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CHAPTER 8

Diffusion of Technological Change and Economic Growth

Yannis M. Ioannides

8.1 INTRODUCTION

One of the most significant developments in neoclassical economic theory is the theory of economic growth. While economic growth as a subject has received a lot of attention from the classical economists, the pioneering work by Solow (1956) gave it new impetus and elevated the subject to great prominence. The literature that has been produced is vast, and it has been eloquently reviewed in many survey articles and books (e.g., Burmeister and Dobell 1970; Dixit 1976; Wan 1971).

A crucial development within growth theory is models of technological change. One of several concepts of technological change (or progress) which have been developed is the notion that innovations may be embodied in new capital goods. Johansen (1959) is the seminal paper that launched the topic. The key idea of that theory is differences in the substitutability between factors of production before and after investment takes place. One of the most interesting features of Johansen's model is that, under certain conditions, per capita production may grow, stagnate, or even decrease over time. Even though the growth rate of production and output per capita do not depend on the savings rate, the latter affects the time-independent factor in these growth rates. This, in particular, implies for countries with different savings rates, which however start at the same initial position, the country with a higher propensity to save will start out with the higher relative growth rate (before "the asymptote" is reached).

We are currently witnessing a renewed interest from economists in the fundamental determinants of economic growth. Lucas (1985), in a provocative reexamination of the performance of neoclassical growth theory in explaining economic development, has concluded that allowing for increasing returns is the only way for that theory to explain observed economic growth across countries and time. An essential element of Lucas' reexamination is that the steady state growth rate per capita magnitudes in standard versions of the neoclassical growth model does not depend on those behavioral parameters which may be interpreted as determinants of the propensity to save. E.g., in the model in Cass (1965), per capita consumption and per capita capital grow at a rate equal to the (exogenous) rate of neutral technological change divided by the share of labor in aggregate production.

The special notion of increasing returns that is invoked by Lucas, namely that they are external to the industry but internal to the economy as a whole, [1] has received support from Romer (1986), who argues that (technological) knowledge is the key factor responsible for such form for increasing returns.[2] Romer (1987) offers a critical review of the traditional growth accounting literature. He interprets the empirical evidence as implying an aggregate production function which contains no autonomous technological change factor, and exhibits capital elasticity close to unity and labour elasticity close to zero.[3] [4]

Several researchers, such as Baumol, et.al. (1985) and Baumol (1986), have emphasized that there are inherent factors in the economy which constrain the economy-wide propagation of cost improvements accomplished in some sectors - such as those associated with information technology. Baumol, et.al. (1985) emphasize the significance of what they call asymptotically stagnant sectors. There inputs from "very progressive sectors" are used jointly with inputs from "stagnant" sectors. They show that the share in total costs of progressive components diminishes continually while that of stagnant components increases both in real terms and as a share of total cost. Computers have brought about an extraordinary increase in labor productivity, but they are intermediate products and thus not consumed directly. They are used with complementary inputs to produce computational services. The argument is the cost of these complementary inputs comes to dominate the total cost - "the progressive component is innovating itself out of its cost-dominating positions" (ibid) - and ultimately the activity assumes all the characteristics of stagnant services.

This paper addresses the impact of the diffusion of technological change on the dynamics of economic growth. This topic has been recognized as worthy of attention by the growth literature of the 1960's, but has received surprisingly little attention. An exception is a recent paper by Shleifer (1986), in which the timing of innovations is endogenous. The innovating firm captures the whole market but prices its product at the marginal cost of inefficient firms. However, the advantages of innovation are rather short-lived. An interesting feature of Shleifer's model is that it leads to cyclical activity. Firms choose to innovate at time of high aggregate demand. Therefore, the resulting synchronization of innovations gives rise to a multiplicity of perfect-foresight equilibria.

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[1]Actually, this concept has been utilized by trade theorists [Chipman (1980)] and is central to the modern urban economics literature [see Henderson and Ioannides (1981) for an application in modeling growth of a system of cities]. Lucas, *op.cit.* also emphasizes the urban aspects of the operation of such externalities - motivated both by the significance of urbanization for economic growth and the presumed importance of externalities for understanding the development of urban economies. Production of knowledge (inventive activity), though not by increasing returns, was investigated by Shell (1967).

