve for labor markets, the job vacancy rate as a function of the unemployment rate, is a well-established and widely researched concept. Its derivation is straightforward.[16] At the steady state with a constant labor force and in either tenure mode, rent and own, we have that:

$$u = \frac{\delta}{\delta + \mu(\frac{v}{u})}, \tag{3.6}$$

where δ is the rate of job destruction and μ is the rate at which the unemployed become employed. Since standard assumptions ensure (as we see further below) that μ is increasing in labor market tightness, $\theta = \frac{v}{u}$, it is easy to show that the unemployment rate is decreasing in the job vacancy rate.

The similarities between the housing and labor markets allow us to develop a Beveridge Curvefor housing. Analogous to vacancies in the labor market, which is unsatisfied demand for workers by firms, there correspond vacancies in the homeownership and rental housing markets, which is unsatisfied demand for buyers by sellers and for renters by landlords. Analogous to unemployed individuals, which is unsatisfied demand for employment by individuals, there are unsatisfied renters who wish to own, and owners who wish to rent. They are prevented from doing so by frictions. Our development of Beveridge Curves for housing markets is adapted to the institutional features of housing markets, where there are owners and renters and is derived, just as the Beveridge Curve in labor markets, as an accounting relationship in the steady state.

We work first with the homeownership market; the vacancy rate, vown, is given by (3.2). We next express it in terms of a quantity that serves as the unemployment counterpart in homeownership market. We allow for mismatch among renters giving rise to unsatisfied homeownership demand. The solutions for n^{WH} and n^{UH} depend on $\bar{\lambda}^H \cdot (1 - \text{msm}^R)$ instead of just λ and thus on the incidence of mismatch. Working with the solution for the homeownership rate, hr (see Appendix A, Eq. (**??**)), and assuming that there is no unsatisfied rental demand, ($\text{msm}^H = 0$), we have that the equilibrium homeownership rate,

[16] Pissarides (1986) is the first joint empirical model of unemployment and job vacancies.

hr, can be expressed as:

$$\text{hr} = n^{WH} + n^{UH} = \frac{\bar{\lambda}^{H}(1 - \text{msm}^{R})}{\bar{\lambda}^{H}(1 - \text{msm}^{R}) + \nu}. \tag{3.7}$$

The equilibrium homeownership rate decreases with the probability of rationing. That is, an increase in msm^R, due to rationed renters, and hence a decrease in the number of individuals searching to buy homes reduces the homeownership rate.

In developing the Beveridge Curve for the homeownership market, we propose the concept of the unfulfilled homeownership rate as the counterpart to the unemployment rate and normalize it appropriately. Consider first the auxiliary quantity

$$\text{uhr} = \frac{N_{u,rent}}{N_{u,rent} + N^{WH} + N^{UH}},$$

the ratio of $N_{u,rent}$, the number of renters who prefer to own but are rationed and remain renters, a quantity that we impute based on a tenure choice estimation, to the number of all participants in the ownership market. This quantity is at most equal to the rental rate, when all renters are unfilled owners, and therefore normalizing it by the actual rental rate yields the unfulfilled homeownership rate,

$$\text{ur}^{H} = \frac{\text{uhr}}{n^{WR} + n^{UR}}. \tag{3.8}$$

This serves as our analog to the unemployment rate for the ownership market: the ownership unemployment rate. It ranges between 0, if no renters would rather own, and 1, if all renters wish to become owners, which Head and Lloyd-Ellis assume, but are rationed.

By solving for $N_{u,rent}$ using Eq. (3.5) and substituting into the definition of uhr we may express uhr in terms of the n's, the probabilities that agents may be found in different states, and msm^R (see Appendix A for the exact derivation). Then by solving the flow equations (see Appendix A) and using Eq. (3.2) yields the analog of the Beveridge Curve for the homeownership market:

$$\text{vown} = 1 - \frac{1}{h} + \frac{1}{h}\frac{\nu}{\lambda^{H} + \nu}\frac{1}{\text{ur}^{H}}. \tag{3.9}$$

The Beveridge Curve for the homeownership market expresses the homeownership vacancy rate as a decreasing function of ur^H, the homeownership unemployment rate. It is thus similar to the Beveridge Curve for the labor market. [17] In Eq. (3.9), the owner-occupied housing stock per capita, h, is endogenous, which may cause the Beveridge Curve to shift and tilt due to the cyclical variation in h. It is also clear from Eq. (3.9) that an increase in the unemployment rate μ, as during an downswing in the business cycle, results in a downward shift in the Beveridge Curvefor the homeownership market.

Turning to the rental market, we introduce the concept of the unfulfilled rental rate as the analog to the unemployment rate for the rental market. We start with the definition of the auxiliary quantity

$$\mathrm{urr} = \frac{N_{u,own}}{N_{u,own} + N^{WR} + N^{UR}},$$

which is defined as the ratio of $N_{u,own}$, the number of owners who prefer to rent but are rationed and remain owners, a quantity that we impute based on the same tenure choice estimation as the one used for renters, to the number of all participants in the rental housing market. This quantity may be at most equal to the homeownership rate, if all owners wish to be renters. Normalizing it by the homeownership rate yields the unfulfilled rental rate,

$$\mathrm{ur}^R = \frac{\mathrm{urr}}{n^{WH} + n^{UH}}. \tag{3.10}$$

Eq. (3.10) serves as our analog to the unemployment rate for the rental market, the rental unemployment rate. ur^R ranges between 0, the assumption made by Head and Lloyd-Ellis, and 1, which would mean that all owners wish to become renters but are rationed.

Using the definition of the rental vacancy rate from Eq. (3.3) and by substituting in for urr we obtain an equation for our analog of the Beveridge Curvefor the rental housing market:

$$\mathrm{vrent} = 1 - \frac{1}{r} + \frac{1}{r}(n^{WH} + n^{UH}) = 1 - \frac{1}{r} + \frac{1}{r}\frac{\mathrm{urr}}{\mathrm{ur}^R}. \tag{3.11}$$

In principle, this may be simplified further but in general both msm^R and msm^H enter the expression for urr. Since renting and owning are interdependent, in the most general case, it is not surprising that the vacancy rates share parameters. The rental vacancy rate depends on r, the rental housing stock per person, which is endogenous and varies procyclically, thus shifting and tilting the rental Beveridge curve.

[17] The expression in Eq. (3.9) is modified if $\mathrm{msm}^H \neq 0$, but its property with respect to ur^H is not affected.

3.4 Linking Housing and Labor Market Vacancies

Our theoretical model suggests that wage bargaining in the labor market leads to different wage rates for renters and owners, $w^H \neq w^R$, but the same equilibrium labor market tightness variable, θ, which determines the unemployment rate and the job vacancy rates at the steady state. This implies a solution for the job vacancy rate. With the wage and employment rates having been determined, the conditional value functions for renters and owners are solved for, which in turn determine the per capita rental and owner-occupied housing stocks, r and h. Finally, the vacancy rates, vown and vrent, defined in Eq. (3.2) and Eq. (3.3), respectively, follow. The detailed derivations are given in Section A.7, Appendix A.

As we detail in Appendix A, section A.7.1, superimposing labor market equilibrium, defined by the job creation condition and the wage curves for owners and renters, Eqns (A.58) and (A.56), respectively, with housing market equilibrium, defined in terms of vacancy rates for the ownership and rental markets, implies that labor market tightness depends on the housing rental and ownership vacancy rates. Thus, the relationship between the job vacancy rate and the unemployment rate is affected by the rental and homeownership rates. It is such an augmented Beveridge Curve that we take to the data; see section 5.1 below.

Not surprisingly, the wage curve for renters does depend on housing market variables: renters become homeowners at the first opportunity. Forward-looking agents anticipate this prospect. This feature implies a spillover effect from the homeownership market to wage setting for renters. An increase in θ, labor market tightness, increases the employment rate for both homeowners and renters, and shifts upwards the wage curves of owners and renters, and therefore the expected wage rate, as well, *cet. par.* This, in turn, shifts the labor market Beveridge Curve upwards, exactly as it was observed during the downturn associated with the Great Recession of 2007-2009 in the US. Therein lies the power of the Beveridge curve tool: it allows us to track structural shifts in the overall economy.

Conceptually, the augmented labor market Beveridge Curve is obtained implicitly from the job creation condition, Eq. (A.59), by expressing the job vacancy rate v in terms of θ. The job creation condition equates the expected wage, which is expressed in terms of the probabilities that a worker is an owner or a renter, to the benefit to the firm from hiring an additional worker. Labor market tightness enters both sides of that equation and is thus a function of both housing vacancy rates, vown and vrent. Unlike in the canonical case of frictional labor markets where one may track the economy's movement on (u, υ) space via changes in θ, here θ depends on the housing vacancy rates via the expected wage rate, which expresses spillovers from the housing to the labor market. Other things being equal, an increase in the ownership (rental) vacancy rate reduces the expected wage rate, produces a downward shift of the wage curve, and thus a negative spillover on job vacancy rate. However, the endogeneity of the per capita housing stock for renters and for owners add complications, and it is for this reason that cannot obtain precise prediction for the spillover effect on the labor market Beveridge Curve from the housing vacancy rates.

The key economic intuition of our entire approach is that the value of housing to owner-occupants and renters defines the demand for the respective type of housing stock, as functions of the respective per capita housing stocks, which when equated to the respective supply functions determine the per capita housing stocks. Thus, rental and homeownership vacancy rates are simultaneously determined and both reflect labor and housing market magnitudes. This justifies the empirical specification of VAR models of vacancy rates in section 5.4 below.

There are, of course, numerous ways in which the model can be extended, in addition to developing fully the case of turnover by owners and its implications for wage determination, unemployment and labor and housing vacancy rates. A particularly interesting feature that is worth exploring is to allow for correlation between residential moves and job changes. As discussed in section 2 above, more than one-third of moves are also associated with a job change.

4 Data

Annual data at the national level on homeownership and rental vacancies is available from the Census Bureau starting in 1950. Data on housing vacancies at the MSA level come from the Census Bureau's Housing Vacancy Survey (HVS). The HVS is a regular part of the Current Population Survey (CPS). Units that are found to be vacant or were otherwise not interviewed are included in the HVS.[18] These data are available from 1986-present on an annual basis for the largest 75 MSAs (though there are less than 75 MSAs in the early years). These data are somewhat problematic since MSA definitions change over time.

Data on monthly job vacancies starting in 1951 come from the Help-Wanted Index for the fifty largest MSAs; these are an aggregate of ads carried by the press that is provided by the Conference Board.[19] However, it is known that since the mid- to late-1990s, this "print"-based measure of vacancy postings has become increasingly unrepresentative as advertising over the internet has become more prevalent. Figure 2 plots the National print Help-Wanted Index starting in 1977 (note that it coincides with the composite index until 1994). One can see the drop-off around 2000. Barnichon (2010) builds a vacancy posting index that captures the behavior of total"print" and "online"-help-wanted advertising, by combining the print with the online Help-Wanted Index published by the Conference Board since 2005. Figure 2 includes our version of the combined index. It closely replicates Barnichon's index which goes through 2009 and extends it through June 2014. The details of our computations are given in Appendix C.

Figure 3 plots the job vacancy data (the composite Help-Wanted Index divided by the

[18] The definition of a vacant housing unit as given by the Census Bureau is "A housing unit is vacant if no one is living in it at the time of the interview, unless its occupants are only temporarily absent. In addition, a vacant unit may be one which is entirely occupied by persons who have a usual residence elsewhere. New units not yet occupied are classified as vacant housing units if construction has reached a point where all exterior windows and doors are installed and final usable floors are in place. Vacant units are excluded if they are exposed to the elements, that is, if the roof, walls, windows, or doors no longer protect the interior from the elements, or if there is positive evidence (such as a sign on the house or block) that the unit is to be demolished or is condemned. Also excluded are quarters being used entirely for nonresidential purposes, such as a store or an office, or quarters used for the storage of business supplies or inventory, machinery, or agricultural products. Vacant sleeping rooms in lodging houses, transient accommodations, barracks, and other quarters not defined as housing units are not included in the statistics in this report. A vacant unit for rent consists of "units offered for rent and those offered both for rent and sale."

[19] Pissarides (1986) for Britain, and Blanchard and Diamond (1989) for the US have used the Help-Wanted Index in studying labor market adjustment.

size of the labor force) along with homeownership and rental vacancy rates for 1956-2014. The correlation coefficients are given below the figure. There is a reasonably strong negative correlation between the job vacancy rate and both the rental and homeownership vacancy rates. This is explained by the following observations: (1) there tend to be more job vacancies when the labor market is "hot," as workers can be more selective, and hence firms find it harder to hire; (2) there are fewer rental or homeownership vacancies when the housing market is hot, as renters are motivated to enter the homeownership market (though there is more churning), and (3) the labor and housing markets tend to be hot at the same time (see Figure 1). We believe the latter fact has not been noticed before. A potential causal mechanism is that as vacancies increase in the housing market, job vacancies decrease since this opens up more residential locations and allows workers to make better job matches (given the joint residential location and job matching decision process).

We have the same job vacancy data at the MSA level for 37 MSAs. We have applied a similar procedure to create a combined Help-Wanted Index (HWI) for each of the MSAs.[20] Details about the construction of our MSA-level combined HWI are given in Appendix C. Summary statistics for the composite HWI for 1986 - 2014 are given in Table 1.

Additional data on monthly job vacancies starting in December 2000 are available from the Bureau of Labor Statistics in the Job Openings and Labor Turnover Survey (JOLTS). These data are only provided at the level of the four Census regions for total nonfarm employment as well as aggregated by a number of industrial categories.

We use the National version of the American Housing Survey (NAHS) to estimate renter's unfulfilled desire to be home owners (and vice versa). The NAHS is an unbalanced panel of more than 50,000 housing units that are interviewed every two years and contains detailed information on dwelling units and their occupants through time, including the current

[20]We have data for 51 MSAs for the HW online index (HWOL) and 49 MSAs for the print index. But Austin, Buffalo, Honolulu, Las Vegas, Orlando, Portland, Providence, San Jose, Tucson, and Virginia Beach are included in the online data but not in the print data, whereas Albany, Allentown, Dayton, Knoxville, Omaha, Syracuse, Toledo, and Tulsa are included in the print data but not in the online data. Also, Houston is missing the print index for 1996.9 to 2003.7 so it is excluded.

owner's evaluation of the unit's market value. We use the NAHS for survey years 1985-2013. Summary statistics for all the variables used in this calculation are given in Table 1.

5 Empirics

5.1 Beveridge Curve Regressions: Labor Market

Recall that the Beveridge Curve plots job vacancies versus the unemployment rate. Movement along the Beveridge curve indicates positions in the business cycle: higher unemployment and lower vacancies in periods of recession, and lower unemployment and higher vacancies in periods of expansion. Shifts in the Beveridge Curve can arise for a variety of reasons: changes in the efficiency of the job matching process, skill mismatch, changes in the labor force participation rate, and others, such as economic and policy uncertainty. See Diamond and Sahin (2014) and Pissarides (2011) for details on shifts in the US and UK Beveridge curves.

Figure 4 plots the National Beveridge Curve for 1951-2014. The job vacancy rate, vjobs, is the composite help wanted index divided by the labor force. The data are split into six episodes that are determined by peaks in the NBER business cycle data, made up of the preceding expansion and succeeding contraction. Figure 5 plots the Beveridge Curve for the most recent episode using the monthly JOLTS data. The curve begins in the upper left corner in January 2001 in a period of low unemployment and a high job vacancy rate. It then moves south east and ends up in the southeast corner at the end of the 2007–2009 contraction, in a weak period of high unemployment and low job vacancies. There appears to be an outward shift in the Beveridge Curve over the next six months followed by a northwest movement to a stronger economic position in 2014.

Our theory establishes spillovers from the wage curves for owners and renters to the labor market Beveridge Curve, which depend on conditions in the housing market.[21] As we detail

[21] This is evident in the last term on the r.h.s. of Eq. (A.59), via γ^H, the rate at which new dwelling units sold by construction firms are matched with potential buyers, and $\mu(\theta)$, the employment rate in the labor

in Appendix A, section A.7.1, the prediction for the full effect follows by substituting in the job creation condition for the wage curves for renters and owners. The wage curves for renters and owners are themselves functions of labor market tightness. As we indicate in section 3.4 above, it is unfortunate that we cannot obtain any specific prediction for the impact of the labor market Beveridge Curve on housing vacancy rates.

