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Dynamic Evolution of the U.S. City Size Distribution.

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ABSTRACT

We present a model of city size distributions that emphasizes the importance of human capital accumulation. We then use it to explore the evolution of city size distributions in the United States by means of a newly constructed data set. The data are from the U.S. Census and cover metropolitan areas from 1900 to 1990. We examine in some detail the dynamics in the evolution, using both a variety of parametric and non-parametric distributional approaches, including the Pareto law, and consider convergence aspects of those dynamics. We show that entry of new cities is an important characteristic of the U.S. experience, and that the U.S. urban system appears to be characterized by divergent growth, if spatial evolution is ignored, and by convergent growth in the presence of very significant regional effects.

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EVOLUTION OF THE U.S. CITY SIZE DISTRIBUTION

1 Introduction

The empirical study of trends in city size distributions has engaged economists since the beginning of this century. It has attracted renewed attention recently, in part because of two reasons. First, the cumulative effect of important contributions by the new urban economics literature [Fujita and Thisse (1998)] have heightened the need for new empirical work. Second, the topic appeals to those enamored by the "universality" of power laws [Gell-Mann (1994)].

Important contributions to the city size distribution literature are found in the work of J. Vernon Henderson, which determines city size by integrating international trade theory with urban economics. As summarized in Henderson [(1974; 1987; 1988)], this fundamental approach proposes a model of optimum city size, which balances the benefits of larger city size with its costs in terms of disamenities. Several of the implications of these theories have been tested, especially by Henderson himself. To the best of our knowledge, however, Henderson's theories and others we discuss below have never been tested directly, that is, in terms of data for an entire systems of cities. Such an endeavor is timely in view of the enormous attention which city sizes have been attracting in the economic geography, regional economics and urban economics literature.

Some of the most recent empirical interest has been rekindled by Eaton and Eckstein (1997), who use data from France and Japan and find urban systems in those two countries to be characterized by parallel growth, that is, city size distributions appear to have remained unchanged over many years.

In Section 2 we review the significant literature on city size distributions from early in the twentieth century to the present. In Section 3 we propose a model of city size distributions within the national economy. Our model emphasizes human capital: each city within the economy is assumed to contribute to the production of national output by means of specialized labor. Demand for national output drives the evolution of cities in the economy, in the simplest possible version of the model. The model is aimed at explaining the endogenous emergence of cities and important stylized facts associated with it, namely that earnings are positively related to city size, which is generally regarded as an important piece of evidence linking productivity and city size.

We examine the pattern of growth rates of U.S. metro areas and find, in contrast to Eaton and Eckstein, that the U.S. urban system appears to be characterized by divergent growth, if spatial evolution is ignored, and convergent growth, if it is not. In order to examine this property of the data further, we look at the data in more detail (and non-parametrically) by constructing transition matrices to track the movement of each city in the distribution relative to the others.

2 Historical Perspective on City Size Distributions

Some economists, as well as geographers and other social scientists, have found it useful to invoke the *rank size rule*, an alleged statistical regularity that is an outgrowth of the application of the Pareto distribution to city size data. Stated in its various forms, the rank size rule – city size multiplied by its rank in its system equals a constant – has been debated, calculated, and dismissed several times over since its first mention (that we can find) in Auerbach (1913). Unlike other scientists, economists are dubious of universal constants and power laws.[1]

Singer (1936)suggested that city sizes follow a Pareto distribution; his *courbe des populations*) parallels Pareto's (*courbe des revenues*) [Pareto (1906)]. If city sizes follow a Pareto distribution, its characteristic exponent α, in effect expresses the degree of concentration of population among cities in a system. Proponents of the rank size rule assert that this coefficient is equal to 1. Strictly speaking, if the Pareto exponent is equal to 1, then the size distribution possesses no finite moments. In that case, the rank size rule may hold on its own, but it is not associated with an interesting distribution function. Whether the critical coefficient, α, is actually 1; whether the α coefficient equal to 1 is the true test of the rank size rule; and whether a Pareto distribution is