[2]The essence of the methodology applied by Romer (1986) is that knowledge is treated as an externality. Given the aggregate level of knowledge, firms' production functions are assumed to be concave in inputs - say firms' own technology and labor input. Then equilibrium is defined by equating the sum of individual firms' demand for knowledge to aggregate technology, which enters into individual production functions. A *critical* assumption is that from a planner's point of view, by treating all firms as identical, an individual firm's production function is convex in knowledge. No accumulation of physical capital, but only accumulation of knowledge takes place. Accumulated knowledge has increasing external effects, but is produced with diminishing returns.

Capital investment can have positive external effects through the creation of new knowledge, which spills over throughout the economy and may be propagated by the introduction of new intermediate goods. The latter is related to the total amount of invested capital, so that investment has an additional effect resembling that of an externality.

[3]Romer, ibid. does not explain why labor's coefficient should be so small, except that it could be explained by a putty-clay capital type of theory. Romer also corroborates the evidence in Baumol et.al. (1985) that computerization has not brought forth the magnitudes of benefits one would expect on the basis of the sheer reduction in the cost of computing machinery.

[4]A small labor elasticity is consistent with putty-clay theories of economic growth [Johansen (1959)].

One of them is always the steady-growth acyclical equilibrium, in which inventions are implemented immediately.[5]

Chari and Hopenhayn (1986) also endogenize the adoption of new technologies. A key, and innovative, characteristic of their model is that new technologies are embodied in the form of *human* capital. Young individuals choose the vintage of capital with which to work and thus acquire capital vintage-specific skills. The paper emphasizes the determination of the distribution of workers over vintage-specific skills, which they show to be single-peaked at the steady state. Thus, in general, it takes time for newer technologies to become popular, and technologies associated with different vintages are used at the same time.

From a modern perspective, the technological change of particular interest is the one associated with computerization. Certain stylized facts about computerization may be constructed as embodied, as in the case of digitally controlled machine tools, or disembodied, as in the case of software improvements. It takes a long time and substantial start-up costs to fully computerize a particular establishment, time during which the new technology cannot be utilized to full advantage. A related issue is that of flexibility.[6] Technological change associated with computerization may improve flexibility, but such flexibility may be utilized in a economy-wide sense only if all firms are computerized. Another crucial stylized fact about computerization is the significance of the network effect (Chow 1969; Katz and Shapiro 1986). The more computers have been installed, the easier it is to utilize an additional computer. However, what is significant here is the adoption of the same standard, or else there is no network effect. The advantages of computerization do not appear only within firms. E.g. firms may pass to one another masses of data in electronic form, but such communication is otherwise very costly.

This paper uses an aggregative neoclassical growth model, which includes explicit assumptions about the diffusion of technological change in order to study the ensuing dynamics. We show that the consideration of non-instantaneous diffusion of technological change in conjunction with embodied capital-augmenting technological change leads to a second-order differential equation in aggregate capital per capita. Even though this equation is difficult to solve, it allows us to conclude that the long-run growth rates of all magnitudes of interest do not depend upon the propensity to save. That conclusion is based on two alternative specific assumptions about the process of diffusion of technological change. Our result depends critically on the assumption that technological change is capital - augmenting, for which it is well-known that it is possible to aggregate across capital vintages. That along with the equalization of effective factor intensities across vintages ensures the validity of our results. The second-order differential equation that we obtain implies interesting dynamics, whose precise properties depend on the characteristics of the process of diffusion. The study of these dynamics is left for future work.

[5] Cost reductions associated with economy-wide computerization may be assumed to diminish at an increasing rate with the percentage of the economy which is computerized. To see this, consider the cost of having bills paid directly from bank accounts. The cost is miniscule in a fully computerized economy, and may be substantial in a partly computerized one.

[6] The issue of flexibility is of particular interest in its own terms [Kulatilaka (1986)]. Very much like a clay-putty model, embodied innovations may allow greater substitution ex post than the substitution possibilities embodied in the technology available *ex ante*. Flexible manufacturing systems are a case in point here. As far as modelling flexibility is concerned, dual models like that of Fuss (1977) [see Ansar, et.al. (1986) for a recent application] are superior for the purposes of empirical investigations to those utilized by Mizon (1974) and Malcomson and Prior (1979).