Denote the unemployment rate in MSA i and time t as $\text{unempl}_{i,t}$, the job vacancy rate as $\text{vjobs}_{i,t}$, the homeownership vacancy rate as $\text{vown}_{i,t}$, and the rental vacancy rate as $\text{vrent}_{i,t}$. Then the augmented Beveridge Curve is specified in logs as

$$\ln \text{vjobs}_{i,t} = \alpha_{0i} + \alpha_1 \ln \text{unempl}_{i,t} + \alpha_2 \ln \text{vown}_{i,t} + \alpha_3 \ln \text{vrent}_{i,t} + \epsilon_{i,t}. \tag{5.1}$$

The results for 1959-2014 are given in Table 2. Columns (1) and (2) present results for the model that includes the natural logs of the unemployment rate and the homeownership and rental vacancy rates as regressors. OLS results are given in column (1). The elasticity of the job vacancy rate with respect to the unemployment rate is -0.641. Both housing vacancy rates are significant. Surprisingly, given that the correlations between the job vacancy rate and the homeownership and rental vacancy rates are both positive, Figure 3 (bottom), the corresponding coefficient estimates are opposite in sign. The coefficient for the rental vacancy rate has a smaller $p-$value and an elasticity near -1. This is in line with the negative relationship between the labor and housing market vacancy rates that we predicted earlier. Given the interdependence of the vacancy rates in the housing and labor markets, the former are likely to be endogenous in this model. We instrument for the homeownership and rental vacancy rates using new housing permits and starts for 1-unit and 2-or more unit structures. These instruments pass the over-identification and weak instrument tests but the test for the exogeneity of the homeownership and rental vacancy rates is not rejected. Not surprisingly, the IV results (reported in column (2) of Table 2) are very similar to the OLS results.

Following Benati and Lubik (2014) we next include indicators of the shifts in the Beveridge Curve (see Figure 4). We allow both the intercepts and the slopes to vary, by including interaction terms. Given its significance, we also allow for different spillover effects for the

market. A larger μ causes a downward shift of the wage curve for renters, thus increasing the respective labor market tightness and causing a downward movement along the corresponding Beveridge Curve.

homeownership and rental vacancy rates for the 2001-2014 period. OLS and IV results are given in columns (3) and (4) of Table 2. Not surprisingly, accounting for these shifts significantly improves the fit of the model. Also not a surprise is the fact that this lowers the significance and magnitude of the homeownership and rental vacancy rate coefficients since it is these shifts that the housing vacancy variables help explain. In this case, the test for the exogeneity of the homeownership and vacancy rates is rejected ($p-$value $= 0.0065$) and the OLS and IV results show some large differences in estimated coefficients. Note that the IV estimates of the homeownership and rental vacancy rates coefficients are now both negative, as expected. Furthermore, the spillover effects are even larger (in magnitude) during the 2001-2014 period.

5.2 Beveridge Curve Regressions: Housing Market

Our theory aims at a symmetric treatment of the labor and housing markets over and above the presence of spillovers. In particular, we develop housing market Beveridge curves. Whereas the vacancy rate concept applies equally well to the housing and labor markets, at this point, no obvious counterpart of unemployment in the housing market has been proposed. In the labor market, the unemployment rate measures the unfulfilled desire of labor market participants to work. We posit in Section A.3.1 a counterpart concept in terms of an unfulfilled desire on the part of renters to become homeowners. Renters may be prevented from owning homes due to the inability to get a mortgage because of down payment constraints, poor credit, because of discrimination in the mortgage credit market, or because of prohibitive transaction costs relative to anticipated length of stay. Similarly, owners may be prevented from becoming renters due to frictions in selling their homes and/or difficulties in coordinating housing and job changes.

5.3 Unfulfilled Rental and Homeownership rates: Estimation via Tenure Choice

In order to calculate the unfulfilled homeownership and rental rate variables, uhr_t and urr_t, defined in (3.8) and (3.10) above, we estimate a tenure choice equation by probit, where the probability of owning is given by $\Phi(\mathcal{X}_{i,m,t}\hat{\alpha})$, and $\Phi(\cdot)$ denotes the cumulative normal distribution, and the vector $\mathcal{X}_{i,m,t}$ includes all characteristics used so far. Details of this estimation are given in Appendix B.

To each renter i, in MSA m, and time t, $\text{own}_{i,m,t} = 0$, we attribute[22] a probability of being rationed (i.e., being an unfulfilled owner) that is obtained from the tenure choice estimation discussed above and detailed in Appendix B. It follows that according to Eq. (3.8) the unfulfilled homeownership rate, "homeownership unemployment rate," is given by:

$$\text{ur}_t^H = 100 \times \frac{\frac{N_{u-rent,}}{N_{u-rent,t}+N_{own,t}}}{\frac{N_{rent,t}}{N_{rent,t}+N_{own,t}}},$$

where $N_{u-rent,t}$ sums up the predicted probabilities of owning imputed to observed renters, $N_{u-rent,t} = \sum_{i,\text{own}_{i,m,t}=0} \Phi(\mathcal{X}_{i,m,t}\hat{\alpha})$, and $N_{rent,t}, N_{own,t}$ are all self-reported renters and owners, respectively.

Working in a like manner according to Eq. (3.10), we define the unfulfilled rental rate, "rental unemployment rate," as:

$$\text{ur}_t^R = 100 \times \frac{\frac{N_{u,own,t}}{N_{u,own,t}+N_{rent,t}}}{\frac{N_{own,t}}{N_{rent,t}+N_{own,t}}},$$

where $N_{u,own,t} = \sum_{i,\text{own}_{i,m,t}=1} [1 - \Phi(\mathcal{X}_{i,m,t}\hat{\alpha})]$, and $N_{rent,t}, N_{own,t}$ all self-reported renters, owners. Note that the estimation of the tenure choice equation $\Phi(\mathcal{X}_{i,m,t}\hat{\alpha})$ is sufficient for imputing $N_{u,own,t}$, in view of the binary nature of the tenure choice here.

We estimate the share of unfulfilled homeowners and renters using the NAHS. The mean

[22] In an earlier version of the paper, we experimented with defining as unfulfilled owners, those with a predicted probability of owning $\Phi(\mathcal{X}_{i,m,t}\hat{\alpha})$ exceeding 0.5, and correspondingly for owners, that is, defining as unfulfilled renters, those with a predicted probability of renter $1 - \Phi(\mathcal{X}_{i,m,t}\hat{\alpha})$ exceeding 0.5. The results we report avoid the arbitrariness of the cutoff probability of 0.5.

of the unfulfilled demand for renting, by homeowners over the sample period is 27%. Correspondingly, the mean number of the unfulfilled demand for owning over the sample period is 42%.

The US housing Beveridge curves for 1985-2013 are plotted in Figure 6. Each curve appears to have shifted outward in the latter half of the period, 2003–2013 vs. 1985–2001. Note that both curves follow fairly similar patterns, and they are roughly distorted "negative" logistic. The homeownership Beveridge Curve is located in the southeast corner in 2005, indicating a "hot" market. It then moves to the northwest corner in 2009, indicating a cold market. There is then a movement back towards the southeast, indicating that this market has rebounded.

While visually, prices in the rental market appear to move in step with those in the housing market (see Figure 7), the correlation between the real price growth in the two markets is only 0.36 over the 1975 to 2015 period. Differences clearly emerge starting in the early 2000s. The rental market did not suffer the huge decline in prices that occurred in the housing market starting around 2005. Another difference between the two markets is that the vacancy rate is significantly higher in the rental market compared to the homeownership market: 7.4% versus 1.6% between 1956 and 2014 (see Figure 3). Still, like the Beveridge Curve in the housing market, the Beveridge Curve in the rental market is in the northwest corner in 2009. Then there was a strong movement towards the southeast, indicating market recovery (see Figure 6). We believe that this is the first time that Beveridge Curves for the housing market have been drawn. [23]

Next, we specify and estimate the augmented Beveridge Curves for the homeownership

[23] Peterson (2009) defines a long-run "Beveridge Curve" in the housing market as the rate of household formation as a decreasing function of the vacancy rate for housing, which he finds to be true for the owner-occupied market, the rental market, and the total market for housing irrespective of homeownership status. He sees this as a long-run supply condition that he explains by assuming that, one, the cost to produce new housing is decreasing in the growth rate of the housing stock, and two, the likelihood of selling a new house is decreasing in the vacancy rate. The first condition clashes with a long-held stylized fact of urban congestion; the second one is, however, consistent with the search model.

and rental markets

$$\ln vx_{i,t} = \beta_{0t} + \beta_1 \ln \text{uxr}_{i,t} + \beta_2 \ln \text{vjobs}_{i,t} + \varepsilon_{i,t}, \tag{5.2}$$

where $x = h(\text{own}), r(\text{rent})$. Results for the Beveridge curves that include the labor market vacancy rate are given in Columns (1) and (3) of Table 3.

Given that we only have 15 observations at the national level to estimate the housing market Beveridge Curves, we move to the CBSA level. We have complete data on 27 CBSAs for 1991, 1993,..., 2011 for a total of 297 observations. Results for the Beveridge Curves that are estimated with CBSA fixed effects and include the labor market vacancy rate are given in Columns (1) and (3) of Table 3. The fixed effects are significant. The coefficient estimate for $\ln \text{vjobs}_{i,t}$ is negative and significant for the ownership Beveridge Curve whereas the coefficient estimate for $\ln \text{urr}_{i,t}$ is positive but not significant for the rental Beveridge Curve. The coefficient estimate for $\ln \text{vjobs}_{i,t}$ is negative and significant (at 1%) in both cases and the elasticities are quite similar, -0.285 and -0.288, for the homeownership and rental Beveridge Curves, respectively. This indicates that when job market vacancy rates increase, the vacancy rates in the housing market decrease. This could arise since the joint decision of job and residential location is now easier given that there are more job vacancies. The results including the shifts in the homeownership and rental market Beveridge Curves are included in columns (2) and (4) in Table 3. Not surprisingly, the coefficient estimates for $\ln \text{vjobs}_{i,t}$ are no longer significant since this variable is included to account for shifts in the housing market Beveridge Curves.

5.4 VAR Models and Impulse Response Functions for Owner, Rental, and Job Vacancy Rates

Following up on the discussion in sections 3.4 and Appendix A.6 of the simultaneous determination of the rental and owner-occupancy vacancy rates, we next specify and estimate VAR models of job and homeownership and rental vacancy rates using the CBSA-level data. The purpose is to establish the interrelationship between the two markets and then to calculate how shocks in one market propagate themselves in the other markets using an impulse response function. These models are extensions of the augmented labor and housing market Beveridge curves that include lags of the explanatory and dependent variables, the CBSA house price index as an additional control variable, and CBSA fixed effects. We have data for 37 CBSAs for 1991–2012.

First, we check for unit roots in each of the time series (using xtunitroot in Stata). the jobs vacancy rate, the unemployment rate (and its inverse) and the owner-occupied house price index are found to have a unit root whereas the homeownership and rental vacancy rates do not have a unit root To be consistent, we run the VAR regressions in first differences for all the variables. Next, we test for Granger Causality. The three regressions for owner, rental, and job vacancy rates include two lags of these variables, fixed effects, and time dummies. We use the Arellano-Bond estimator since there are lagged dependent variables along with CBSA fixed effects. When we run these tests in levels the only evidence of causality is from the rental vacancy rate to the owner vacancy rate ($p-$value $= 0.017$).When we run the tests in first differences, the only (mild) evidence of causality is from the owner vacancy rate to the job vacancy rate ($p-$value $= 0.0639$).

The VAR equations are reduced form (there are no contemporaneous variables included as explanatory variables). That is:

$$\Delta \ \text{vx}_{i,t} = \alpha_{0,x} + \sum_{j=1,2} \alpha_{1,j,x} \Delta \text{vown}_{i,t-j} + \sum_{j=1,2} \alpha_{2,j,x} \Delta \text{vrent}_{i,t-j}$$

$$+ \sum_{j=1,2} \alpha_{3,j,x} \Delta \text{vjobs}_{i,t-j} + \sum_{j=1,2} \alpha_{4,j,x} \Delta \mathbf{X}_{i,t-j} + u_{t,x} + v_{i,x} + \varepsilon_{it,x}, \quad (5.3)$$

where vx = own, rent, job vacancy rates, that is, o, r, j, $\mathbf{X}_{i,t-j}$ is a vector containing the inverse of the unemployment rate and the house price index. We estimate these three equations (5.3) with two lags included. We use the Arellano-Bond estimator since there are lagged dependent variables along with CBSA fixed effects. The results are given in Table 4 below.

We then calculate responses to shocks to vjobs, vown, and vrent. We do so by adding a one standard deviation increase in each (values given in Table 1) and following the changes in $\Delta \text{vx}_{i,t}, x = o, r, j$ over time. This produces three sets of impulse response functions (IRFs);

with shocks to the first-differences in owner, rental, and job vacancy rates. Note that this means that the ordering of the variables in not necessary. The IRFs for the three cases are given in Figure 8 (upper and lower 95% confidence intervals are also presented). These are cumulative in the levels of vown, vrent, vjobs. The responses to the owner and rental vacancy rates due to a shock to job vacancies are small and not significantly different from zero. The responses to the owner vacancy rates due to a shock to rental vacancies are small and not significantly different from zero (and vice versa). But the shocks to the owner and rental vacancies do have negative and significant impacts on job vacancies (note that the upper 95% confidence interval estimates are barely above 0 in the case of rental vacancies). In the case of the shocks to owner and rental vacancies, there is a long-term negative and significant impact of about -0.15 and -0.10 standard deviation in the job vacancies variable, respectively. Both of these long-term effects are a reasonably large impact. These results support the notion that in the Great Recession it was the downturn in the housing market that resulted in the subsequent decline in the real economy.

6 Conclusions

This paper explores the interdependence between the housing and labor markets by means of a DMP-type model. The model gives rise naturally to equilibrium vacancy rates in housing and labor markets. The labor market model with frictions produces as an outcome the Beveridge Curve. We use the model to develop a Beveridge Curve for the homeownership and rental markets. We propose a housing market counterpart for the concept of unemployment, namely the unfulfilled homeownership and rental rates. Movement along the housing Beveridge Curves is opposite that of the labor market Beveridge Curve. That is, there is a movement to the southeast as the housing market (and economy) improves. Our model predicts negative spillovers from the housing market to the labor market and vice versa. In the case of the labor market, the mechanism is that an increase in housing vacancies increasing the matching efficiency as it easier for workers to move to new jobs given that it is easier to find new housing. This result implies that the increase in vacancies in the housing market results in an inward shift in the Beveridge Curve in the labor market. Despite this inward shift, the labor market Beveridge Curve shifted outward during the Great Recession of 2007–2009.

We estimate the model using data at the MSA level on housing vacancies from the US Census Bureau's Housing Vacancy Survey starting in 1986 and data on job vacancies from the Help-Wanted Index starting in 1951 and the online version starting in 2005. We first estimate a Beveridge Curve for labor markets that includes rental and homeownership vacancies as explanatory variables as predicted by the model. We find that rental vacancies have a significant negative impact on job vacancies whereas the impact of homeownership vacancies is not significant. We also estimate Beveridge curves for the homeownership and rental markets. We use the 1985-2013 waves of the National AHS to calculate our counterpart measure of "unemployment" in the housing market, the unfulfilled desire by renters to own and by owners to rent. This results in 15 observations so the results are only illustrative. Still, we find that labor market vacancies have a negative and significant impact on both homeownership and rental vacancies with an elasticity of around -0.4 in both cases. Again, the mechanism is that given that many residential moves are joint with job decisions, more job vacancies mean that households are able to better match their housing needs given the greater availability of job openings.

The results from the VAR models for labor market and housing vacancies are used to study how shocks to either the housing or labor markets will propagate themselves in the other market. We find evidence that spillovers from the rental and ownership markets affect labor market vacancies and not vice versa. This is consistent with the notion that the Great Recession of 2007-2009 started in the housing market and spilled over into the real economy.

7 References

Arnott, Richard. 1989. “Housing Vacancies, Thin Markets, and Idiosyncratic Tastes.” *Journal of Real Estate Finance and Economics*. 2:5–30.

Bachmann, Rüdiger and Daniel Cooper. 2014. “The Ins and Arounds in the U.S. Housing Market.” Federal Reserve Bank of Boston working paper No. 14-3. July.

Barnichon, Regis. 2010. “Building a composite Help-Wanted Index.” *Economics Letters* 109:175-178.

Benati, Luca, and Thomas A. Lubik. 2014. “The Time-Varying Beveridge Curve.” In: Schleer-van Gellecon, Frauke. *Advances in Non-linear Economic Modeling.* Berlin and Heidelberg: Springer-Verlag.

Beugnot, Julie, Guy Lacroix, and Olivier Charlot. 2014. “Homeownership and Labour Market Outcomes: Micro versus Macro Performances.” IZA DP No. 8599. October.

Blanchard, Olivier J., and Peter A. Diamond. 1989. “The Beveridge Curve.” *Brookings Papers on Economic Activity.*

Coulson, N. Edward and Lynn Fisher. 2009. “Housing Tenure and Labor Market Impacts: The Search Goes On.” *Journal of Urban Economics* 69: 252-264.