[1] There are several writers, including Auerbach (1913), who see the rank size rule as part of an overall scheme of things. Lotka (1925) noted the rule as he sought similarity in the "laws" of various disciplines, including economics and physics. Zipf (1949) made the rule an example of his overriding "Principle of Least Effort" in the conduct of human behavior. See also Beckmann (1958). For the latest in this history of fascination with the rank size rule see Gell-Mann (1994) and Krugman (1993; 1994; 1996). Krugman suggests that the rank size rule may be evidence of complexity theory at work and links the "self-organization properties" of complex systems to the statistical regularity implied by the rank-size rule. Krugman's papers fit within the larger explanation of the relationship between increasing returns and path dependence. See also Arthur (1994).

at all an appropriate law of city size distributions, are all questions addressed in the literature. The higher the threshold city size, the higher would be the estimated Pareto exponent. Madden (1956) provides an interesting non-parametric analysis of urban growth in the United States. He emphasizes stability features in the distribution of growth rates and their evolution over time, where he notes that great dispersion coexists with considerable intertemporal variation for individual cities. We continue this line of inquiry by performing rank size regressions with our data set. We look, however, at Standard Metropolitan Areas—as Singer would have said, "conurbations".

The question of city size distribution also underlies Eaton and Eckstein (1997), which uses a different method, but asks the same question: how has city size distribution changed over a long historical period? Using nonparametrically data from France for 1876 to 1990, and from Japan for 1925 to 1985, they find evidence of "parallel" growth; that is to say, city size distributions in those countries have remained almost the same over time.

Eaton and Eckstein's selection of France and Japan was motivated by their roughly stable geographical boundaries and the consistent availability of data. In contrast to such old countries as France and Japan, the United States has grown by continuously expanding its land mass into a well defined hinterland. New regions and cities have been brought into the U.S. urban system during the nineteenth and twentieth centuries, older regions have grown and declined, and the spatial distribution of economic activity has undergone some remarkable changes. In Europe, almost no new cities were created during the twentieth century. The economic forces at work may well be the same as in other economies. However, to the extent that "history matters", the U.S. urban system has developed with initial conditions quite different from those of other countries. It is for this reason, too, that a fresh approach to the U.S. case is of particular interest. Our choice of the time span of this study, the entire twentieth century, would allow us in principle to address some of these phenomena. In contrast, Crihfield and Panggabean (1995) use data for U.S. metro areas for 1960-1982. The peculiarities of cities and their openness is a natural subject during the revival at present of empirical growth theories.

3 Theoretical models

The dynamics of city size distributions when cities of different sizes and types coexist are still not very well understood. Several seminal studies by Henderson [Henderson (1974; 1983; 1987; 1988)] rest on the notion that cities differ because of the demand for their products either as final goods or intermediate goods. However growth in a Henderson-type system of cities would consist of the economy's producing an increasing number of cities, with the number of each city type growing at the rate of growth of national population [Henderson and Ioannides (1981)]. A drawback of this approach, which is not mitigated in the present paper, is the fact that national space is ignored.[2] When that is brought into consideration, the location of new cities matter, as we shall discuss further below. Eaton and Eckstein (1997) also work with the assumption that the price of non-urban land use remains constant over time, or at least is exogenous as far as urban growth is concerned, and thus exclude national space considerations as well.

The dynamics of city growth in Eaton and Eckstein (1997) are assumed to depend critically on knowledge flows across a given number of cities. Each individual's learning productivity depends on a linear combination of the average levels of human capital. The assumption of a nationwide capital market is a second source of the dependence of each city's growth on all others'. Equilibrium city sizes depend critically on the condition that, at the steady state, residents of different cities have no incentives to migrate. At one extreme, human capital is general; at the other, it is perfectly city-specific. Eaton and Eckstein show that the general case where human capital is partly city-specific implies lower and upper bounds on city size distributions. These bounds share some common determinants, including, in particular, the ratio of human capital at the corresponding cities at the steady state, which is of course endogenous.

Models of the Henderson *genre*, on the other hand, imply a theory of city size distribution that directly reflects preferences. For similar sets of reasons, such a theory implies that all determinants are highly interdependent.