8.2. NEOCLASSICAL ECONOMIC GROWTH WITH DIFFUSION OF TECHNOLOGICAL CHANGE

8.2.1 Diffusion of Technological Change

Consider a neoclassical economy consisting of an infinity of firms with an aggregate measure of unity. At the beginning of time all firms have identical technologies. Their production function exhibits constant returns to scale. As time evolves some firms innovate. The proportion of firms which have innovated by time t is given by $y(t)$. The function $y(\cdot)$ satisfies the following conditions:

$$\dot{y} > 0; \ \lim_{t \to \infty} y(t) = 1.$$

We have now yet made any specific assumption about the nature of the process which described the diffusion of technological change through the economy. For the purposes of comparison we could consider a number of alternative processes. An assumption made frequently is that the spread of innovations across the economy is described by the so-called logistic growth equation:[7]

$$\frac{\dot{y}(t)}{y(t)} = c(1-y(t)),$$

where c is a positive constant. That is, the growth rate of $y(t)$, the proportion of all firms which have adopted the innovation by time t, is proportional to the number of firms which have not yet innovated. The higher that number the higher is the growth rate. It is an interesting property of logistic growth that innovation spread initially at increasing rate but ultimately at a decreasing one. Integrating the logistic growth equation yields:

$$y(t) = \frac{y(0)e^{ct}}{1-y(0) + y(0)e^{ct}}.$$

If innovations were to spread at a constant growth rate, then they would spread through the entire economy in finite time. This case is not particularly interesting in our context.

However, the case of a continuously decreasing growth rate is particularly interesting. In this negative exponential case we have:

$$y(t) = 1 - e^{-ct}.$$

Innovation takes the form of knowledge that is newly acquired and allows better utilization of productive factors.

8.2.2 Embodied Technological Change With Diffusion

As we described in Section A above, innovation takes the form of knowledge about how to implement capital-augmenting technological progress. For a firm which innovates at

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[7]See Allen (1982) for a justification of the logistic growth equation by means of a behavioral model of information transmission (involving Gibbs states with nearest neighbor potential), and Chow (1969) for an application on the growth of demand for computers).

time θ, one unit of capital yields $e^{\gamma\theta}$ efficiency units of capital, where γ is an exogenous rate of technological change. We further assume that technological progress is embodied in capital goods and that the technology is putty-putty. That is, a firm that has innovated at time θ and invests $I(\theta)$ units of capital, has an effective capital equal to $e^{\gamma\theta}I(\theta)e^{-\delta(t-\theta)}$ at any time $t \geq \theta$, where δ denotes a constant exponential rate of deterioration. Thus, the rate of output at t by a firm that innovated at time θ is given by:

$$F(\theta,t) = F(e^{-\delta t}e^{\beta\theta}I(\theta),L_i(\theta,t)), \qquad t \geq \theta. \tag{1}$$

where $\beta = \gamma + \delta$, and $(F\cdot,\cdot)$ is a variable-proportions production function which exhibits constant returns to scale. The subscript i is a mnemonic for *innovation.* After it has innovated, a firm may vary its output only by varying its labor input at time t, $L_i(\theta,t)$, $t \geq \theta$. We assume throughout that all existing vintages of capital are used at any point in time. This in effect, is an assumption about the curvature of the production function.[8]

Equilibrium in the labor market implies that all firms face the same wage rate. Thi requires that the marginal product of labor be equalized across all firms, that is, firms which have already innovated and thus operate with capital stocks of different vintages, as well as firms which have yet to innovate. This, in turn, along with the constant returns to scale assumption implies that the capital-labor ratio in firms which have not yet innovated, $k_s(t) = K_s(t)/L_s(t)$, is equal to the effective capital-labor ratios across all currently existing vintages:

$$k_s(t) = \frac{K_s(t)}{L_s(t)} = \frac{e^{-\delta t}e^{\beta\theta}I(\theta)}{L_i(\theta,t)}, \qquad \theta \leq t. \tag{2}$$

The subscript s is a mnemonic for *stagnant.*

Equilibrium in the labor market requires that the labor supply, N(t), be equal to the demand for labor by all firms, that is by those which have yet to innovate and by those which have already innovated. A proportion 1-y(t) belong to the former category and y(t) to the latter. To aggregate the labor demands by the firms that belong to the latter group we must take into consideration when they innovated. Therefore, we have:

$$N(t) = (1-y(t))L_s(t) + \int_{\theta\leq t} L_i\,(\theta,t)\dot{y}(\theta)d\theta. \tag{3}$$

Let N(t), the exogenous labor force (which is equal to total labor supply), grow at a constant exponential rate n. Equations (2) and (3) yield the total demand for capital:

$$k_s(t)N(t) = (1-y(t)K_s(t) + e^{-\delta t} \int_{\theta\leq t} e^{\beta\theta}I(\theta)\dot{y}(\theta)d\theta. \tag{4}$$

The total rate of output is obtained as follows:

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[8]That capital becomes obsolete in finite time is an assumption which has crucial consequences for the dynamics of the model in Johansen (1959).