Diamond, Peter A., and Ayşegül Şahin. 2014. “Shifts in the Beveridge Curve.” Staff Report No. 68. Federal Reserve Bank of New York.

Daz, Antonia, and Belén Jerez. 2013. ”House Prices, Sales, and Time on the Market: A Search Theoretic Framework.” *International Economic Review* 54 (3): 837-72.

Elsby, Michael W. L., Ryan Michaels, and David Ratner. 2015. “The Beveridge Curve: A Survey.” *Journal of Economic Literature*. 53(3):571–630.

Genovese, and L. Han. 2012. “Search and Matching in the Market for Existing Homes.” *Journal of Urban Economics* 72: 31-45.

Ghent, Andra, and Michael Owyang. 2010. "Is Housing the Business Cycle? Evidence from US Cities." *Journal of Urban Economics* 67: 336-351.

Glaeser, Edward L., Joseph Gyourko, Eduardo Morales, and Charles G. Nathanson. 2014. "Housing Dynamics: An Urban Approach." *Journal of Urban Economics* 81:45–56.

Head, Allen, and Huw Lloyd-Ellis. 2012. "Housing Liquidity, Mobility, and the Labour Market." *Review of Economic Studies.* 79:1559–1589.

Head, Allen, Huw Lloyd-Ellis, and H. Sun. 2014. "Search, Liquidity, and the Dynamics of House Prices and Construction." *American Economic Review.* 104(4):1172–1210.

Henderson, J. Vernon, and Yannis M. Ioannides. 1986. "Tenure Choice and the Demand for Housing." *Economica.* 53(210):231–246.

Igarashi, Masahiro. 1991. "The Rent-Vacancy Relationship in the Rental Housing Market." *Journal of Housing Economics.* 1: 251–270.

Ioannides, Yannis M. 1975. "Market Allocation through Search: Equilibrium Adjustment and Price Dispersion." *Journal of Economic Theory.* 11:247–262.

Ioannides, Yannis M. 1987. "Housing Tenure Choice and Residential Mobility." *Regional Science and Urban Economics.* 17:265-287.

Ioannides, Yannis M., and Kamhon Kan. 1996. "Structural Estimation of Residential Mobility and Housing Tenure Choice." *Journal of Regional Science.* 36(3):335-363.

Ioannides, Yannis M., and Giulio Zanella. 2008. "Searching for the Best Neighborhood: Mobility and Social Interactions."

http://www.tufts.edu/˜yioannid/IoannidesZanellaMobility08.pdf

Karahan, Y. Fatih, and Serena Rhee. 2014. "Geographical Reallocation and Unemployment and Unemployment During the Great Recession." working paper, University of Hawaii.

Leamer, Edward. 2007. "Housing *IS* the Business Cycle." Federal Reserve Bank of Kansas City Jackson Hole Symposium.

Limnios, A. Christopher. 2014. "Can Frictions in the Housing Market Help Explain Frictions in the Labor Market?" Working paper, UC Santa Cruz, September.

Mian, Atif and Amir Sufi. 2014. "House Price Gains and U.S. Household Spending from 2002 to 2006." Unpublished mimeo.

Mian, Atif, Amir Sufi, and Francesco Trebbi. 2014. "Foreclosures, House Prices, and the Real Economy." Unpublished mimeo.

Moen, Espen R. 1997. "Competitive Search Equilibrium." *Journal of Political Economy* 105 (2):385-411.

Ngai, L. Rachel, and Kevin Sheedy. 2013. "The Ins and Outs of Selling Houses." LSE mimeo.

Ngai, L. Rachel, and Kevin Sheedy. 2015. "Moving House." CEPR Discussion Paper No. 10346. January.

Oswald, Andrew. 1997a. "Thoughts on NAIRU (Correspondence)." *Journal of Economic Perspectives* 11: 227-228.

Oswald, Andrew. 1997b. "Theory of Homes and Jobs." Working Paper, University of Warwick

Peterson, Brian M. 2009. "The Beveridge Curve in the Housing Market: Supply and Disequilibrium." working paper, Center for Applied Economics and Policy Research, Indiana University. June.

Petrovsky-Nadeau, Nicolas, and Etienne Wasmer. 2015. *Advanced in Search and Matching Models: A Macroeconomic Approach.* Unpublished manuscript. Sciences Po, Paris.

Pissarides, Christopher A. 1985. "Short-run Equilibrium Dynamics of Unemployment, Vacancies, and Real Wages." *The American Economic Review.* 75(4):676–690.

Pissarides, Christopher A. 1986. "Vacancies and Unemployment in Britain." *Economic Policy.* 1(3): 499-559.

Pissarides, Christopher A. 2000. *Equilibrium Unemployment Theory.* Cambridge, MA: MIT Press.

Pissarides, Christopher A. 2011. "Equilibrium in the Labor Market with Search Frictions." Nobel Lecture. *American Economic Review.* 101: 1092-1105.

Pissarides, Christopher A. 2013. "Unemployment in the Great Recession." *Economica.* 80:385-403.

Poterba, James 1984. "Tax Subsidies to Owner-Occupied Housing: An Asset-Market Approach." *Quarterly Journal of Economics.* 99(4):729–752.

Rosen, Kenneth T., Laurence B. Smith. 1983. "The Price-Adjustment Process for Rental Housing and the Natural Vacancy Rate." *American Economic Review.* 73(4):779–786.

Rupert, Peter and Etienne Wasmer. 2012. "Housing and the Labor Market: Time to Move and Aggregate Unemployment." *Journal of Monetary Economics.* 59: 24-36.

Wasmer, Etienne, and Philippe Weil. 2004. "The Macroeconomics of Labor and Credit Market Imperfections." *American Economic Review.* 94(4):944–963.

Wheaton, William. 1990. "Vacancy, Search, and Prices in the Housing Market Matching Model." *Journal of Political Economy,* 1270-1292.

8 Figures

Figure 1

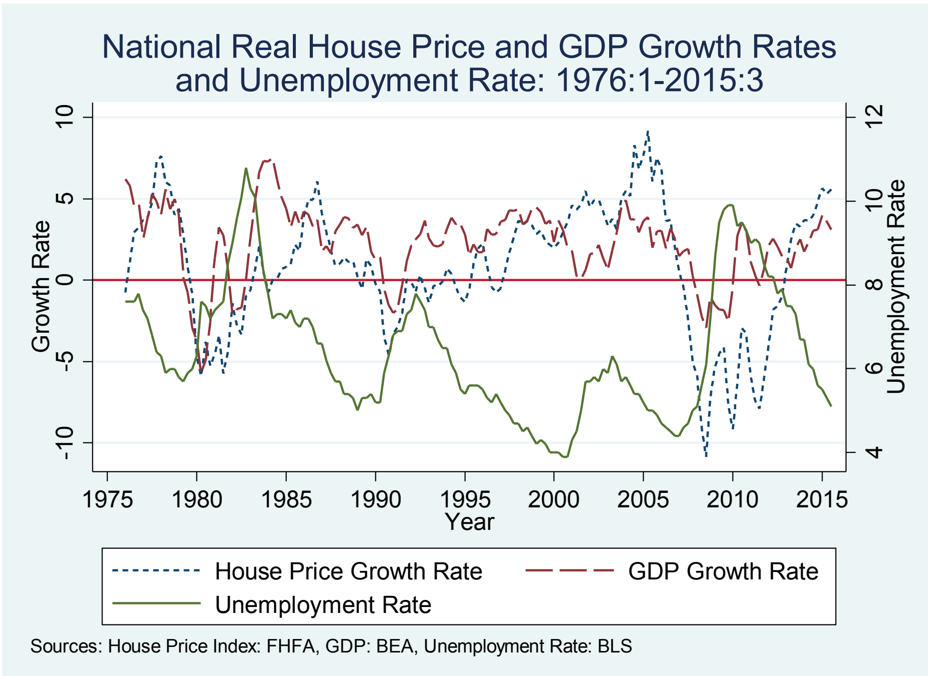
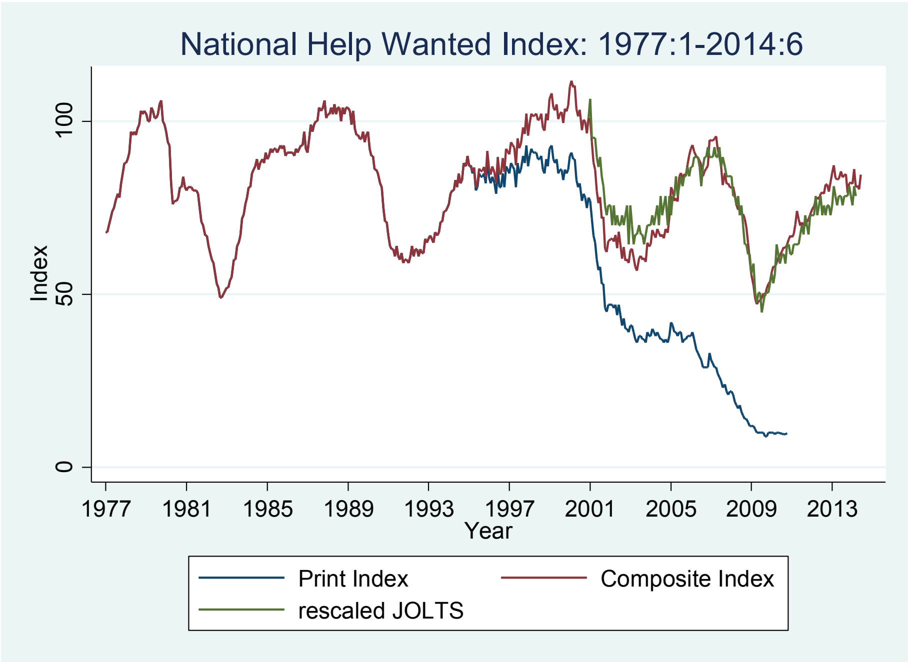
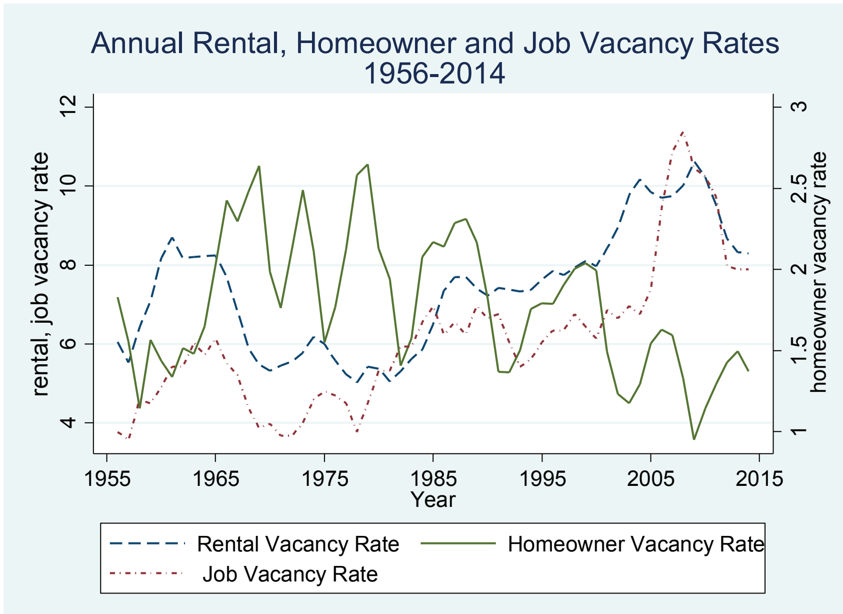

National Real House Price and GDP Growth Rates
and Unemployment Rate: 1976:1-2015:3
Growth Rate
Unemployment Rate
Year
-10
-5
0
5
10
4
6
8
10
12
1975
1980
1985
1990
1995
2000
2005
2010
2015
House Price Growth Rate
GDP Growth Rate
Unemployment Rate
Sources: House Price Index: FHFA, GDP: BEA, Unemployment Rate: BLS

Figure 2



National Help Wanted Index: 1977:1-2014:6
Index
0
50
100
1977
1981
1985
1989
1993
1997
2001
2005
2009
2013
Year
Print Index
Composite Index
rescaled JOLTS

Figure 3



Correlations

	Rental Vacancy	Home Vacancy	Job Vacancy
Home vacancy	0.805		
Job Vacancy	-0.591	-0.495	
Unemployment Rate	0.003	0.294	-0.468

Figure 4

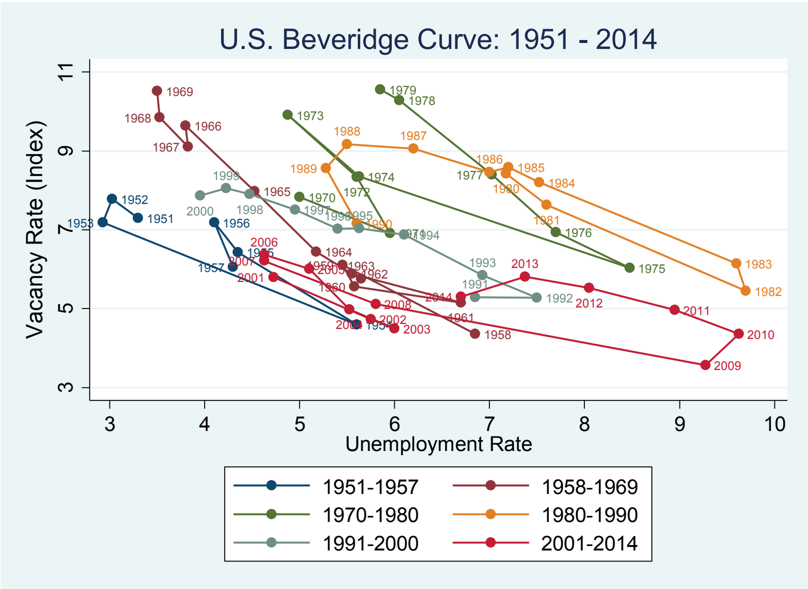

U.S. Beveridge Curve: 1951 - 2014
Vacancy Rate (Index)
Unemployment Rate
1951-1957
1958-1969
1970-1980
1980-1990
1991-2000
2001-2014

Figure 5

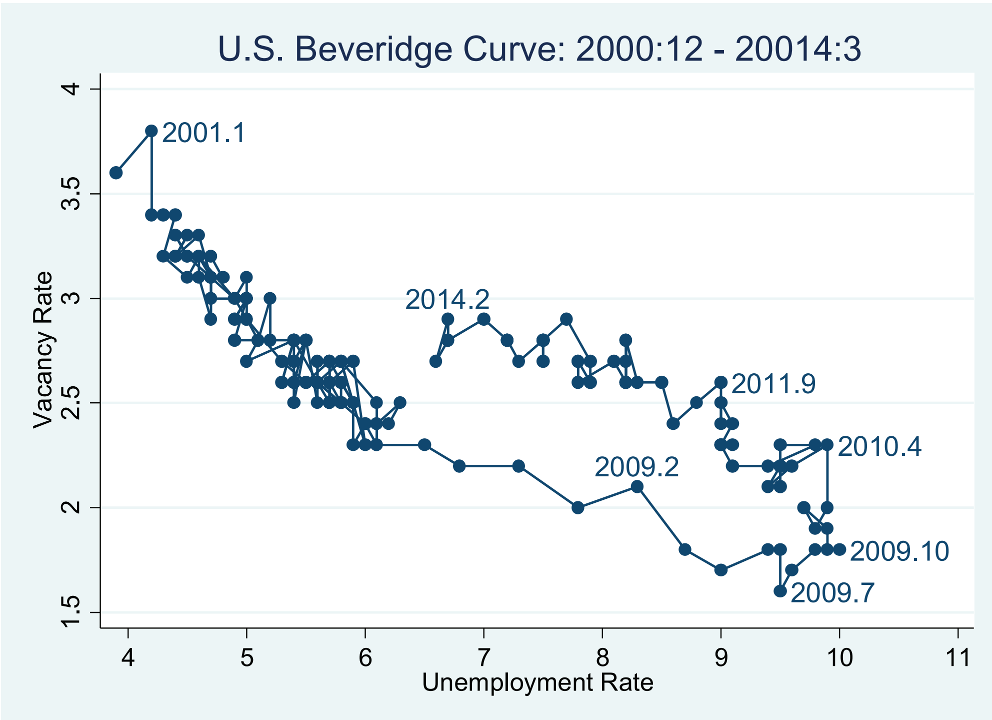
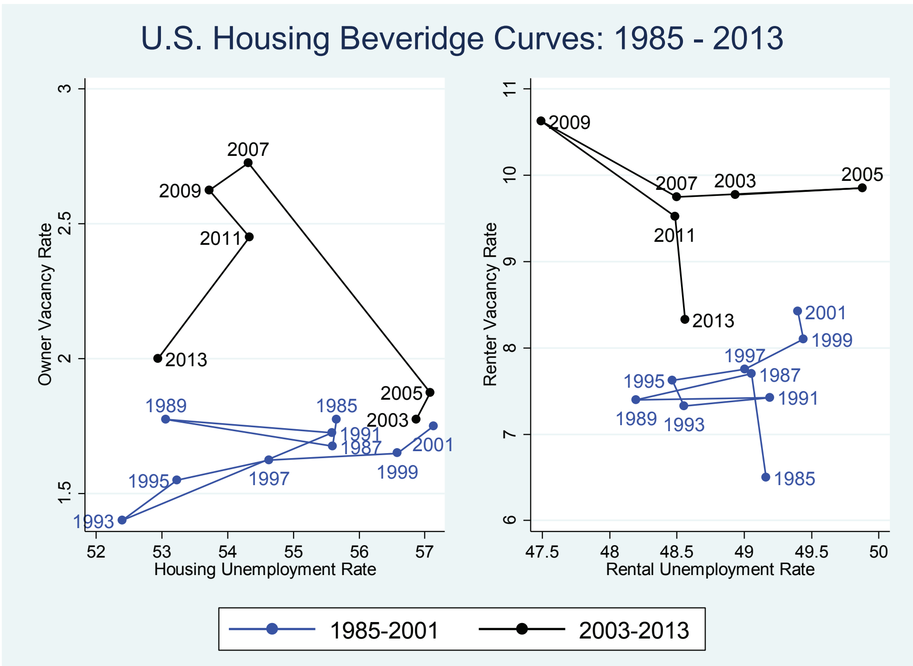