Henderson explains that the types of goods produced in cities help determine the size of those

[2]This is also true of Ioannides (1994), where the Dixit-Stiglitz-Ethier-Krugman monopolistic competition model is employed to motivate the existence of many city types. Even though that model is symmetric, it is straightforward to see that a model of asymmetric preferences would produce different city types but not very different dynamics. Ioannides (1997b) mitigates this. See also Fujita and Thisse (1998) for a comprehensive review of monopolistic competition models of urban structure.

cities; if the types of goods currently in vogue change, then we would expect urban concentrations to change. Thus city-specific factors combine with aggregate ones to determine the distribution of city sizes. Abdel-Rahman and Fujita (1993) show that diversified and specialized cities can co-exist in a system, with diversified cities being larger. If industrial structures change to favor smaller, more specialized cities, we would see less concentration in large cities, *ceteris paribus.* If industrial structures remain the same over time, we might see a parallel growth pattern, as in Eaton and Eckstein. And if industrial structures change to favor larger cities, or demand conditions change to favor the larger, more diversified cities, we might well see increasing concentration.

3.1 The Model

We propose a model that is aimed at explaining a number of very important stylized facts, such as that urbanization is closely associated with economic growth and that earnings are positively related to city size, which is generally regarded as important evidence linking productivity and city size. Our model emphasizes human capital: each city within the economy is assumed to contribute to the production of national output by means of specialized labor. Demand for national output drives the evolution of cities in the economy, in the simplest possible version of the model. Localization and other spatial factors have not been explicitly brought into the analysis here.

Let there be I_t different cities at time t. Let P_{it} denote the size (in terms of possibly different though not equivalent measures, such as population, or employment, or labor force) of city i at time t, and assume that data are available for cities $i = 1, \ldots, I_t$, and time periods $t = 1, \ldots T$. Let $P_{Ut}, t = 1, \ldots, T$, be the total urban population, $P_{Ut} = \sum_{i=1}^{I_t} P_{it}$, and $\bar{P}_{Ut} = \frac{P_{Ut}}{I_t}$ the mean city size at time t. We shall suppress the time subscript when no confusion arises. The distribution of city sizes is a way to approximately study the density distribution of economic activity over space when its actual geographical features may be safely ignored. If we think of the distribution of economic activity as a mathematical surface over physical space, urban areas may be identified with regions where a certain threshold is exceeded.

City i uses raw labor P_i with capital and land to produce *skilled* labor of type i. The quantity of skilled labor of type i is denoted by X_i, and its price by W_{X_i}. We neglect in this paper the urban use of land. [3] We assume that city i's demand for capital is independent of its size and equal to κ. This assumption is justified at equilibrium, as we shall see, where city size depends entirely on the parameters of the technology with which skilled labor is produced in the city. [4] r_L and r_K denote the rental rates of land and capital, respectively.

National output is produced by using as inputs the quantities of specialized labor produced by the economy's set of cities, $\{X_1, X_2, \ldots, X_I\}$, along with land, L, and capital, K :

$$Y = G(\{X_1, X_2, \ldots, X_I\}, L, K). \tag{1}$$

National production takes place according to constant returns to scale in terms of all inputs. National land in the RHS of (1) represents constraints that major features of national geography represent for national production, whereas capital stands for producible means of production. E.g., doubling of the land input would require that all convenient sites for port facilities, etc., also be doubled. We assume the following form for $G(\cdot)$:

$$Y = G_0 \left[\sum_{i=1}^{I} X_i^{\chi} \right]^{\frac{\alpha}{\chi}} L^{\beta} K^{\gamma}, \tag{2}$$

where $0 < \chi < 1$, $\alpha + \beta + \gamma = 1$, and $\alpha, \beta, \gamma > 0$. The larger is χ the greater the substitutability among different kinds of skilled labor in national production The aggregate production function (2) is invoked to represent the use of specialized labor inputs $\{X_1, X_2, \ldots, X_I\}$ by a large number of different firms in the economy in order to produce a final good which may be consumed or invested.

Derived demands for land, capital and the specialized labor inputs are determined in perfectly competitive conditions. Let R_L, R_K, R_Y, and W_{Xi} denote prices (rental rates) for land, capital, the price of national output, and the price of specialized input i. The derived demands for X_i, $i = 1, \ldots, I$, readily follow:

$$X_i = \bar{G}_0 \frac{1}{R_Y^{\frac{\beta+\gamma}{\alpha}} R_L^{-\frac{\beta}{\alpha}} R_K^{-\frac{\gamma}{\alpha}}} \left(\frac{W_{X_i}}{R_Y^{\frac{1}{\alpha}} R_L^{-\frac{\beta}{\alpha}} R_K^{-\frac{\gamma}{\alpha}}} \right)^{-\frac{1}{1-\chi}} Y, \tag{3}$$

[3] It would be consistent with some historically observed patterns of urbanization during economic growth to ignore initially the impact of population growth on the demand for land, and/or alternatively, the demand for capital. However, at some point a stage will be reached where national production will compete with the needs of urban production, which would set in a qualitatively new phase in national economic growth [Ioannides (1997b)].