Kurz (1963) criticises Johansen for failing to account for the endogeneity of obsolescence. As labor productivity might rise over time, certain vintages of capital might have to be retired before they have physically depreciated. Kurz defines as a terminal path the situation where each variable develops through time at a constant relative rate of change. He shows that on a terminal path, capital becomes obsolete after a constant length of time has elapsed from when first installed, $\theta = -\frac{1-\omega}{\lambda}\log(1-\omega)$, where $1-\omega$ is labor's relative share of output and λ is equal to the rate of technical progress in the vintage production function.

$$(1-y(t))F(K_s(t),L_s(t)) + \int_{\theta\leq t} F(e^{-\delta t}e^{\beta\theta}I(\theta),L_i(\theta,t))\dot{y}(\theta)d\theta$$

$$= f(1,k_s^{-1}(t))[(1-y(t))L_s(t) + e^{-\delta t} \int_{\theta\leq t} e^{\beta\theta}I(\theta)\dot{y}(\theta)d\theta]$$

$$= k_s(t)N(t)F(1,k_s^{-1}(t)). \tag{5}$$

Thus, output per capita is equal to $F(k_s(t),1)$ and depends only on the capital-labor ratio in the stagnant sector, $k_s(t)$. (5) exploits a well-known result of aggregation from growth theory, namely that constant returns to scale together with capital-augmenting embodied technological progress allow aggregation of capital of all vintages (Solow, 1959; Dixit 1976). A new feature in our analysis is that this aggregation still holds when a certain portion of the economy operates under different production conditions.

In order to complete the analysis, we must develop the equation of motion of the system and the determination of the rate of investment when innovation occurs. Even though it pays to adopt the innovation, the process of competition bids away all rents, reducing profits of innovating firms to zero, in the current period as well as in all future ones. Were it not for this, we should characterize the optimal amount of investment in terms of the trade off between the future stream of extra profits and the present cost of investment. It is thus natural to assume that the innovating firms simply convert their current capital into one that embodies the new technology. We assume that this conversion process is costless. Therefore, for the innovating firms we have for all t:

$$K_s(t) = I(t). \tag{6}$$

We retain the neoclassical growth theory assumption of a constant gross savings rate. We may obtain the equation of motion of the system by equating savings per unit of time to additional investment needed by the stagnating sector, $(1-y(t))\dot{K}_s(t)$, plus what is needed to make up for depreciation of capital in the stagnating sector, $\delta K_s(t)$. Capital in the stagnating sector that is freed by the diffusion of technological change is transformed into capital that embodies technological change. We thus have:

$$(1-y(t))\dot{K}_s + \delta K_s(t) = sN(t)f(k_s(t)), \tag{7}$$

where $f(k_s) \equiv F(k_s,1)$ denotes output per capita. Note that because technological change is embodied, no additional investment is made by firms which have already innovated, nor is depreciated capital replaced.

We shall transform (7) in order to make it tractable. First we divide both sides of (7) by N(t) and then use the auxiliary transformation $\psi = K_s/N$ and (6) to get:

$$(1-y(t))\dot{\psi}(t) + (\delta + (1-y(t))\psi$$

$$= sf((1-y(t))\psi(t) + e^{-(n+\delta)t} \int_{\theta\leq t} e^{\beta\theta}\psi(\theta)\dot{y}(\theta)d\theta). \tag{8}$$

Next we perform an integral transformation in order to convert the integro-differential equation (8) into a differential one. We define:

$$J(t) = e^{-(n+\delta)t} \int_{\theta \leq t} e^{(\beta+n)\theta}\psi(\theta)\dot{y}(\theta)d\theta. \tag{9}$$

From (9) by differentiating we obtain an expression for $\psi(\cdot)$ in the terms of $J(\cdot)$:

$$\psi(t) = Y(t)(\dot{J}(t) + (n+\delta)J(t)),$$

where $Y(t) \equiv (1/\dot{y}(t)e^{(\beta-\delta)t})$, a known function.