U.S. Beveridge Curve: 2000:12 - 20014:3
Vacancy Rate
Unemployment Rate
4
3.5
3
2.5
2
1.5
4
5
6
7
8
9
10
11
2001.1
2014.2
2011.9
2010.4
2009.2
2009.10
2009.7

Figure 6



U.S. Housing Beveridge Curves: 1985 - 2013
Owner Vacancy Rate
Housing Unemployment Rate
Renter Vacancy Rate
Rental Unemployment Rate
1985-2001
2003-2013

Figure 7

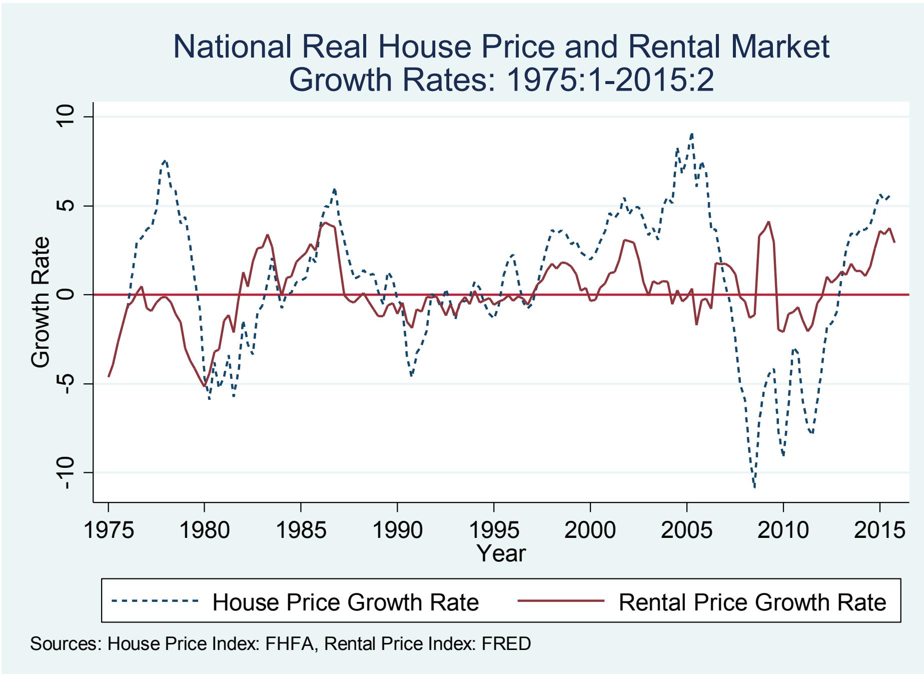
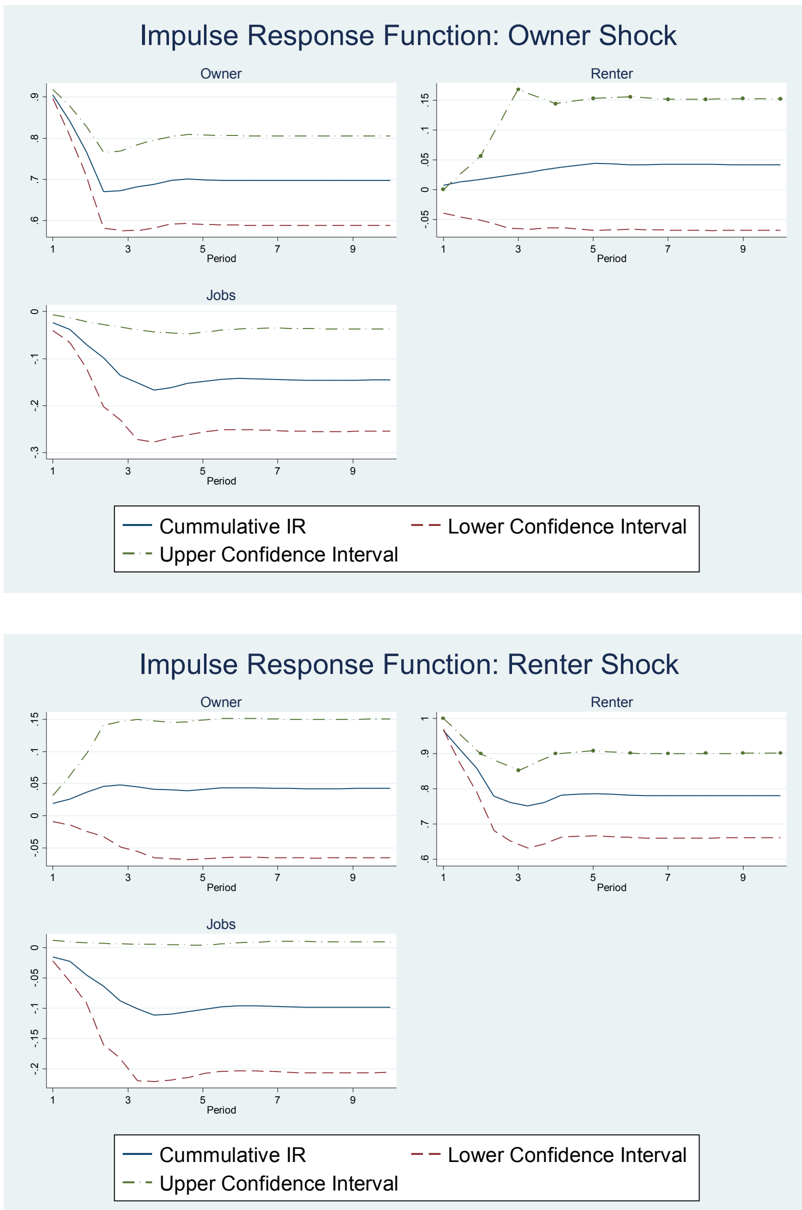
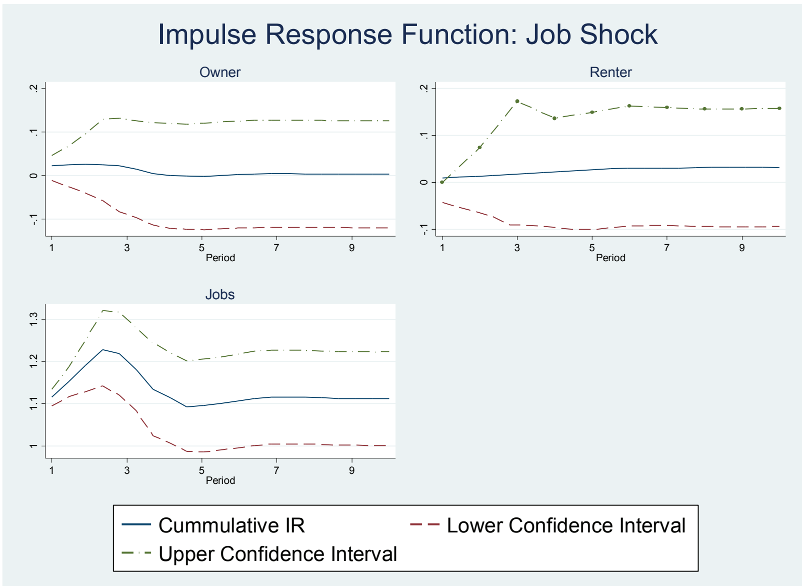

National Real House Price and Rental Market
Growth Rates: 1975:1-2015:2
Growth Rate
10
5
0
-5
-10
1975
1980
1985
1990
1995
2000
2005
2010
2015
Year
House Price Growth Rate
Rental Price Growth Rate
Sources: House Price Index: FHFA, Rental Price Index: FRED

Figure 8: Impulse Response Functions




Impulse Response Function: Job Shock
Owner
Renter
Jobs
Period
Cummulative IR
Lower Confidence Interval
Upper Confidence Interval

9 Tables

Table 1: Summary Statistics

Variable	Nobs	Mean	Std. Dev.	Minimum	Maximum
HVS Sample					
Single Family House Price Index	1423	144.44	48.26	67.31	333.53
Single Family Housing Permits (in hundreds)	1423	85.76	90.35	0.52	615.58
Owner Occupied Vacancy Rate (HVS)	1423	1.67	1.00	0.10	6.30
Natural Vacancy Rate (HVS)	1423	8.56	4.21	2.50	21.10
Employment (1,000s)	1351	1006.96	1174.33	16.79	7737.40
Fair Market Rent	1423	715.03	231.19	370.16	1791.65
Population (1,000s)	1423	2439.26	2847.47	105.18	19069.80
Per Capita Income (1,000s)	1150	34.31	8.65	15.59	80.14
Age Adjusted Ownership Rate	1423	56.32	6.97	37.75	78.30
Unemployment Rate	1421	5.62	2.21	1.56	15.87
Unemployment Compensation (millions)	1423	396.46	793.45	2.31	11456.67
Wages (1,000s)	1150	39.08	9.75	19.73	94.75
ACS Sample					
Single Family House Price Index	2465	181.32	37.07	105.03	362.87
Single Family Housing Permits (in hundreds)	2465	20.19	45.64	0.10	615.58
Owner Occupied Vacancy Rate (HVS)	2465	2.34	1.31	0.10	11.90
Employment (1,000s)	2112	283.90	653.85	14.79	7737.40
Fair Market Rent	2465	726.51	191.21	356.17	1730.00
Population (1,000s)	2465	716.26	1596.95	70.26	19069.80
Per Capita Income (1,000s)	2458	35.29	6.79	17.29	80.14
Age Adjusted Ownership Rate	2465	61.05	6.22	38.90	78.30
Unemployment Rate	2463	6.82	2.99	2.07	29.67
Unemployment Compensation (millions)	2458	180.48	567.83	0.59	11456.67
Wages (1,000s)	2458	38.91	6.81	24.44	94.75
Wages – Construction (1,000s)	2443	34.81	8.79	10.48	67.48
Monthly Composite Help Wanted Index					
HWI	13680	98.40	67.27	11.35	628.13
JOLTS Data					
Jobs Vacancy Rate – National	160	2.65	0.41	1.6	3.8
Jobs Vacancy Rate – Northeast	160	2.52	0.36	1.7	4
Jobs Vacancy Rate – Midwest	160	2.39	0.39	1.4	3.8
Jobs Vacancy Rate – South	160	2.82	0.49	1.7	3.9
Jobs Vacancy Rate – West	160	2.77	0.5	1.6	4.3

Table 2: Beveridge Curve Results: 1956-2014
Dependent Variable is Natural Log of Job Vacancy Rate

Variables	OLS (1)	IV (2)	OLS (3)	IV (4)
ln(Unem)	-0.641***	-0.679***	-0.936***	-0.923***
	(0.064)	(0.086)	(0.070)	(0.067)
ln(Owner Vacancy; OV)	0.308***	0.412**	0.088	-0.107
	(0.104)	(0.196)	(0.085)	(0.199)
ln(Rental Vacancy; RV)	-1.101***	-1.321***	-0.294**	-0.483**
	(0.128)	(0.280)	(0.118)	(0.188)
1 if 1970-1979* ln(Unem)			0.152***	0.113***
			(0.023)	(0.022)
1 if 1980-1990* ln(Unem)			0.036	0.009
			(0.042)	(0.035)
1 if 1991-2000* ln(Unem)			0.139	0.203
			(0.101)	(0.135)
1 if 2001-2014* ln(Unem)			1.582***	2.113***
			(0.177)	(0.634)
1 if 2001-2014*ln(OV)			-0.625***	-0.667
			(0.219)	(0.518)
1 if 2001-2014*ln(RV)			-1.029***	-1.473***
			(0.160)	(0.463)
1 if 1970-1979			0.182***	0.163***
			(0.034)	(0.034)
1 if 1980-1990			0.093**	0.183***
			(0.038)	(0.065)
1 if 1991-2000			-0.003	0.095
			(0.076)	(0.077)
1 if 2001-2014			-0.390**	-0.348
			(0.177)	(0.227)
Constant	5.115	5.575	4.009***	4.407***
	(0.266)***	(0.582)***	(0.194)	(0.326)
		IV Test Statistics		
Over ID: p-value		0.21		0.63
Endogeneity: p-value		0.26		0.0065
1st Stage F stat:				
Owner Vacancy		45.67		45.67
Rental Vacancy		21.33		21.33
Observations	56	56	56	56
R-squared	0.74	0.73	0.94	0.92

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1
Instruments: natural logs of: 1 unit permits, 2 or more unit permits, 1 unit starts, 2or more unit starts

Table 3: Housing Beveridge Curve Results: 1991-2011

	Dependent Variable in Logs			
	Owner Vacancy Rate		Rental Vacancy Rate	
Variables	(1)	(2)	(3)	(4)
ln(Unfulfilled Ownership)	-2.503 (0.674)***	-1.072 (0.759)		
ln(Unfulfilled Rental)			0.190 (0.413)	0.765 (0.326)**
ln(Job Vacancy Index)	-0.285 (0.075)***	-0.064 (0.067)	-0.288 (0.062)***	-0.122 (0.073)
1 if 2005-2011		0.456 (0.081)***		
1 if 2003-2011				0.246 (0.058)***
Constant	10.363 (2.691)***	4.613 (3.047)	1.217 (1.604)	-1.017 (1.265)
R-squared	0.12	0.27	0.15	0.27
Observations	297	297	297	297
Number of CBSAs	27	27	27	27

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 4 - VAR Regression Results

	Dependent Variable: Vacancy Rates (if First-Differences)		
	Ownership	Rental	Jobs
Vacancies in First Diff			
$Ownership_{t-1}$	-0.328	-0.019	-0.061
	(0.039)**	(0.043)	(0.039)
$Ownership_{t-2}$	-0.108	0.071	-0.120
	(0.038)**	(0.044)	(0.039)**
$Rental_{t-1}$	0.054	-0.186	-0.041
	(0.039)	(0.038)**	(0.037)
$Rental_{t-2}$	0.026	-0.113	-0.062
	(0.039)	(0.038)**	(0.036)
$Jobs_{t-1}$	0.055	-0.015	0.313
	(0.043)	(0.045)	(0.039)**
$Jobs_{t-2}$	-0.041	0.070	-0.208
	(0.043)	(0.046)	(0.039)**
Other Variables I in First-Diff			
$Unpl^{-1}_{t-1}$	-1.997	-1.179	0.368
	(1.978)	(2.103)	(1.880)
$Unpl^{-1}_{t-2}$	-2.237	-4.018	-1.320
	(1.907)	(2.018)*	(1.816)
$House\ Price\ Index_{t-1}$	0.004	-0.003	-0.019
	(0.005)	(0.006)	(0.005)**
$House\ Price\ Index_{t-2}$	0.008	0.017	0.013
	(0.005)	(0.006)**	(0.005)**
Constant	0.608	-0.466	-0.076
	(0.184)**	(0.198)*	(0.173)
Observations	740	740	740

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

FOR ONLINE PUBLICATION

A A Model of Housing and Labor Market Vacancies as Joint Outcomes

This appendix details our extension of the theory of housing markets with frictions of Head and Lloyd-Ellis (2012). We extend the model by means of the following components. One, we introduce a frictional labor market along the lines of Pissarides (1985; 2000). In contrast, the labor market in Head and Lloyd-Ellis (2012) is Walrasian. Two, we introduce a frictional rental housing market, which allows us to introduce vacancy rates for the rental segment of the housing market. Three, we allow for rationing of individuals who wish to enter the ownership market, and explain how a similar concept may be introduced for owners who wish to enter the rental market. Four, these extensions allow us to develop a novel concept of "unemployment" for the rental and ownership housing markets, which in turn allows us to derive Beveridge curves for housing markets. Five, we identify the existence of spillovers between the Beveridge curves for housing and labor markets. These extensions are critical in helping us structure the empirical investigations reported in the main body of the paper.