[4] Since each city in our analysis is assumed to have a homogeneous labor force, the level of schooling represents the total effect, and thus subsumes the impact on an individual's productivity of the average level of schooling in the community of his residence [Rauch (1993)]. Rauch (1993) finds that cities with higher levels of human capital have higher wages and higher land rents. He finds that each additional year of SMSA average education can be expected to raise total factor productivity by 2.8%.

where $\bar{G}_0$ is a function of parameters. The derived demands for L and K are given by: $L = \bar{\beta}\frac{R_Y}{R_L}Y$ and $K = \bar{\gamma}\frac{R_Y}{R_K}Y$, where $\bar{\beta}$ and $\bar{\gamma}$ are functions of parameters.

City i is assumed to be host to the sole producer of specialized labor of type i, who takes the demand function, from (3), as given, and decides on its price. This modelling choice readily leads to a determinate solution for the range I of the types of specialized labor, that is the number of cities in our framework [Ioannides (1994)]. We assume that production of specialized labor by city i occurs under increasing returns to scale in the style of Dixit and Stiglitz (1977).

Specifically, let the labor requirements function for city i production be denoted by

$$P_i = \Pi + cX_i, \tag{4}$$

where Π and c denote, respectively, the fixed and average variable cost of city i production. Optimal pricing assumes the average wage rate, W_i, at which labor is hired as given and implies the familiar constant markup formula $W_{X_i} = \frac{1}{\chi}cW_i$. Free entry, in turn, leads to zero profits for all firms producing differentiated inputs. This implies $X_i = \frac{\chi}{1-\chi}\frac{\Pi}{c}$ which along with the labor requirements function (4) yields:

$$P_i = \frac{1}{1-\chi}\Pi. \tag{5}$$

3.2 General Equilibrium

With overall symmetry the prices of all intermediate inputs are equal. At equilibrium, the price of national output must be equal to its marginal cost: $R_Y = g_0 R_L^{\beta} R_K^{\gamma}\left(\sum_{i=1}^{I} W_{X_i}^{-\frac{\chi}{1-\chi}}\right)^{-\frac{1-\chi}{\chi}\alpha}$, where g_0 is a function of parameters. It is convenient to normalize by choosing national output as the numeraire, $R_Y = 1$ which yields a relationship between the three unknown prices R_L, R_K, and W. Equilibrium conditions for land and capital yield two additional equations: $R_L = \bar{\beta}\frac{Y}{L}$; $R_K = \bar{\gamma}\frac{Y}{\bar{K}-\kappa(1-\chi)\frac{N}{I}}$. where κ denotes city i's demand for capital, which has been assumed to be inelastic. National output is given by:

$$Y = y_0 \frac{W^{\beta+\gamma}}{R_L^{\beta} R_K^{\gamma}} c^{\beta+\gamma} \Pi I^{\beta+\gamma+\frac{\alpha}{\chi}}. \tag{6}$$

Equilibrium in the national labor market requires:

$$I\frac{1}{1-\chi}\Pi = N, \tag{7}$$

where N denotes the national labor force. Condition (7) determines the equilibrium number of cities I. The model is closed by specifying the demand for national output, which in turn determines the only remaining unknown, namely the equilibrium wage rate W.