Furthermore,

$$\dot{\psi}(t) = \dot{Y}(t)(\dot{J}(t) + (n+\delta)J(t)) + Y(t)(\ddot{J}(t) + (n+\delta)\dot{J}(t)).$$

$J(\cdot)$ plays here the same role as aggregate capital per capita in vintage capital models (*c.f.*, Dixit *op.cit.*). It is now clear that (8), the equation of motion of the system, may be transformed into a second-order differential equation in $J(t)$ with non-linear coefficients:

$$\begin{aligned}(1-y(t))Y(t)(\ddot{J}(t) &+ [(1-y(t))(n+\delta)Y(t) + (1-y(t))\dot{Y}(t) \\ &+ (\delta + (1-y(t))n]\dot{J}(t) + [(n+\delta)\dot{Y}(t)(1-y(t)) + (\delta + (1-y(t))n]J(t) \\ = sf[(1-y(t))Y(t)\dot{J}(t) &+ ((1-y(t))Y(t)(n+\delta) + 1)J(t)].\end{aligned} \tag{10}$$

We have thus obtained a generalization of the law of motion in the neoclassical model of economic growth when the diffusion of technological change is not instantaneous (*c.f.*, Dixit, *op.cit..*, 91-94). This equation is expressed in terms of an aggregator for the capitals of different vintages, where the aggregation is weighted by the efficiency factors, multiplied by the speed of innovation diffusion. The fact that the law of motion is a second-order rather than a first-order differential equation is, of course, entirely due the non-instantaneous diffusion of technological change. Therefore, it could, in principle, give rise to much more complicated dynamics than the standard neoclassical growth model. This differential equation, however, is rather difficult to solve because it is not linear in terms of the derivatives of $J(t)$.

It would be interesting to compare the above solution with the case where technological change is disembodied. Unfortunately, as it is clarified in the Appendix, this case is intractable. Therefore in the following section we restrict ourselves to a comparison with the textbook case of embodied technological change, that is when diffusion is instantaneous.

The differential equation (10) is linear only if $f(\cdot)$ is linear in its argument. This may occur only if $F(K,N) \equiv AK + BN$, where A and B are constants. In that case, (10) becomes:

$$\begin{aligned}&(1-y(t))Y(t)(\ddot{J}(t) + \{[(1-y(t))(n+\delta-sA) + (\delta + (1-y(t))n)]Y(t) \\ &+ \ (1-y(t))\dot{Y}(t)\}\dot{J}(t) + [(n+\delta)(1-y(t))\dot{Y}(t) + \delta(n+\delta)Y(t) \\ &+ \ (n-sA)(1-y(t))(n+\delta)Y(t)]J(t) = sB.\end{aligned} \tag{10'}$$

For particular specifications of $y(t)$, this second-order differential equation which describes the law of motion of the system may be solved in closed form. (10') is a linear second-order differential equation with non-linear coefficients.

It is easy to see that the long-run rate of growth of aggregate capital depends upon the specification of the process which describes the diffusion of technological change.

However, it can be shown, by working with (10), that both for the logistic as well as the negative exponential class the asymptotic growth rate of aggregate capital per capita is independent of the propensity to save. This demonstration is rather tedious and uninteresting and is thus deleted here. It involves assuming a long-run rate of growth for capital per capita and substituting in (10) so as to compute the unknown asymptotic growth rate.

8.2.3 Embodied Technological Change and Instantaneous Diffusion

In order to assess the impact of the slow diffusion and adoption of technological change, we compare with the vintage capital model when the diffusion occurs uniformly throughout the economy. The equation of motion for the putty-putty model with embodied technological change is:

$$\psi(t) = sf(J(t)), \tag{11}$$

where $\psi(t) \equiv I(t)/N(t)$ denotes investment per capita, and $J(t)$ is defined as:

$$J(t) = e^{-(\delta+n)t} \int_{\theta\leq t} e^{(n+\beta)\theta}\psi(\theta)d\theta.$$

$J(t)$ is, again, an aggregate measure of capital per capita across vintages. As we mentioned earlier, such an aggregation is possible because technological change is capital-augmenting. Working as before, we obtain the equation of motion in terms of $J(t)$:

$$\dot{J}(t) + (n+\delta)J(t) = e^{\gamma t}sf(J(t)). \tag{12}$$

Equation (12) is the counterpart of Equation (8. It is again easy to solve (12) for a variety of specifications of the aggregate production function. If $F(\cdot,\cdot)$ is Cobb-Douglas, then (12) may be solved as a Bernoulli equation. Specifically, if $f(J) \equiv J^{\alpha}$, then by multiplying both sides of (12) by $(1-\alpha)J^{-\alpha}$ we obtain a linear differential equation in terms of $u \equiv J^{1-\alpha}$ which may then be solved in closed form.