A.1 Preferences

Let W^j, U^j, denote the conditional value functions, that is expected lifetime utility, conditional on being employed (W^j) and unemployed (U^j), for a renter (R) and a homeowner (H), $j = R, H$ respectively, which are expressed in real terms, and under the assumption of unrestricted borrowing or lending at a fixed rate of interest, ρ.[24] These are generated by flow of utility per unit of time, denoted by π^j, and defined in terms of non-housing consumption c^j, labor supply, l^j, and housing consumption, z^j, per unit of time. Following Head *et al.* (2014), Eq. (3), we let the flow of utility be linear in non-housing consumption, housing

[24] Wasmer and Weil (2004) extend the Pissarides model to account for credit frictions and Petrosky-Nadeau and Wasmer (2015) extend it further to account for credit multipliers.

consumption, and leisure, $1 - l^j$:

$$\pi^j(c, l, z) = c^j - l^j + z^j, j = R, H. \tag{A.1}$$

We assume that a person is either employed, earning w^j, or unemployed, receiving $b < w^j$. Note also that we allow for the possibility that bargaining between firms and workers may lead to wage rates that are different between renters and owners, w^H, w^R, respectively. More on this below.

In defining the flow of indirect utility (A.1), we allow for housing costs to depend on tenure. Let non-housing consumption be the numeraire, with its price set equal to 1, and let κ be rent per unit of rental housing. Ignoring commuting costs, the quantity of housing consumed by renters in a particular area is given by rent expenditure divided by κ. Let p_h be the annual user cost of owner-occupied housing. This is defined as the annualized user cost of housing per unit of housing value [*c.f.* Poterba (1986); Henderson and Ioannides (1986)]: a dwelling unit of value V^H, generates an annualized user cost of $p_h V^H$, and superscript H accounts for the fact that dwelling units for owner occupancy or renting are distinct.[25] The user cost of housing reflects the implications of the tax treatment of housing as well as its durability.[26] The respective quantity of housing consumed, that is, housing services, is given

[25]This definition maintains consistency between the rental and the homeownership sectors. However, this could be modified so as to be based on transactions values instead of the vacant unit value. Also property tax rates, maintenance and depreciation rates as well as housing price appreciation rates may be area-specific.

[26]Following Poterba (1986) and Henderson and Ioannides (1987) the user cost of housing reflects mortgage payments at a rate of interest ι, times the portion of the value of a dwelling unit that is financed, $1 - \text{equity}$, and adjusted for the tax deductability of mortgage interest associated with the portion of the value of owner-occupied housing that is leveraged, by multiplying by 1 minus the marginal US income tax rate, $1 - \tau$. Property taxes, denoted by rate τ_p here, are also deductible for US income tax purposes. In addition, allowing for maintenance and depreciation, at rates maint and depr, respectively, and deducting the rate of expected housing price appreciation, appr^e, yields the annual user cost of housing as:

$$p_h = [(1 - \tau)[\iota(1 - \text{equity}) + \tau_p] + \text{depr} + \text{maint} - \text{appr}^e].$$

This definition maintains consistency between the rental and the homeownership sectors. However, this could be modified so as to be based on transactions values instead of the vacant unit value. Also property tax rates, maintenance and depreciation rates as well as housing price appreciation rates may be area-specific.

by $\frac{p_h V}{p}$, where p is a house price index. Suppose that there are no property or taxes, nor maintenance, depreciation, and appreciation, and an individual borrows at the real rate of interest ρ to finance living in a house of value V. She would thus incur housing costs per unit of time equal to the opportunity cost of housing of value V, ρV. Equivalently, since housing is durable, services from an actual housing stock V are given by $\rho \frac{V}{p}$.

Under the assumption of perfect capital markets, with individuals' being able to borrow against their expected future income or to save at rate ρ, the Bellman equations for the conditional value functions W^j, U^j, may be defined once we have defined the respective flows of real utility, π^j, π^j. For a homeowner, from (A.1), ignoring the disutility of work and allowing for institutional considerations to enter through the definition of p_h, flow utility may be written as the sum of the flow of housing and non-housing consumption, defined as the real wage rate (or unemployment compensation, as appropriate) plus dissaving:

$$\pi^H(w^H) = \frac{p_h V}{p} + w^H - \rho V + \text{dissaving}, \tag{A.2}$$

where $-\rho V$ denotes the opportunity cost (dissaving) associated with holding (durable) housing stock of value V. For a renter, we have correspondingly:

$$\pi^R(w^R) = \frac{\text{rent expenditure}}{\kappa} + w^R - \text{rent expenditure} + \text{dissaving}. \tag{A.3}$$

For an unemployed individual, b takes the place of $w^j,\ j = H, R$, on the right hand sides of Eq.'s (A.2) and (A.3).

We now provide the implications of this formulation, in the simplest possible case at the steady state, with renters and owners retaining their housing tenure status forever. Let δ denote the exogenous job destruction rate and μ the job finding rate (which will be specified in section A.4 below as a function of labor market tightness). The Bellman equations for the conditional value functions are, first for employed and unemployed owners:

$$\rho W^H = \pi^H(w^H) + \delta[U^H - W^H]; \tag{A.4}$$

$$\rho U^H = \pi^H(b) + \mu[W^H - U^H]; \tag{A.5}$$

In the absence of taxation, maintenance and capital gains, and with $\iota = \rho$, the above definition yields $p_h = \rho$ as a special case of the user cost concept. All these quantities may be defined per appropriate unit of time.

and correspondingly for renters:

$$\rho W^R = \pi^R(w^R) + \delta[U^R - W^R]; \tag{A.6}$$

$$\rho U^R = \pi^R(b) + \mu[W^R - U^R]; \tag{A.7}$$

where the flow utilities π^H and π^R, are specified in Eq.'s (A.2)-(A.3) above, except that the term dissaving is of course dropped when we integrate from the flow to the stock (to arrive at the respective value functions). From now on, we will use $\pi^j, j = H, R$ without the term dissaving.[27] Below we solve for the expressions for the conditional value functions under more general conditions. We also use the resulting solutions to motivate a housing tenure choice estimation, which we detail in section B below, and use it to estimate the probabilities of unfulfilled renters and owners in section 5.3.

The associated steady-state unemployment rate is given by: $\frac{\delta}{\delta+\mu}$. It may vary across MSAs because of differences in their industrial compositions. The job finding rate is typically specified in terms of the the job matching process and labor market tightness, to which we come further below. It can reflect individual characteristics, which is relevant at the empirical stage. Housing spells of homeowners are initially assumed to last forever, if job market events and housing tenure events are independent. We assume that housing units for renters and owners are perfect substitutes.

A.2 Frictions in Housing Markets

Both housing and labor markets are subject to frictions. The individual (or household, the two terms will be used interchangeably) is subject to the risk of job loss: jobs break up at a Poisson rate δ, and the unemployed individual finds a job at a Poisson rate μ, per unit of time. Dwelling units, either for owner-occupancy or renting may be occupied or vacant. Frictions are present in the matching of dwelling units and individuals via search, which leads to the

[27] In contrast to Head and Lloyd-Ellis, our definition of $\pi^H(w)$ in (Eq. A.2) above makes it dependent on V, in general, which is endogenous. We will ignore this endogeneity from now on, when we derive the equilibrium value of V below. However, V cancels out of the expressions for the $\pi^j, j = H, R$ if we assume that $p = 1$, and $p_h = \rho$.

determination of vacancy rates for the homeownership and rental housing markets. Suppose first, like Head and Lloyd-Ellis (2012), that rental units may be found instantaneously and thus frictionlessly, while units for owner-occupancy involve a matching process, i.e. frictions. Consequently, the values of vacant units as assets may differ from the transaction prices at which they change owners. We extend the model to allow for frictions in the rental housing market as well In section A.2.5 below.

Specifically, let γ^H denote the rate at which new dwelling units sold by construction firms match with prospective homeowners. Head and Lloyd-Ellis specify γ^H as the product of the rate at which prospective homeowners match with dwelling units, $\bar{\lambda}^H$, times the ratio of prospective homeowners to vacant units in the homeownership segment of the market, ϕ^H:

$$\gamma^H = \bar{\lambda}^H \phi^H. \tag{A.8}$$

Clearly, ϕ^H and thus γ^H may vary across areas, and we may introduce a subscript i, when it is necessary for clarity. This definition may be generalized by specifying, in the standard Pissarides fashion, a matching function for individuals and vacant dwelling units.[28] It may also be generalized to account for the time it takes owner-occupied houses to be transferred from one household to another, when turnover in owner-occupied units is allowed; see section 3.2 below. Because matching in housing markets involves frictions, it means that housing is, to some extent, illiquid; its value when vacant depends on how fast buyers may be found for dwelling units on the market. The model highlights this fact.

The population consists of N individuals whose number is assumed to grow at a rate ν. Individuals may be found in one of four different states, employed and unemployed home-

[28]Let $I_{b,t}, I_{s,t}$, denote the stock of buyers searching for houses and the stock of sellers searching for buyers, respectively. Let the matching process be specified in the standard fashion in terms of the Poisson rate of contacts generated, denoted by $\Gamma_t = \Gamma\left(I_{b,t}, I_{s,t}\right)$. So, in general, the rate of arrivals of contacts to the typical dwelling unit in MSA i is: $\gamma = \frac{1}{I_{s,t}}\Gamma\left(I_{b,t}, I_{s,t}\right)$, which under the assumption of constant returns to scale, this may be written as:

$$\gamma = \Gamma(\phi, 1) = \Gamma\left(\frac{I_{b,t}}{I_{s,t}}, 1\right).$$

This differs from the Head and Lloyd-Ellis assumption, (A.8) above, only because of the nonlinearity of Γ, but is consistent with the assumptions typically made about matching models. Parameter λ is subsumed in this formulation.

owners, N^{WH} and N^{UH}, and employed and unemployed renters, N^{WR} and N^{UR}, respectively. Therefore:

$$N^{WR} + N^{UR} + N^{WH} + N^{UH} = N. \tag{A.9}$$

It is more convenient to work with the relative numbers of agents, that is their proportions in different states. By using lower case n's, i.e. $n^{WH} = \frac{N^{WH}}{N}$, Eq. (A.9) becomes:

$$n^{WH} + n^{UH} + n^{WR} + n^{UR} = 1. \tag{A.10}$$

Let R, H denote the total housing stock, in the rental and homeownership segments of the housing market, respectively, and let r, h denote the corresponding per capita values. Given that $\phi^H = \frac{N^{WR}+N^{UR}}{H-N^{WH}-N^{UH}}$, Eq. (A.8) can be expressed as

$$\gamma^H = \bar{\lambda}^H \frac{n^{WR} + n^{UR}}{h - n^{WH} - n^{UH}}. \tag{A.11}$$

A.2.1 Housing Supply: The Rental Housing Market

Following Glaeser *et al.* (2014) and Head and Lloyd-Ellis, *op. cit.*, we assume free entry in the housing construction-real estate business and specify a supply equation for rental housing units: the present value of rents equals the asset value of their unit construction costs, that is:

$$\frac{\kappa}{\rho} = c_0 + c^R r,$$

where c_0 denotes fixed construction costs, and $c^R r$ variable costs that depend linearly on the rental housing stock per person, r, in order to express the cost of land due to congestion. We assume initially that the entire stock of rental units are occupied as soon as they are produced, that is, the rental housing market is not subject to frictions and rental vacancy rates are equal to 0. Since all rental units are occupied, $r = n^{WR} + n^{UR}$, the above equation may be rewritten instead in terms of n^{WR} and n^{UR} :

$$\frac{\kappa}{\rho} = c_0 + c^R \left(n^{WR} + n^{UR}\right). \tag{A.12}$$

Housing is assumed to last forever, and the rental price κ may be assumed to include maintenance costs. Under the assumption of free entry, we need not worry about the profits of owners of rental housing stock. We modify this equation further below in section A.2.5 in order to introduce frictions in the rental housing market.

A.2.2 Housing Supply: The Homeownership Market

The value of units for owner-occupancy when vacant must compensate their producers:

$$V^H = c_0 + c^H h, \tag{A.13}$$

where $c^H h$ denotes unit variable costs, that depend linearly on the per capita housing stock for homeownership, h, in order to express the cost of land due to congestion. It is also possible to allow for (costly) conversion of dwelling units from one mode of tenure to another.

We note that the supply equations (A.12)-(A.13) link "prices," that is rents and values of vacant units, to their respective stocks relative to the numbers of individuals. Next, we relate supply equations to demand conditions by specifying the decision problems of individuals.

A.2.3 The Value of Vacant Housing in the Owner-Occupied Market

The value of vacant dwellings in the home ownership market must, at asset equilibrium, reflect the fact that dwelling units may be purchased by either employed or unemployed renters, whose willingness to pay may be different. The return per unit of time to holding an asset of value V is equal to the probability per unit of time that it may be sold either to an employed renter, at price P^W, or an unemployed renter, at price P^U, whichever of the two bids is higher:

$$\rho V = \gamma^H \mathcal{E}\left[\max_j \left\{P^j - V\right\}\right], j = W, U. \tag{A.14}$$

The expectation on the rhs of Eq. (A.14), the arbitrage equation for V, may be written out by recognizing that a unit may either be purchased by an employed renter, if $P^W > P^U$, an event that occurs with probability equal to the proportion of those employed among all renters, $\alpha = \frac{N^{WR}}{R}$; or by an unemployed renter, if $P^W < P^U$, an event that occurs with probability $1 - \alpha = \frac{N^{UR}}{R}$.

Consistent with the literature of markets with frictions, a seller and a buyer of a dwelling unit who come into contact engage in Nash bargaining and split the surplus from the transaction, with a share σ of V going to the seller and $(1-\sigma)(W^H - W^R)$ to the buyer, if employed, or $(1-\sigma(U^H - U^R)$, if unemployed. So, the prices paid by employed and unemployed households satisfy:

$$P^W = \sigma V + (1-\sigma)\left[W^H - W^R\right]; P^U = \sigma V + (1-\sigma)\left[U^H - U^R\right]. \tag{A.15}$$

Solving for V from Eq.'s (A.14) and (A.15) gives:

$$V = \frac{(1-\sigma)\gamma^H}{\rho + (1-\sigma)\gamma^H}\left[\alpha[W^H - W^R] + (1-\alpha)[U^H - U^R]\right]. \tag{A.16}$$

Recall that we assume here that once renters purchase dwelling units and become homeowners they remain so forever. Their conditional value functions are given by (A.4)–(A.5) above. Renters, on the other hand, are faced with opportunities, at a rate γ^H, to purchase dwelling units and become homeowners. Thus, the respective Bellman equations, the counterparts of (A.6) and (A.7), for the conditional value functions become:

$$\rho W^R = \pi^R(w^R) + \delta[U^R - W^R] + \gamma^H\left[W^H - P^W - W^R\right]; \tag{A.17}$$

$$\rho U^R = \pi^R(b) + \mu[W^R - U^R] + \gamma^H\left[U^H - P^U - U^R\right]. \tag{A.18}$$

We may modify the model to allow for the interdependence between employment and housing tenure mode transitions, but so far, such transitions are assumed to be independent. However, conditions in the housing market have a profound effect on the conditional value functions.

Next, we may use Eq. (A.16) in (A.15) in order to express the transaction prices, P^W, P^R, in terms of the conditional value functions, W^H, W^R, and U^H, U^R. We substitute back into the Bellman equations, (A.4-A.5) for owners, and (A.17-A.18) for renters, and solve for the conditional value functions, namely for W^H, W^R, U^H, U^R, as functions of the real wage rate and unemployment compensation, on the one hand, and of labor market and housing market tightness, on the other. Labor market tightness enters the job finding rate for owners and renters, as we discuss in more detail in section A.4 below.

A.2.4 Housing Market Flows and Conditional Value Functions

Given labor market magnitudes, that is wages, unemployment, and job vacancy rates, which are determined from the model of labor markets with frictions, we may proceed as follows. In view of the value of vacant units for sale from Eq. (A.16), the transaction prices for owner-occupied units P^W and P^R, may be expressed in terms of the four conditional value functions, W^H, W^R, U^H, U^R, that enter their definitions. There are seven unknowns, the per capita stocks for owner-occupancy and renting, h, r, the rent, κ, and the relative numbers of agents in the four different states, $n^{WR}, n^{UR}, n^{WH}, n^{UH}$. By solving Eq. (A.4), (A.5), (A.17), and (A.18) for the conditional value functions, we can express the value of a vacant home, V^H, in terms of the four unknown relative numbers of agents in different states, the unknown rental price, κ, the wage rates w^H and w^R, and the unemployment compensation rate, b.