The model may be solved as follows:

$$Y = \upsilon L^{\beta} \left(K - \kappa(1-\chi)\frac{N}{\Pi} \right)^{\gamma} N^{\frac{\alpha}{\chi}} \Pi^{\alpha(1-\frac{1}{\chi})}; \tag{8}$$

$$W = \omega \frac{1}{c} L^{\beta} \left(K - \kappa(1-\chi)\frac{N}{\Pi} \right)^{\gamma} N^{\frac{\alpha}{\chi}-1} \Pi^{\alpha(1-\frac{1}{\chi})}; \tag{9}$$

$$R_L = \bar{r}_L L^{\beta-1} \left(K - \kappa(1-\chi)\frac{N}{\Pi} \right)^{\gamma} N^{\frac{\alpha}{\chi}} \Pi^{\alpha(1-\frac{1}{\chi})}; \tag{10}$$

$$R_K = \bar{r}_K L^{\beta} \left(\bar{K} - \kappa\frac{N}{\Pi} \right)^{\gamma-1} N^{\frac{\alpha}{\chi}} \Pi^{\alpha(1-\frac{1}{\chi})}, \tag{11}$$

where $\upsilon, \omega, \bar{r}_L$, and $\bar{r}_K$, denote functions of parameters. These expressions for equilibrium output and factor prices reveal the impact of increasing returns. Not surprisingly, output and the rental rates for land and capital increase with the size of the national labor force, were we to ignore the increased demands for land and capital caused by such a change. But, so does the wage rate, provided that $\alpha > \chi$, namely that the higher the share of specialized labor in national output, the higher the substitutability among different kinds of specialized labor which would be consistent with generation of increasing returns in the manner suggested by this model. In that case, increase in the national labor force increases national output more than proportionately. We also note that output and factor prices are all decreasing functions of Π, the fixed costs in the production of specialized labor inputs.

Two remarks are in order. First, the positive association between total population and the wage rate reflects directly the impact of increasing returns in the production of skilled labor. In contrast, the modern urban economics literature in the Henderson *genre* has obtained a positive association by endowing city production with Marshallian external effects: city size confers advantages to city production which are external to each firm and internal to the city economy. Second, it would be appropriate to interpret cities here as the smallest urban centers which are consistent with exploitation of the advantage of increasing returns. This smallest city size may be identified with what Krugman (1996) calls "lumps." Appearance of increasing returns here is not getting something out of nothing. The maintenance of "lumps" requires resources, which cuts into the amounts of land and capital available for national production.

3.3 Dynamics

Equ. (7) implies that when the labor force grows the number of cities also grows, unless the capacity of each city may grow through an increase in fixed costs Π. This could be accommodated if we were to assume that although Π is a fixed cost relative to the production of a specialized input, it may be decreased by means of capital investment and use of land, under decreasing returns to scale.

The dynamic evolution of the economy may be described once we have elaborated on the capital accumulation process. We invoke, for simplicity, a simple neoclassical descriptive (Solow) growth setting, where the economy saves a constant fraction s, $0 < s < 1$, of aggregate national output. Savings, sY_t, are invested in capital used in national production. The law of motion for our model of an urbanized economy readily follows from (8): $K_{t+1} = svL^{\beta}\left(K_t - \kappa(1-\chi)\frac{N_t}{\Pi}\right)^{\gamma} N_t^{\frac{\alpha}{\chi}} \Pi^{\alpha(1-\frac{1}{\chi})}$, where v_u denotes a function of parameters. We see that sufficiently strong increasing returns, i.e. $\alpha > \chi$, would make up for the fixity of land in national production. The law of motion when transformed in intensive form becomes:

$$k_{t+1} = s\frac{v}{1+\eta}L^{\beta}\left(k_t - \frac{\kappa(1-\chi)}{\Pi}\right)^{\gamma} N_t^{\frac{\alpha}{\chi}-(\alpha+\beta)} \Pi^{\alpha(1-\frac{1}{\chi})}, \tag{12}$$

where k_t denotes the national capital labor ratio, $k_t \equiv \frac{K_t}{N_t}$.

At a cost of $\frac{\kappa(1-\chi)}{\Pi}$ per capita, the economy avails itself of growth according to (12), which reflects in effect an endogenous source of technological change, $N_t^{\frac{\alpha}{\chi}-(\alpha+\beta)}$. Provided that $\frac{\alpha}{\chi} > 1-\gamma$, the more important capital is in aggregate production, the more likely it is that a given degree of substitutability among specialized labor inputs will cause increasing returns sufficiently strong to overcome the decreasing returns caused by the fixity of land.