By working in this fashion we have:

$$\dot{u}(t) + (1-\alpha)(n+\delta)u(t) = (1-\alpha)se^{\gamma t}. \tag{13}$$

The general solution of (13) has the form:

$$u(t) = (1-\alpha)\left[\bar{k} + s\int_0^t e^{\gamma t}e^{(1-\alpha)(n+\delta)t}dt\right]e^{-(1-\alpha)(n+\delta)t},$$

which yields the aggregate capital per capita as

$$J(t) = (1-\alpha)^{1/1-\alpha}\left[\bar{k} - \frac{s}{\gamma+(1-\alpha)(n+\delta)} + se^{(\gamma+(1-\alpha)(n+\delta))t}\right]^{1/1-\alpha} e^{-(n+\delta)t}. \tag{14}$$

It readily follows from (14) that the asymptotic rate of growth of aggregate capital per capita does not depend on the savings rate.

Therefore we conclude that the complicated dynamics characterizing the growth model with slow diffusion of embodied technological change are entirely due to our assumption about how innovations spread in the economy.

APPENDIX

Disembodied Technological Change

It is somewhat surprising that the case where technological change is disembodied and diffusion of technological changes is not instantaneous is intractable. To show this, consider that technological progress is implemented as soon as news of it reaches firms. Any additional investment made after the innovation has been adopted also benefits from it. The assumption means that the adoption of technological change augments capital by a factor $e^{\gamma(t-\theta)}$, where θ is the point in time when the innovation was adopted. Let $L_s(t)$ and $L_i(\theta,t)$ denote the quantities of labor employed by a stagnant and an innovating firm, respectively. For equilibrium in the labor market, it is required that the marginal product of labor be equalized across both kinds of firms. Thus we have:

$$k(t) = \frac{K_s(t)}{L_s(t)} = \frac{e^{\gamma(t-\theta)}K_i(\theta,t)}{L_i(\theta,t)}, \; t \geq \theta. \tag{A.1}$$

and

$$N(t) = (1-y(t))L_s(t) + \int_{\theta \leq t} \dot{y}(\theta)L_i(\theta,t)d\theta. \tag{A.2}$$

As before, we assume that firms convert their capital into a form suitable for production after the innovation has been adopted:

$$K_i(t,t) = K_s(t). \tag{A.3}$$

For the total amount of capital in efficiency units we have:

$$K^*(t) = (1-y(t))K_s(t) + \int_{\theta \leq t} \dot{y}(\theta)e^{\gamma(t-\theta)}K_i(\theta,t)d\theta = k(t)N(t). \tag{A.4}$$

As for the amount of physical capital we have

$$K(t) = (1-y(t))K_s(t) + \int_{\theta \leq t} K_i(\theta,t)\,\dot{y}(\theta)d\theta. \tag{A.5}$$

The equation of motion for the system is obtained by equating savings per unit of time to additional investment needed by the stagnating sector, $(1-y(t))\dot{K}_s(t) - \dot{y}(t)K_s(t)$, plus new investment in the innovating sector, $\dot{y}(t)K_s(t)$, plus what is need to make up for depreciation. That is:

$$(1-y(t))\dot{K}_s(t) + \int_{\theta \leq t} \frac{\partial K_i(\theta,t)}{\partial t} \dot{y}(\theta)d\theta + \delta K(t) = sN(t)f(k(t)). \tag{A.6}$$

If the dynamics of this model could be analyzed in a simple manner, then we could analyze the economic value of flexibility which is implied by disembodied technological change relative to embodied. To the extent that the value of flexibility is positive, it would be interesting to require that firms could costlessly implement the embodied technology but would have to incur an extra cost in order to adopt the disembodied technology. Unfortunately, the only tractable way in which disembodied technological change may be handled is to assume that the technological change capital-augmenting factor for firms which have adopted the innovation takes the form $e^{\gamma t}$. Consequently, factor use by such firms is independent of the point in time in the past when they adopted the innovation.

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