It is more convenient to think of the model in a steady state, with the number of individuals growing at an exogenous rate ν. Along the steady state, all stocks of agents grow at the same rate leaving the relative numbers of agents constant. This leads to four relationships in terms of the relative numbers of agents. First, Eq. (A.10) expresses that all individuals may find themselves in one of the four states, so that their respective relative numbers sum up to 1. We derive next the other three flow equations that express transitions across different states. The four equations allow us to solve for $n^{WR}, n^{UR}, n^{WH}, n^{UH}$.

Second, the change in the number of employed renters in a given city, $\frac{dN^{WR}}{dt}$, equals the number of unemployed renters who become employed, μN^{UR}, minus the measure of employed renters whose jobs are destroyed, δN^{WR}, and minus those renters who become owners, $\bar{\lambda}^H N^{WR}$. That is:

$$\frac{dN^{WR}}{dt} = \mu N^{UR} - (\delta + \bar{\lambda}^H) N^{WR}.$$

Imposing the condition that for a steady state, $\frac{dN^{WR}}{dt} = \nu N^{WR}$, and rewriting the above condition in terms of relative numbers of agents yields:

$$(\nu + \delta + \bar{\lambda}^H) n^{WR} - \mu n^{UR} = 0. \tag{A.19}$$

Third, working in a like manner, the change in the relative number of unemployed homeowners, νn^{UH}, is equal to minus those unemployed homeowners who find jobs, μn^{UH}, plus the number of those employed homeowners who lose their jobs, plus the number of unemployed renters who become homeowners, $\bar{\lambda}^H n^{UR}$. Rewriting, we have:

$$(\mu+\nu)n^{UH} - \delta n^{WH} - \bar{\lambda}^H n^{UR} = 0. \tag{A.20}$$

Fourth, the increase in the number of employed homeowners, νn^{WH}, is equal to the number of unemployed homeowners getting jobs, μn^{UH}, plus the number of employed renters who become homeowners, $\bar{\lambda}^H n^{WR}$, minus those employed homeowners who become unemployed. Rewriting, we have:

$$\nu n^{WH} + \delta n^{WH} - \bar{\lambda}^H n^{WR} - \mu n^{UH} = 0. \tag{A.21}$$

Rewriting the above equations in matrix form gives:

$$\begin{bmatrix} \delta+\bar{\lambda}^H+\nu & -\mu & 0 & 0 \\ 0 & -\bar{\lambda}^H & -\delta & \mu+\nu \\ -\bar{\lambda}^H & 0 & \delta+\nu & -\mu \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} n^{WR} \\ n^{UR} \\ n^{WH} \\ n^{UH} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}. \tag{A.22}$$

The matrix on the l.h.s. of Eq. (A.22) depends on the parameters $(\delta, \bar{\lambda}^H, \mu, \nu)$ only. This yields the solution:

$$n^{WR} = \frac{\mu\nu}{(\bar{\lambda}^H+\nu)(\bar{\lambda}^H+\nu+\delta+\mu)}, n^{UR} = \frac{\nu(\bar{\lambda}^H+\nu+\delta)}{(\bar{\lambda}^H+\nu)(\bar{\lambda}^H+\nu+\delta+\mu)}; \tag{A.23}$$

$$n^{WH} = \frac{\bar{\lambda}^H\mu(\bar{\lambda}^H+\nu+\delta)+\bar{\lambda}^H\mu(\mu+\nu)}{(\bar{\lambda}^H+\nu)(\delta+\mu+\nu)(\bar{\lambda}^H+\nu+\delta+\mu)}, n^{UH} = \frac{\bar{\lambda}^H\delta(\bar{\lambda}^H+\nu+\delta+\mu)+\bar{\lambda}^H\nu(\delta+\bar{\lambda}^H+\nu)}{(\bar{\lambda}^H+\nu)(\delta+\mu+\nu)(\bar{\lambda}^H+\nu+\delta+\mu)}. \tag{A.24}$$

With these results, the share of employed renters, $\alpha = \frac{n^{WR}}{n^{WR}+n^{UR}}$, and the unit matching rate introduced in Eq. (A.11) are given by:

$$\alpha = \frac{\mu}{\bar{\lambda}^H+\nu+\delta+\mu}; \tag{A.25}$$

$$\gamma^H = \bar{\lambda}^H \frac{\frac{\nu}{\bar{\lambda}^H+\nu}}{h - \frac{\bar{\lambda}^H}{\bar{\lambda}^H+\nu}}. \tag{A.26}$$

In equilibrium, the denominator in Eq. (A.26) above must be positive. The model implies steady state equilibrium homeownership and rental rates given by:

$$\text{hr} = n^{WH} + n^{UH} = \frac{\bar{\lambda}^H}{\bar{\lambda}^H+\nu}, \ \text{rr} = \frac{\nu}{\bar{\lambda}^H+\nu}. \tag{A.27}$$

We note that these rates depend critically on the rate of growth of the population. Below, we derive the equilibrium unemployment rate in the presence of population growth, Eq. (A.49), which also depends on ν.

Clearly, we may arrive at a more general model by specifying churning within the housing market. We go some way further in this direction below in sections A.2.5 and 3.2 where we relax the assumption that all renters seek to become homeowners, which boosts the equilibrium rental rate. We rework the system of equations in Eq. (A.22) accordingly.

The conditional value functions for homeowners may be obtained by solving Eq. (A.4) and (A.5). Thus, we have:

$$W^H = \frac{1}{\rho(\delta+\mu+\rho)}\left[(\rho+\mu)\pi^H(w^H)+\delta\pi^H(b)\right]. \tag{A.28}$$

$$U^H = \frac{1}{\rho(\delta+\mu+\rho)}\left[\mu\pi^H(w^H)+(\delta+\rho)\pi^H(b)\right]. \tag{A.29}$$

Recall that we have allowed for bargaining between firms and workers to lead to different wage rates for renters and owners, w^H, w^R, respectively which is natural in the context of our model. Wage setting is defined in terms of the improvement in utility from an unemployed owner expects from accepting a job. By subtracting (A.29) from (A.28), we obtain:

$$W^H - U^H = \frac{w^H - b}{\delta+\mu+\rho}. \tag{A.30}$$

Under our assumptions, transitions from employment to unemployment and vice versa occur at a Poisson rate $\delta+\mu$. Thus, the expected discounted net benefit for a homeowner of moving from unemployment to employment is simply the increase in pay times the expected length of stay in employment, which also allows for discounting.

We can solve for the conditional value functions for renters, Eq.'s (A.17) and (A.18), after we have expressed the transaction prices for vacant units in terms of the conditional value functions. Recall Eq. (A.15), which gives give transaction prices via Nash bargaining. From Eq.'s (A.15) and (A.16), and (A.17) and (A.18), we can solve for W^R and U^R. However, it is more directly useful that we solve instead for $W^R - U^R$, which is the quantity that enters wage setting below. That is, by subtracting Eq. (A.18) from Eq. (A.17) we obtain an equation that contains $P^W - P^U$. From Eq. (A.15), by subtracting the solution for P^U from that for P^W we may express $P^W - P^U$ in terms of $W^H - U^H$ and $W^R - U^R$, and substituting into the equation for $W^R - U^R$, a solution readily follows:

$$W^R - U^R = \frac{w^R - b}{\delta + \mu + \rho + \gamma^H \sigma} + \frac{\gamma^H \sigma (w^H - b)}{(\delta + \mu + \rho)(\delta + \mu + \rho + \gamma^H \sigma)}. \tag{A.31}$$

For unemployed renters, there are two types of transitions: transition to employment while remaining a renter, with the expected length of stay being equal to $(\delta + \mu + \rho)^{-1}$, and the expected increase in pay given by $(\delta + \mu + \rho)(\delta + \mu + \rho + \gamma^H \sigma)^{-1}(w^R - b)$; and transition to employment and homeownership, with the expected length of stay being equal to $(\delta+\mu+\rho)^{-1}$, and the expected increase in pay given by $\gamma^H \sigma(\delta + \mu + \rho + \gamma^H \sigma)^{-1}(w^H - b)$.

As we discuss further below, if renters and owners are perfect substitutes in production and are treated symmetrically in wage setting, then they receive equal wage rates, $w^H = w^R = w$, and Eq. (A.31) yields: $W^R - U^R = \frac{w-b}{\delta+\mu+\rho} = W^H - U^H$. That is, the improvement in expected lifetime utility resulting from becoming employed is equal for renters and owners. At the steady state equilibrium, individuals who are identical in production experience the same expected utility from becoming employed.

To recapitulate, the conditional value functions (W^H, U^H, W^R, U^R) have been solved in terms of the wage rates, w^H, w^R, the unit matching rate, γ^H, which in view of Eq. (A.26) depends on h, and the labor market tightness that enters via the employment rate, μ, as we see further below. From Eq. (A.12) and in view of Eq. (A.23), the rent κ is determined as a function of the share of renters $n^{WR} + n^{UR} = \frac{\nu}{\bar{\lambda}^H + \nu}$, and thus is exogenous. Finally, from Eq. (A.16), V may be expressed, via the conditional value functions, in terms of the wage rates, w^H, w^R, labor market tightness (via the job finding rate μ) and the unit matching rate $\gamma^H = \bar{\lambda}^H \frac{\frac{\bar{\lambda}^H}{\bar{\lambda}^H + \nu}}{h - \frac{\nu}{\bar{\lambda}^H + \nu}}$, which depends on h. These derivations when used in Eq. (A.13) yield an equation for the relative stock of owner-occupied units, h. Finally, the equilibrium is fully determined once the wage rates are set. We turn to this below, which requires looking at the labor market with frictions, after we introduce frictions in the rental housing market. Recall that Head and Lloyd-Ellis (2012) assume frictionless rental housing and labor markets. When the model is extended to allow transitions also from owning to renting, the equations for the conditional value functions and the above solutions have to be amended accordingly.

A.2.5 Allowing for Frictions in Rental Markets

The model so far treats the rental housing market as frictionless. However, rental housing units may be vacant, for reasons that are identical to those in the ownership market. In fact data on rental market vacancies are also available and are generally higher than homeownership vacancy rates. The purpose of our generalization of the Head and Lloyd-Ellis model is to introduce rental market vacancies, which helps structure the use of such data in our empirical analysis. To the variables denoting the relative stocks of individuals in different labor market states, $(n^{WR}, n^{UR}, n^{WH}, n^{UH})$, and the per capita housing stocks for owner-occupancy and renting, h and r, we need to add as unknowns the stocks of vacant units, υ^H and υ^R, respectively.

We extend the model of housing market frictions by also allowing for rationing of owners, that is, given their circumstances some owners would rather be renting. Similarly, for renters, given their circumstances some renters would rather be owning. Both types of rationing express financial and mobility frictions. Let the numbers of rationed individuals be $N_{u,rent}$ and $N_{u,own}$; unfulfilled owners and renters, respectively. Correspondingly, let the respective shares of mismatched renters, who would rather own, and mismatched owners, who would rather rent, denoted by msm^R and msm^H, respectively, be defined as follows:

$$\text{msm}^R = \frac{N_{u,rent}}{N^{WR} + N^{UR}},\ \text{msm}^H = \frac{N_{u,own}}{N^{WH} + N^{UH}}, \tag{A.32}$$

If only rationed renters (alternatively, unfulfilled owners) are introduced, $msm^R \neq 0$, and not rationed owners (alternatively, unfulfilled renters), $msm^H = 0$, the first three equations in Eq. (A.22) continue to hold with the modification that instead of $\bar{\lambda}^H$, the rate at which the non-rationed renters find dwelling units, we now have $\lambda^H = \bar{\lambda}^H(1 - \text{msm}^R)$.

By definition, the rental housing stock may be occupied by employed or unemployed renters or be vacant. Rental housing per capita thus satisfies

$$r = n^{WR} + n^{UR} + \frac{\upsilon^R}{R} r. \tag{A.33}$$

This allows us to rewrite Eq. (A.12), the supply equation for rental housing stock, for the expected value of a vacant rental unit, V^R,

$$V^R = c_0 + c^R \left(n^{WR} + n^{UR} + \frac{\upsilon^R}{R} r \right). \tag{A.34}$$

The matching model for rental housing units, to be developed shortly, allows us to obtain an expression for the "demand" for rental housing units.

A.3 Owner Rationing

Introducing owner rationing in the form of owners who wish to rent but cannot (alternatively, unfulfilled renters), $msm^H \neq 0$, requires a greater modification of the model. That is, there are now transitions of owners, unemployed and employed, into renters. This needs to be accounted for in the Bellman equations and in the system of equations Eq. (A.22) that determine the equilibrium distribution of agents across states. The number of employed and unemployed renters must account for inflow from employed and unemployed owners who are mismatched and would rather be renters.

The algebra however tedious is straightforward. Specifically, it involves two steps. First, in Eq.'s (A.19)–(A.21), $\bar{\lambda}^H$ is replaced by $\lambda^H \equiv \bar{\lambda}^H(1 - \text{msm}^R)$. Second, the term $\lambda^R n^{WH}$, where $\lambda^R \equiv \bar{\lambda}^R(1 - \text{msm}^H)$, is added to the lhs of Eq. (A.19), the term $\lambda^R n^{UH}$ is added to the lhs of Eq. (A.20), and the term $-\lambda^R n^{WH}$ is added to the rhs of Eq. (A.21), where $\bar{\lambda}^R$ denotes the rate at which owners make contacts with dwelling units for renting. The resulting counterpart of Eq. (A.22) is more complicated but still linear and thus straightforward to solve.[29] This extended model has the advantage that owning is no longer an absorbing state and a nonzero probability of renting is possible even if $\nu = 0$, which removes a drawback of the previous model.

[29] System (A.22) becomes:

$$\begin{bmatrix} \delta + \lambda^R + \nu & -\mu & \lambda^H & 0 \\ 0 & -\lambda^R & -\delta & \mu + \nu + \lambda^H \\ -\lambda^R & 0 & \delta + \nu - \lambda^H & -\mu \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} n^{WR} \\ n^{UR} \\ n^{WH} \\ n^{UH} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}. \tag{A.35}$$

These definitions allow us to complete the determination of the value functions for rental housing units, V_v^R and V_o^R. At equilibrium, vacant and occupied rental units must earn the market return, that is:

$$\rho V_v^R = \text{maint} + \gamma^R \left[V_o^R - V_v^R \right]; \tag{A.36}$$

$$\rho V_o^R = \kappa + \bar{\lambda}^R \frac{\text{msm}^R (N^{WR} + N^{UR})}{H - N^{WH} - N^{UH}} \left[V_v^R - V_o^R \right], \tag{A.37}$$

where the matching rates with dwelling units of prospective renters and of prospective owners, γ^R and γ^H, are now defined as:

$$\gamma^R = \bar{\lambda}^R \frac{(1 - \text{msm}^H)(n^{WH} + n^{UH})}{r - n^{WR} - n^{UR}}; \; \gamma^H = \bar{\lambda}^H \frac{(1 - \text{msm}^R)(n^{WR} + n^{UR})}{h - n^{WH} - n^{UH}}. \tag{A.38}$$

By solving the system of linear equations (A.36)– (A.37) in terms of (V_v^R, V_o^R) we obtain an expression for the expected value of a vacant rental unit, V^R, from the demand side:

$$V^R = \frac{\upsilon^R}{R} V_v^R + \left(1 - \frac{\upsilon^R}{R} \right) V_o^R. \tag{A.39}$$

By equating V^R from Eq. (A.34) and Eq. (A.39), and in view of Eq. (A.33), which expresses the allocation of the per capita rental stock into employed and unemployed renters and vacant units, along with the system of Eq.'s (A.36) and (A.37), the remaining endogenous variables, that is, the vacancy rate and the rental capital stock per capita, $(\frac{\upsilon^R}{R}, \frac{R}{N})$, are determined. Here we take κ, the housing rent as given.[30] Noting that msm^H, msm^R, and maint are given, the solutions for $(n^{WR}, n^{UR}, n^{WH}, n^{UH})$ are obtained from the augmented eq. (A.22), now eq. (A.35) in footnote 29, explicitly for the vacancy rate in the rental housing market, $\frac{\upsilon^R}{R}$.