We contrast with economic growth in a non-urbanized economy, in which case the counterpart of (12) is:

$$k_{t+1} = s\frac{v_n}{1+\eta}L^{\beta}k_t^{\gamma}N_t^{-\beta}, \tag{13}$$

where v_n is a function of parameters. It is well-known that if a productive factor is available in fixed supply and no exogenous source of technological change is present, standard neoclassical growth with a constant returns to scale aggregate production function admits no steady state. Aggregate output grows at a rate which is less than that of population. An economy growing along these lines will find it advantageous to urbanize as soon as it is feasible in order to avoid further decrease in per capita income.[5]

It would be consistent with the spirit of this approach if we were to assume that sites can cluster into forming larger metro areas, as long as this process does not affect the technology of production within each city. However, since we have not addressed spatial aspects, it is impossible to say anything about where new cities would locate. Therefore, even though cities in this model are identical, they may cluster into forming metropolitan areas of different sizes.

In the presence of population growth, the model of the urbanized economy will be associated with unceasing growth, in spite of the absence of exogenous technological change. Unless Π_0 changes, the number of identical cities grows in proportion to population.

3.4 Different Types of Cities

The above model implies that cities in the economy are of the same type and size. We know, however, that economies are made of cities of different types [Henderson (1974; 1987; 1988)] and sizes. It would be straightforward to assume a non-symmetrical CES aggregator in the RHS of (2). In a growing economy, the number of cities grows, but there are at least two drawbacks of such a formulation: one, there would in principle be only one city of each type (unless the CES aggregator is defined to be of the mixed continuous-discrete type); two, the properties of the CES aggregator assumed determine entirely the characteristics of the urban system.

In an effort to draw a greater distance between assumptions and conclusions, we propose a model that takes advantage of increasing returns in a "vertical," i.e., *hierarchical* sense. The intuition here is to emulate the manner in which the productivity of raw labor is enhanced when it is transformed into varieties of differentiated skilled labor which are used as inputs by a higher-level production process. Let subscripts 0 denote variables and parameters associated with the production process described above. I_{0_t} denote the number of lumps, minimum-size cities, at equilibrium. We assume that level-0 output, whose quantity Y_{0_t} is given by (8) above, may be used as capital input along with land and differentiated products via an constant returns to scale production process, just as

[5] This will occur at the smallest value of t for which

$$\upsilon_u L^\beta \left(s\upsilon_n L^\beta K_t^\gamma N_t^\alpha - \kappa(1-\chi)\frac{N_{t+1}}{\Pi}\right)^\gamma N_{t+1}^{\frac{\alpha}{\chi}} \Pi^{\alpha(1-\frac{1}{\chi})} \geq \upsilon_n L^\beta \left(s\upsilon_n L^\beta K_t^\gamma N_t^\alpha \right)^\gamma N_{t+1}^\alpha .$$

It may shown that if the effective increasing returns are sufficiently strong, then a transition to the urbanized phase is feasible in finite time.

above, to produce output Y_{1_t}, and so on at level-2, etc. If we assume that the corresponding fixed costs for producing specialized labor inputs satisfy $\Pi_0 < \Pi_1 < \Pi_2 < \ldots$, then we can analyze the relative magnitudes of the numbers $I_{0_t}, I_{1_t}, I_{2_t}, \ldots$, of the varieties of differentiated outputs required by each ascending level of the hierarchy.

We assume that the output of the highest-level extant is used for consumption and investment; all other outputs are used as intermediate products. The number of levels in the hierarchy evolves endogenously, so that the highest-level output is used for final consumption and for investment in the capital used by the level-0 process. That is, output at level-n requires I_{n_t} differentiated products and occurs according to:

$$Y_{n_t} = \upsilon_n \left(L_{n_t}\right)^{\beta} \left(Y_{n-1_t} - \kappa(1-\chi)\frac{1}{\Pi_n} N_{n_t}\right)^{\gamma} N_{n_t}^{\frac{\alpha}{\chi}} \Pi_n^{\alpha\left(1-\frac{1}{\chi}\right)}, \; n \geq 1; \tag{14}$$

$$Y_{0_t} = \upsilon_0 \left(L_{0_t}\right)^{\beta} \left(K_t - \kappa(1-\chi)\frac{1}{\Pi_0} N_{0_t}\right)^{\gamma} N_{0_t}^{\frac{\alpha}{\chi}} \Pi_0^{\alpha\left(1-\frac{1}{\chi}\right)}, \tag{15}$$

where $L_{0_t}, \ldots, L_{n_t}$ denote land allocated to level-0, ..., level-n production, respectively, $N_{0_t}, \ldots, N_{n_t}$, denote raw labor used to produce the differentiated skilled labor used in level-0, ..., level-n production, respectively, and K_t denotes capital used in level-0 production. Finally, capital accumulation evolves according to

$$K_{t+1} = sY_{n_t}. \tag{16}$$

By substituting back from the corresponding production function for Y_{n-1_t}, and by working iteratively backwards we get an expression that contains the quantities of land and raw labor used by each level of the hierarchy. E.g., for a hierarchy with level-0 and level-1, we have:

$$Y_{1_t} = \upsilon_1 \left(L_{1_t}\right)^{\beta} \left(\upsilon_0 \left(L_{0_t}\right)^{\beta} \left(K_t - \kappa\frac{1-\chi}{\Pi_0} N_{0_t}\right)^{\gamma} N_{0_t}^{\frac{\alpha}{\chi}} \Pi_0^{\alpha\left(1-\frac{1}{\chi}\right)} - \kappa\frac{1-\chi}{\Pi_1} N_{1_t}\right)^{\gamma} \left(L_{0_t}\right)^{\gamma\beta} N_{1_t}^{\frac{\alpha}{\chi}} \Pi_1^{\alpha\left(1-\frac{1}{\chi}\right)}, \tag{17}$$

and $K_{t+1} = sY_{1_t}$.

A key question which our framework must address is how many are the levels of the hierarchy. We take up this question under the simplifying assumption that the capital requirements for city production are not very large, $\kappa \approx 0$. In the process we examine how our hierarchical setting improves upon the identical city case. First note that competition for land implies that : $L_{0_t} = \frac{\gamma}{1+\gamma}L$; $L_{1_t} = \frac{1}{1+\gamma}L$. Competition for raw labor determines the allocation of N_t to different city types: $N_{0_t} = \frac{\gamma}{1+\gamma} N_t$; $N_{1_t} = \frac{1}{1+\gamma} N_t$. We now note that an effect of hierarchical production is to increase effectively the elasticities for both total land and total raw labor in the production of the final good used for consumption and investment. That is, these elasticities with respect to land and labor,respectively, are: $\beta(1+\gamma)$, and $\frac{\alpha}{\chi}(1+\gamma)$. We also note that employment is higher the higher the level of the hierarchy. As a result, the hierarchical organization of urban production strengthens the increasing returns to scale that urban production with one city type makes possible in the first place. In fact, the larger is the number of the levels of the hierarchy, the stronger is this effect. In the limit, when $n \to \infty$, the elasticity of raw labor in the production of final output tends to $\frac{\alpha}{\chi}\frac{1}{1-\gamma}$, the elasticity of land tends to $\beta\frac{1}{1-\gamma}$, and the elasticity of capital used by the level-0 production tends to 0 faster, the smaller is γ. Consequently, $\beta\frac{1}{1-\gamma} + \frac{\alpha}{\chi}\frac{1}{1-\gamma} > 1$, and therefore increasing returns generated by the hierarchical model persist in the limit. It is this strengthening of increasing returns which explains why the economy would be better off by adding an additional level in the hierarchy.

Turning now to the model's implications for city size distributions, we note that the numbers of cities of type 0, and 1 are equal, respectively, to: $I_{0_t} = \frac{1-\chi}{\Pi_0} N_{0_t}$, $I_{1_t} = \frac{1-\chi}{\Pi_1} N_{1_t}$. These imply a frequency distribution:

$$f_{0_t} = \frac{\frac{\gamma}{\Pi_0}}{\frac{\gamma}{\Pi_0} + \frac{1}{\Pi_1}}, f_{1_t} = \frac{\frac{1}{\Pi_1}}{\frac{\gamma}{\Pi_0} + \frac{1}{\Pi_1}}, \tag{18}$$

and a total number of cities given by:

$$I_t = (1-\chi)\frac{\frac{\gamma}{\Pi_0} + \frac{1}{\Pi_1}}{1+\gamma} N_t. \tag{19}$$

We note that the relative proportion of larger cities would decrease with city size if $\gamma > \frac{\Pi_{i-1}}{\Pi_i}$, $i = 1, \ldots, n$.