A.3.1 Housing Beveridge Curves

The Beveridge Curve for labor markets is a well-established and a widely researched concept. See Pissarides (1985; 1986) and Blanchard and Diamond (1989). The intuitive similarities

[30] Rental housing transactions also involve landlords and prospective tenants coming into contact. For symmetry with our treatment of the ownership market, we could specify the determination of the rent, κ, by means of bargaining between landlords and prospective tenants. But the agreements typically lead to spells of stay which are shorter than ownership spells [Henderson (1987)], and it is thus appropriate to assume that κ is determined competitively.

between housing and labor markets motivates us to exploit analogies in order to obtain a Beveridge Curve for housing. Indeed, it is remarkable that this has not been done to date. Analogous to vacancies in labor markets, which is unsatisfied demand for workers by firms, there correspond prospective buyers and prospective renters in housing markets, which is unsatisfied demand by individuals for housing. Analogous to unemployed individuals, which is unsatisfied demand for employment by individuals, there are unsatisfied renters who wish to own, and unsatisfied owners who wish to rent. They are prevented from doing so by frictions. Our development of Beveridge Curves for housing markets is adapted to the institutional features of housing markets, where there are owners and renters, and adheres to the notion of the Beveridge Curve as an accounting relationship at the steady state.

We work first with the homeownership market; the vacancy rate, vown, given by (3.2) and repeated here:

$$\text{vown} = \frac{\upsilon^H}{H} = \frac{H - N^{WH} - N^{UH}}{H} = 1 - \frac{1}{h}\left(n^{WH} + n^{UH}\right). \tag{A.40}$$

. We next express it in terms of a new concept which serves as the unemployment counterpart in housing markets. Allowing for mismatch for renters gives rise to unsatisfied homeownership demand. The respective solutions for n^{WH} and n^{UH} depend on $\bar{\lambda}^H(1-\text{msm}^R)$ instead of just $\bar{\lambda}^H$ and $\bar{\lambda}^R$ in the augmented case, and thus on the incidence of mismatch. Working with the solution (A.27) for the homeownership rate and assuming that $\text{msm}^H = 0$, we have that the equilibrium homeownership rate is now:

$$hr = n^{WH} + n^{UH} = \frac{\bar{\lambda}^H(1-\text{msm}^R)}{\bar{\lambda}^H(1-\text{msm}^R) + \nu}. \tag{A.41}$$

The equilibrium homeownership rate decreases with the probability of mismatch. That is, an increase, due to the mismatch of renters, in the number of individuals searching to buy homes reduces the homeownership rate.[31]

[31] In view of the generalization of the matching model in footnote 3 above, the rate at which buyers contact dwelling units, λ, may be written in terms of the matching function $\Gamma(.,.)$, and the ratio of potential buyers to vacant units, ϕ. That is:

$$\lambda = \Gamma(1, \phi^{-1}).$$

In developing a Beveridge Curve for the homeownership market, we propose the concept of the unfulfilled homeownership rate as the counterpart of the unemployment rate and normalize it appropriately. We start with the definition of the unfulfilled homeownership rate

$$\text{uhr} = \frac{N_{u,rent}}{N_{u,rent} + N^{WH} + N^{UH}}.$$

This quantity is at most equal to the rental rate, and therefore normalizing it by the rental rate yields the relative unfulfilled homeownership rate,

$$\text{ur}^H = \frac{\text{uhr}}{n^{WR} + n^{UR}}. \tag{A.42}$$

This serves as our analog of the unemployment rate for the ownership market. It ranges between a minimum of 0 and a maximum of 1, if all renters wish to become owners, which Head and Lloyd-Ellis assume.

By manipulating the definitions we may express ur^H in terms of the n's. That is:

$$\text{ur}^H = \frac{\text{uhr}}{n^{WR} + n^{UR}} = \frac{\text{msm}^R}{\text{msm}^R(n^{WR} + n^{UR}) + n^{WH} + n^{UH}}.$$

Solving the flow equations while using Eq. (A.41), expressing msm^R in terms of ur^H, and substituting back into the definition of the vacancy rate for owners (A.40) yields the analog of the Beveridge Curve for the home ownership market:

$$\text{vown} = 1 - \frac{1}{h} + \frac{1}{h} \frac{\nu}{\bar{\lambda}^H(1 - \text{msm}^R) + \nu} \frac{1}{\text{ur}^H}. \tag{A.43}$$

Thus, the Beveridge Curve for the homeownership market is a decreasing function of ur^H, the respective homeownership "unemployment rate," a result that agrees with the Beveridge Curve for labor markets.[32] In this expression, the owner-occupied housing stock per capita, h, is endogenous, which may cause the Beveridge Curve to shift and tilt by the cyclical variation in h. From Eq. (A.43), it follows that when the matching rate of prospective homeowners with dwelling units increases, as during an upswing in the housing cycle, the housing Beveridge Curve shifts downwards, implying greater efficiency in the housing markets (just as with the labor market Beveridge Curve).

[32]The expression in Eq. (A.43) is modified if $\text{msm}^H \neq 0$, but its relationship to respect to ur^H is not affected.

Turning next to the rental market, we propose the concept of the unfulfilled rental rate as the analog of the unemployment rate for the rental market. We start with the definition of the auxiliary quantity:

$$\text{urr} = \frac{N_{u,own}}{N_{u,own} + N^{WR} + N^{UR}},$$

which may be at most equal to the home ownership rate, if all owners wish to be renters: $N_{u,own} = N^{WH} + N^{UH}$. and therefore normalizing it by the home ownership rate yields the unfulfilled rental rate,

$$\text{ur}^R = \frac{\text{urr}}{n^{WH} + n^{UH}}. \tag{A.44}$$

This serves as our analog of the unemployment rate for the rental market. ur^R ranges between 0, which is the case for the assumption made by Head and Lloyd-Ellis namely that owners never leave that mode, and 1, which would mean that all owners wish to become renters. By manipulating the definitions we may express ur^R in terms of the n's. That is:

$$\text{ur}^R = \frac{\text{msm}^H}{\text{msm}^H(n^{WH} + n^{UH}) + n^{WR} + n^{UR}}.$$

From the solution of the flow equations (A.22) we have expressions for the n's in terms of parameters, including the imputed shares of mismatched renters and owners, msm^R and msm^H. Unlike in the case of the homeownership and rental rates when there is mismatch of owners only, the case with renter mismatch as well leads (as noted above) to a much more complicated modification of the flow equations. However, definition (3.3) still holds and would allow us to obtain an expression for the Beveridge Curve for the rental housing market,

$$\text{vrent} = 1 - \frac{1}{r} + \frac{1}{r}(n^{WR} + n^{UR}), \tag{A.45}$$

once the generalized flow equations have been solved. In general, both msm^R and msm^H enter the expressions for the housing unemployment rates, ur^R and ur^H and thus enter the expressions for both vacancy rates, as well. Since renting and owning are interdependent, in the most general case, it is not surprising that the vacancy rates share parameters. As with the vacancy rate in the homeownership market, the rental vacancy rate depends on r, the per capita rental housing stock, which is endogenous and varies procyclically, thus shifting and tilting the rental Beveridge Curve. Because the two housing vacancy rates share common determinants, in the most general case, it is appropriate to treat them as a system when we come to estimation.

A.4 The Labor Market with Frictions

So far the models have taken the wage rate and the employment rate as given. The treatment that follows completes the analysis by employing the same preference structure to examine symmetrically the labor market with frictions. Since housing market magnitudes enter the analysis, it follows that housing market outcomes show up as determinants of wages and the unemployment rate. That is, we embed the above model of individuals into a DMP model, by following the pared down approach of Pissarides (1985), as presented in Pissarides (2000), the canonical equilibrium model of search unemployment. By doing so, we also extend Head and Lloyd-Ellis (2012) by including a frictional labor market and a frictional rental housing market.

A.4.1 Labor market flows

Consider a labor market in a steady state with a fixed number of labor force participants, L who are either employed or unemployed. Time is continuous and agents have infinite time horizons. Recalling the basic details, jobs are destroyed at the exogenous rate δ; all employed workers thus lose their jobs and enter unemployment at the same rate. Unemployed workers enter employment at the rate μ which is endogenously determined, as we see shortly below. Frictions in the labor market are modeled by a matching function of the form

$$M = \mathcal{M}(uN, \upsilon N), \tag{A.46}$$

where uN, the number of unemployed workers, and υN, the number of job vacancies, are both stocks. The matching function is taken as increasing in both arguments, concave and exhibiting constant returns to scale.

Unemployed workers find jobs at the rate

$$\mu = \frac{\mathcal{M}(uN, \upsilon N)}{uN} = \mu(\theta),$$

where $\theta \equiv \frac{\upsilon}{u}$ is labor market tightness. It follows that firms fill vacancies at the rate

$$q = \frac{\mathcal{M}(uN, \upsilon N)}{\upsilon N} = \mathcal{M}\left(\left(\frac{\upsilon}{u}\right)^{-1}, 1\right) = q\left(\frac{\upsilon}{u}\right) = q(\theta), \tag{A.47}$$

and[33] that:

$$\mu'(\theta) > 0,\ q'(\theta) < 0.$$

The intuition is straightforward: the tighter the labor market, the easier it is for workers to find a job, and the more difficult it is for firms to fill a vacancy. A steady state in the labor market requires that the unemployment rate is constant over time. This occurs when the inflow from employment into unemployment, $\delta(1-u)N$, equals the outflow from unemployment to employment, $\mu(\theta)uN$. The steady-state unemployment rate[34] is thus given

[33]By definition:

$$\mu(\theta) = \theta q(\theta).$$

Differentiating with respect to θ we have:

$$\mu'(\theta) = q(\theta) + \theta q'(\theta).$$

From the definition of q the second term above becomes:

$$\theta q'(\theta) = -\theta \frac{\partial \mathcal{M}}{\partial(uN)}\left(\frac{\upsilon}{u}\right)^{-2} = -\frac{\partial \mathcal{M}}{\partial(uN)}(\frac{\upsilon}{u})^{-1} < 0.$$

Greater labor market tightness reduces the rate at which firms fill their vacancies. Therefore,

$$\mu'(\theta) = \mathcal{M}\left(\theta^{-1}, 1\right) - \theta^{-1}\frac{\partial \mathcal{M}}{\partial(uN)} > 0.$$

The inequality follows from the concavity of $\mathcal{M}(\cdot,\cdot)$ by a simple geometric argument, provided that $\lim_{uN\to 0} \frac{\partial \mathcal{M}(\sqcap\mathcal{N},\upsilon\mathcal{N})\partial(uN}{\to}\infty$.

[34]The full dynamic equation for the unemployment rate readily follows: At any point in time, $(1-u)N$ people are employed. Of these, per unit of time, $(1-u)N\delta dt$ people lose their jobs and enter unemployment. Again, during time dt, $uN\mu\left(\theta\right)$ are finding jobs, thus reducing the ranks of the unemployed and νdtN people enter the economy and become unemployed. Consequently,

$$d(un) = Ndu + udN = (1-u)N\delta dt - uN\mu\left(\theta\right)dt + \nu N dt.$$

Using the fact that $\frac{dN}{N} = \nu dt$ and rewriting this as a differential equation we have:

$$N\dot{u} + u\nu N = (1-u)N\delta - uN\mu\left(\theta\right) + \nu N. \tag{A.48}$$

The Beveridge Curve follows if we impose the condition that the unemployment rate remains constant, as:

$$u = \frac{\delta + \nu}{\delta + \mu(\theta) + \nu}. \tag{A.49}$$

Since $\mu(\theta)$ is increasing in its argument, Eq. (A.49) also implies a negative relationship, at the *steady state*, between unemployment and vacancies known as the Beveridge Curve, typically depicted on an unemployment rate – vacancy rate, (u, v) space. In an important sense, this is a mechanical accounting relationship, the consequences of flow balance, and it is this feature that we sought to emulate in Section A.3.1 above in defining Beveridge Curves for housing markets.

A deterioration of matching efficiency, i.e., a decline in job finding given a certain level of tightness, results in an outward shift of the Beveridge curve in the (u, v) space. An increase in the job destruction rate, possibly induced by faster sectoral reallocation of jobs, is also associated with an outward shift of the Beveridge curve. The Beveridge curve, computed using U.S. monthly data on unemployment and vacancies, is regularly reported by the BLS and is based on its Job Openings and Labor Turnover Survey (JOLTS) [`www.bls.gov/ljt`]. The movements in the unemployment rate, u, are measured here as unemployment divided by the labor force, and in the job openings (vacancy) rate, v, are measured here as openings divided by employment plus openings. Monthly observations are used to track the business cycle. See Figures 4 and 5, main text.

During the Great Recession, a marked outward shift in the Beveridge Curve has been observed. Earlier recessions were also associated with such shifts, though not as pronounced.[35] The reasons for this shift are not yet fully understood. However, it is clear that the curve is turning around, exactly as predicted by Pissarides' theory. We come to that shortly below. This feature of the observed Beveridge Curve has consequences for the housing market, and it is one of the aims of the present paper to explore it fully.

equilibrium unemployment $\dot{u} = 0$.

[35] For recent discussions of shifts in the Beveridge Curve for labor markets, see Elsby *et al.* (2014) and Diamond and Sahin (2014).

A.4.2 Hiring by Firms and the Job Creation Condition

Jobs are created by firms that decide to open new positions. Job creation involves some costs and firms care about the expected present value of profits, net of hiring costs. The unit price of a firm's output is p_g, which for consistency with the earlier part of the paper can be set equal to 1, as the good is the numeraire. Assume, as is standard in this literature, that firms are small, in the sense that each firm has only one job that is either vacant or occupied by a worker. There is a flow cost, associated with a vacancy, defined in terms of the value of the output, $p_g c$, per unit of time. Let V_u denote the expected present value of having a vacancy unfilled and V_{fH} and V_{fR}, the corresponding values of having a vacancy filled, by a worker who is an owner and a renter. Although owners and renters are perfect substitutes in production, the logic of the bargaining model suggests that their tenure status be taken into consideration. A job vacancy is an asset from which the firm expects to earn profit. A job vacancy is filled with an owner, or a renter, at the rate $(n^{WH} + n^{UH})q(\theta)$, or $(n^{WR} + n^{UR})q(\theta)$, respectively, whereas an occupied job is destroyed at the rate δ. The value functions associated with a vacancy and a filled job satisfy, respectively, the following equations:

$$\rho V_u = -p_g c + (n^{WH} + n^{UH})q(\theta)(V_{fH} - V_u) + (n^{WR} + n^{UR})q(\theta)(V_{fR} - V_u). \tag{A.50}$$

$$\rho V_{fH} = p_g - w^H + \delta(V_u - V_{fH}), \rho V_{fR} = p_g - w^R + \delta(V_u - V_{fR}). \tag{A.51}$$

The l.h.s. of Eq. (A.50) is the opportunity cost per unit of time of a vacancy. Its r.h.s. is the expected return, when costs are incurred per unit of time, $p_g c$, plus the expected capital gain from a job vacancy's being filled by an owner, $(n^{WH} + n^{UH})q(\theta)(V_{fH} - V_u)$, or a renter, $(n^{WR} + n^{UR})q(\theta)(V_{fR} - V_u)$. Similarly, the l.h.s.'s of the equations in (A.51) are the opportunity cost per unit of time of a filled vacancy, ρV_{fj}; their r.h.s.'s are the expected return, which consist of output minus the wage rate, profit per unit of time, $p_g - w^j$, plus the expected capital gain from a job's becoming vacant, $\delta(V_u - V_{fj}),\ j = H, R$.

Hiring by firms is done indirectly by opening vacancies. Firms open vacancies as long as it is profitable to do so. As firms open up vacancies, the value of a vacancy decreases. At the free entry equilibrium, $V_u = 0$. Using this in Eq. (A.51), and solving for V_{fH} and V_{fR}

yields:

$$V_{fH} = \frac{p_g - w^H}{\rho + \delta},\ V_{fR} = \frac{p_g - w^R}{\rho + \delta}, \tag{A.52}$$

Substituting into Eq. (A.50) yields

$$(n^{WH} + n^{UH})w^H + (n^{WR} + n^{UR})w^R = p_g - (\rho + \delta)\frac{p_g c}{q(\theta)}. \tag{A.53}$$

Once filled, each job produces a unit of output per unit of time. It is equal to the expected wage rate plus the capitalized value of the firm's hiring cost. A vacancy once created is expected to last for $q(\theta)^{-1}$ periods of time, generating costs $\frac{pc}{q(\theta)}$. Each vacancy is created with probability δ per unit of time and the hiring cost incurs an interest cost at a rate ρ. The capitalized value of the firm's hiring cost is given by $(\rho + \delta)\frac{p_g c}{q(\theta)}$. From this relationship, since q is decreasing in labor market tightness, θ, the higher the expected wage rate, $(n^{WH} + n^{UH})w^H + (n^{WR} + n^{UR})w^R$, the lower the labor market tightness.

Equ. (A.53) will be referred to as the *job creation* condition. It plays the role of the demand for labor in the standard model of a labor market without frictions, where the quantity of labor is represented by labor market tightness, $\theta = \frac{v}{u}$, the ratio of the vacancy rate to the unemployment rate. Note, in equilibrium, from Equ. (A.53), that given p_g and the expected wage rate, the incentive to create vacancies is reduced by a higher real interest rate, a higher job destruction rate and a higher vacancy cost. Vacancy creation is encouraged by improved matching efficiency that exogenously increases the rate at which the firm meets job searchers.

A.5 Wage bargaining

Since the labor market is characterized by frictions and bilateral meetings, the standard wage determination mechanism is not appropriate. The main approach that has been used by the markets with frictions literature assumes that bargaining between the employer and the worker determines the wage rate. In the remainder of this section we distinguish first the logic of our model which requires that we distinguish between homeowners and renters in their bargaining with employers.[36] We then return to examine the implications of assuming that wage bargaining is not conditional to housing tenure.

A.6 Wage bargaining distinguishing owners and renters

The logic of our model suggests that if firms bargaining with job seekers could distinguish between homeowners and renters, then we would expect that wage bargain be conditional on tenure choice. This delivers, as we see shortly, a more general model allowing for richer interactions between labor and rental and homeownership housing markets. After we have developed the model we discuss how this outcome may be sustained in the light of the fact that homeowners and renters as workers are perfect substitutes in production.

A.6.1 Homeowners' bargaining and labor market equilibrium

The expected capital gain for an unemployed homeowner from becoming employed is equal to $W^H - U^H$. A firm, on the other hand, gives up $V_u = 0$, in order to gain V_{fH}. Following generalized Nash bargaining, the wage rate is determined so as to split the total surplus,

$$\text{Total Surplus}^H = W^H - U^H + V_{fH} - V_u, \tag{A.54}$$

in order to

$$\max_{w^H} : \left(W^H - U^H\right)^{1-\sigma_L} \left(V_{fH} - V_u\right)^{\sigma_L}, \tag{A.55}$$

where $1 - \sigma_L$ is a measure of the worker's relative bargaining power. With free entry of vacancies, $V_u = 0$, and thus: $V_{fH} = \frac{p_g - w^H}{\rho+\delta}$. Note that the threat points in the Nash bargaining are taken to be what the worker and the firm would receive upon separation from each other. As Hall and Milgrom (2008) note, the job-seeker then returns to the market and the employer waits for another applicant. A consequence is that the bargained wage is a

[36] Our approach to both housing and labor markets is based on the original formulation of labor markets with frictions due to Pissarides (1985). It can be extended by means of competitive search models, along the lines of Diaz and Jerez (2013), which is applied to housing markets, and Moen (1997), which aims at job market applications.

weighted average of the applicant's productivity on the job and the value of unemployment. That latter value, in turn, depends in large part on the wages offered by other jobs.[37]

The first-order condition for the maximization of the total surplus is:

$$W^H - U^H = (1-\sigma_L)\left[W^H - U^H + V_{fH}\right],$$

which yields $\sigma_L(W^H - U^H) = (1-\sigma_L)V_{fH} = (1-\sigma_L)\frac{p_g - w^H}{\rho+\delta}$. From Eq. (A.30) we have $W^H - U^H = \frac{w^H - b}{\delta+\rho+\mu}$, which along with the first order condition above allows to solve for w^H :

$$w^H = \frac{\delta+\rho}{\delta+\rho+(1-\sigma_L)\mu(\theta)}\sigma_L b + \frac{\delta+\rho+\mu}{\delta+\rho+(1-\sigma_L)\mu(\theta)}(1-\sigma_L)p_g, \tag{A.56}$$

the *wage curve* for owners. Not surprisingly, it does not depend on housing market conditions, provided that once individuals become homeowners, they stay as homeowners and do not move. Of course, this would no longer be the case were we to modify the model and allow for turnover for homeowners, while staying either in the owneship mode or transiting to the rental mode. It can be verified that the r.h.s of Eq. (A.56) is increasing in θ, the labor market tightness.

A.6.2 Renters' bargaining and labor market equilibrium

Working in like manner, we formulate the bargaining problem for renters in order to obtain the wage curve for renters. Because renting is a transitional state, as individuals look forward to becoming owners, the wage curve reflects conditions both for renters and owners. The bargaining model is defined as maximizing

$$\max_{w^R} : \left(W^R - U^R\right)^{1-\sigma_L}\left(V_{fR} - V_u\right)^{\sigma_L},$$

[37] Some researchers have made alternative assumptions about the threat points. Hall and Milgrom (2008) assume that the threat point is to delay and postpone bargaining and agreement instead of threatening to walk out of the deal, as Pissarides does. "The bargainers have a joint surplus, arising from search friction, that glues them together." Hall and Milgrom (2008). They assume that the threats are to extend bargaining rather than to terminate it. The result is to loosen the tight connection between wages and outside conditions of the Mortensen–Pissarides model. When the labor market is hit with productivity shocks, the Hall–Milgrom bargaining model delivers greater variation in employer surplus, employer recruiting efforts, and employment than does the Nash bargaining model.

subject to a total surplus condition, like (A.54), yields:

$$\sigma_L(W^R - U^R) = (1-\sigma_L)V_{fR} = (1-\sigma_L)\frac{p_g - w^R}{\rho+\delta}. \tag{A.57}$$

where we used Eq. (A.51). By using the solution for $W^R - U^R$ from Eq. (A.31) in Eq. (A.57) we obtain the wage curve for renters:

$$\frac{\delta+\rho+(1-\sigma_L)\mu+(1-\sigma_L)\gamma^H\sigma}{\delta+\mu+\rho+\gamma^H\sigma}(w^R - b) + \frac{\sigma_L\gamma^H\sigma}{(\delta+\mu+\rho)(\delta+\mu+\rho+\gamma^H\sigma)}(w^H - b) = \frac{1-\sigma_L}{\rho+\delta}p_g. \tag{A.58}$$

In contrast to Eq. (A.56), the wage curve for owners, the wage curve for renters exhibits spillovers from the labor market for owners. This solution holds even if we allow for owner rationing, that is, if $\text{msm}^R \neq 0$. In all expressions with $\bar{\lambda}^H$, $\bar{\lambda}^H(1-\text{msm}^R)$ takes its place. As we discussed earlier, a greater modification of the model is called for if we also introduce renter rationing, that is, if $\text{msm}^H \neq 0$. In that case, the wage curve for owners would also reflect the fact that there are transitions from ownership to renting, which makes both wage rates and labor market tightness to be simultaneously determined, and would thus reflect both housing markets and labor market parameters. The resulting solutions enter the determination of the relative stocks of individuals in different states and therefore the expressions for vacancy rates in the ownership and rental markets, Eq.'s (3.2) and (3.3) in the main text. It is for this reason that we use the housing vacancy rates in the regressions for the job vacancy rate.

A.7 Linking Housing and Labor Market Vacancies

The theory we have developed suggests that the labor market determines the wage rates, conditional on tenure status, (w^H, w^R), and labor market tightness, θ, which in turn determines the employment rate, $\mu(\theta)$, and the unemployment rate at the steady state, $u = \frac{\delta+\nu}{\delta+\mu(\theta)+\nu}$. This in turn implies a solution for the job vacancy rate. The wage and employment rates then enter the conditional value functions, which allows us to solve for the per capita rental and owner-occupied housing stocks, r, h. Finally, the housing vacancy rates, vown and vrent, defined in Eq.'s(3.2) and (3.3), respectively, follow. The details are tedious but the derivations are quite elementary.

Specifically, in view of Eq.'s (A.30) and (A.31), Eq. (A.16) is simplified to yield:

$$V^H = \frac{(1-\sigma)\gamma^H}{\rho + (1-\sigma)\gamma^H}[U^H - U^R].$$

Using this expression on the rhs of (A.15), substituting for P^W and P^R in Eq.'s (A.17) and (A.18), and using Eq.'s (A.28) and (A.29) allows us to solve for V^H. This solution for V^H contains h. Substituting into the lhs of Eq. (A.13) gives an equation in h, the per capita owner-occupied housing stock. This determines the per capita ownership housing stock.

Working in a like manner we can solve for the rental housing stock, R, and its per capita value r. Specifically, from the Bellman equations for the conditional value functions for rental units, Eq.'s (A.36) and (A.37), we can solve for V_v^R and V_o^R, and by plugging into Eq. (A.39) we obtain an expression for the expected value of a vacant rental unit, V^R. This expression includes both per capita housing stocks, r and h. By substituting into the lhs of the supply equation for rental housing stock, Eq. (A.34), we obtain an equation for r, which includes h, and the rental vacancy rate, $\frac{v^R}{R}$. Finally, this equation together with Eq. (A.33) as a simultaneous system determine the per capita rental housing stock, r, and the rental vacancy rate, $\frac{v^R}{R}$. This solution takes the rental rate, κ, as given. If instead of this simplifying assumption we could, following the logic of the DMP model, alternatively assume that housing rental rate is determined by bargaining between a prospective tenant and a landlord. The bargaining model will introduce an additional equation which would determine κ. We think that for our purposes it would be unnecessary to complicate the model even further.

A.7.1 An Augmented Beveridge Curve

The job creation condition, Eq. (A.53), which equates the expected wage rate to the net expected benefit to the firm from hiring, along with the definitions of owner and rental vacancy rates, Eq. (3.2) and (3.3), respectively, takes the form of a relationship between labor market tightness and the housing vacancy rates. Specifically, by solving for the homeownership rate, $n^{WH} + n^{UH}$, from Eq. (3.2), and for the rental rate, $n^{WR} + n^{UR}$, from (3.3), and by substituting into Eq. (A.53), the resulting equation that follows expresses labor market tightness as a function of the owner and rental vacancy rates. By substituting into this structural relationship for (w^H, w^R) from Eq.'s (A.56) and (A.58), the wage curves for owners and renters, we obtain a reduced form which we may take to the data.

That is, the job creation condition, Eq. (A.53), may be written as,

$$h(1 - \text{vown})w^H + r(1 - \text{vrent})w^R = p_g - (\rho + \delta)\frac{p_g c}{q(\theta)}. \tag{A.59}$$

Labor market tightness enters not only on the r.h.s., but also on the l.h.s., because $\mu(\theta)$ enters the wage curves, Eq.'s (A.56) and (A.58), which depend on $\mu(\theta)$. The intuition of this result is straightforward. In posting vacancies, firms firms recognize that they may attract either unemployed renters or unemployed owners. Since wage rates are set via firm-worker bargaining, they do depend on workers' housing tenure status. Therefore, firms' equating the expected contribution to profit from an additional unit of employment to the expected wage naturally generates a dependence between labor market tightness and housing market vacancy rates. It is this spillover between labor and housing Beveridge curves that is highlighted in our empirical analysis.

B Tenure Choice Estimation

To obtain ur^H and ur^R, the "unemployment rates" for homeowners and renters, respectively, we estimate the following equation for the propensity for household head i in MSA m, and year t to be a homeowner:

$$\text{own}^*_{i,m,t} = \alpha_0 + \alpha_1 \frac{\text{index}^{\text{value}_{imt}}}{\text{index}^{\text{rent}_{imt}}}$$
$$+\alpha_2 \text{income}^p_{imt} + \alpha_2 \text{income}^T_{imt} + \mathbf{X}_{imt}\alpha_4 + \epsilon_{it}, \tag{B.1}$$

where the discrete indicator $\text{own}_{i,m,t} = 1$, if

$$\epsilon_{it} \geq -\left(\alpha_0 + \alpha_1 \frac{\text{index}^{\text{value}_{imt}}}{\text{index}^{\text{rent}_{imt}}} + \alpha_2 \text{income}^p_{imt} + \alpha_2 \text{income}^T_{imt} + \mathbf{X}_{imt}\alpha_4 \right), \tag{B.2}$$

and renting otherwise: $\text{own}_{i,m,t} = 0$, where $\text{index}^{\text{value}}_{imt}, \text{index}^{\text{rent}}_{imt}$ are rental and house value indices (to be explained below).[38] The value to rent ratio is included in the housing tenure equation to capture the relative cost of owning versus renting. The variables income^p_{imt} and income^T_{imt} are permanent and transitory annual household income, respectively. Due to mortgage market imperfections, they have different impacts. In the absence of suitable data, income^p_{imt} proxies for wealth. The vector $\mathbf{X}_{i,m,t}$ includes socioeconomic characteristics, like individual education, gender, race, age, and household size.

We generate the auxiliary variables $\text{index}^{\text{rent}}_{mt}, \text{index}^{\text{value}}_{mt}$ from the following hedonic equations, for renters and owners, respectively:

$$\ln(\text{rent}_{imt}) = \alpha_{0,m} + \alpha_1 \mathbf{Y}_{1,i,m,t} + \epsilon_{1,i,t}, i = \text{renter}, \tag{B.3}$$

[38]One could think of this estimation approach as a reduced form corresponding to a structural form, like in Henderson and Ioannides (1986). That is, the tenure choice probability is evaluated in terms of a comparison of indirect utility values, including shocks, associated with renting and owning. Individuals, however, may not attain their optimal mode of tenure, because of rationing associated with financial frictions and the like.

where rent_{imt} is reported monthly rent paid, and

$$\ln(\text{price}_{imt}) = \beta_{0,m} + \beta_1 \mathbf{Y}_{1,i,m,t} + \beta_2 \mathbf{Y}_{2,i,m,t} + \epsilon_{2,i,t}, i = \text{owner}, \tag{B.4}$$

where price_{imt} is the respondent's estimate of the property's market price, $\mathbf{Y}_{1,i,m,t}$ denotes a vector of dwelling unit characteristics, and $\mathbf{Y}_{2,i,m,t}$ property tax and lot size. The intercepts of the above hedonic equations vary by MSA, m. Then the rent and value indices are calculated as follows:

$$\text{index}_{mt}^{\text{rent}} = 100 \times \exp[\alpha_{0,m}]; \tag{B.5}$$

$$\text{index}_{mt}^{\text{value}} = 100 \times \exp[\beta_{0,m}]. \tag{B.6}$$

The predicted values of the permanent and transitory components of household incomes, income_{imt}^{p}, income_{imt}^{T} are obtained as the predicted value and residual, respectively, from the following equation:

$$\ln(\text{income}_{imt}) = \gamma_{0,m} + \gamma_1 \mathbf{Z}_{i,m,t} + \epsilon_{2.m,t} \tag{B.7}$$

where income_{imt} denotes reported household income and $\mathbf{Z}_{i,m,t}$ denotes a vector that includes functions of education, age, race, and gender.

C Generating the Composite Help Wanted Index

The method we use to construct the full National help wanted index up through 2014 is similar to Barnichon (2010) but not as complicated. It consists of the following 4 steps.

Step 1. 1951-1994: online help-wanted (HWOL) index, O_t does not exist, $O_t = 0$. As in Barnichon, we use the HWI print index through 1994; $H_t = P_t$ where H_t and P_t are the composite and print Help-Wanted advertising indices, respectively.

Step 2. 1995-2005:5: $O_t > 0$, but not observed. Step 2 is also the same as in Barnichon. To get the composite index, we inflate the print index by the estimated print share, $\hat{S}_t^p$. That is: $H_t = P_t/\hat{S}_t^p$. But the procedure we use to estimate this is a simpler version that in Barnichon. That is, we fit a quartic polynomial to P_t over 1951-2010:10 (the last month that we have the National HWI print index), and estimate H_t as the ratio of the polynomial's value at time t to the polynomial's value in 1994:12. Figure 2 reproduces Figure 2 in Barnichon. One can see that the print share based on the polynomial trend fits the Sp-JOLTs print share very well. What is key here is that, unlike in Barnichon, the Sp-JOLTs print share DOES exhibit a constant rate of decline and does NOT appear to follow an $S-$curve. Hence, we use the polynomial trend in the above calculation to estimate H_t and not the more complicated method used by Barnichon.

Step 3. 2005:6-2010:10: both O_t and P_t are observed. Same as in Barnichon: H_t is constructed from

$$\frac{dH_t}{H_{t-1}} = S^p_{t-1}\frac{dH_t}{P_{t-1}} + (1 - S^p_{t-1})\frac{dO_t}{O_{t-1}}$$

where O_t is the online help-wanted advertising index.

What we have from the online data is the total number of ads (seasonally adjusted and not seasonally adjusted) and the total number of new ads (seasonally adjusted and not seasonally adjusted). We use the seasonally adjusted total number of ads to construct O_t.

Step 4. 2010:11-2014:6: Only O_t is observed. We construct H_t from from $d\ln H_t = d\ln O_t$. That is, we assume that $S^p_t = 0$ starting in 2010:11 (the estimated value from the polynomial trend is 0.008). The composite index, the print index and the rescaled JOLTS index are plotted on Figure 5.

We can use the same procedure at the MSA level. The one complication is that the last date that the print index is observed varies across MSAs and can be prior to 2010.10; the earliest date for this is June 2005. Between this date and 2010:11 (call this Step 3.1), we use the inflated value of O_t to construct H_t from $d\ln H_t = \ln\frac{O_t}{1-\hat{S}^p_t} - \ln\frac{O_{t-1}}{1-\hat{S}^p_{t-1}}$.