In sum, our model delivers an explanation for different city types, within a hierarchical model with an endogenous number of levels. This is significant in the context of the literature, [6] as the dispersion in city sizes continues to to attract a lot of attention. The extraordinary performance of power laws as descriptions of size distribution of cities has not been satisfactorily explained, in spite of recent attempts, notably by Krugman (1996), but also Gabaix (1997).[7] Our emphasis has

[6] See Matsuyama (1995) for a hierarchical model with an exogenous number of levels.

[7] Gabaix (1997) emphasizes the emergence of power laws for city size distributions as an outcome of the statistical properties of city growth rates.

been on developing models based on explicit economic economic models. It is for these reasons that further research in this area is warranted.

The fact that a hierarchical model yields a more complex set of outcomes should not be surprising in view of the fact that it involves more complex interactions. In general, cities interact both because of their geographical proximity and also because of economic proximity. E.g., the Boston area and Silicon Valley (and other high-tech industry areas), or New York and Los Angeles (in the world of entertainment) may be geographically apart but economically quite close. Our hierarchical model imposes strong restrictions on the pattern of economic interactions. These restrictions should be relaxed in future work.[8]

We can imagine a variety of empirical exercises that would allow us to investigate the conclusions of our model. The limited availability of data on cities, both over time, and in city-specific detail, imposes constraints on such tests, even in the context of our newly constructed data set. The theory we developed may be summarized in terms of a sequence of evolving distributions of city sizes, $\{f_{0_t}, f_{1_t}, \ldots, f_{n_t}\}$, which will be referred to as f_t, for short. We note whereas it is a drawback of our theory that city sizes are proportional to exogenous parameters, $\Pi_0, \Pi_1, \ldots,$ their frequencies are endogenous.

4 Data

Cities pose special definitional problems for data.[9] We define cities as geographic areas of great concentration (density) of economic activity. Density of economic activity is not, of course, unambiguously defined; it could be in terms of value-added, employment, population, etc. City boundaries change, and changes in transportation technology and investment have altered the effective economic boundaries of metro areas.

U.S. cities are defined by the Office of Management and Budget (OMB) based on data provided by the U.S. Bureau of the Census. The OMB moved to the Standard Metropolitan Area (SMA) concept in 1950, to Standard Metropolitan Statistical Areas (SMSAs) after 1959, and, in 1983, to the Metropolitan Statistical Area (MSA)–Primary Metropolitan Statistical Area (PMSA)–

[8] We take up spatial interaction in Dobkins and Ioannides (1997).

[9] Technically, a metropolitan area must contain either a city of at least 50,000, or an urbanized area of at least 50,000 and total metropolitan population of 100,000 (75,000 in New England). See Bureau of the Census (1990).

Consolidated Metropolitan Statistical Area (CMSA) classification.

Most data available prior to 1950 are for "city proper" sizes, reflecting legal city boundaries. Such data are still available, but ignore the very real fact of suburban integration. Cities do not, in a real economic sense, necessarily coincide with their legal boundaries and metropolitan area data reflect this fact. Bogue (1953) used the 1950 SMA definitions to *reconstruct* what populations would have been in those areas in each of the decennial years from 1900 to 1990. Most of the metropolitan area data identify city units by counties. (In New England metropolitan area definitions may involve parts of counties.) The most cumbersome issue involves these changing definitions within the metropolitan area structure.

This state of affairs suggests three approaches which are appropriate for assembling "consistent" data. First, it would be appropriate for some purposes to have populations for past years drawn up under a consistent set of rules, such as the 1990 standards of the OMB. A second way of generating consistent data would be to use the areas as defined at the time of the appropriate census. That is, we use the 1960 definitions for 1960; 1970 definitions for 1970 data, etc. This requires returning to original data sources for those years. A third way to generate the data would be to pick a *geographical* area that defines a metro area, and to use it consistently. For example, we might use the counties that define a metro area in 1990, and then assign those counties to the cities for each year from 1960 forward. This is essentially what Bogue did in 1953. The issue here is to make this method fit Bogue's work, since the only source we have for the 1900-1940 data is Bogue. In other words, we do not want a pronounced jump in the data between 1950 and preceding years on one hand, and 1960 and succeeding years on the other hand. Because of the latter consideration, we opt for the second method.

A major problem that arises in using each year's data in contemporaneous definition is in the span between the most recent censuses, from 1980 to 1990. Because the Bureau of the Census redefined SMSAs as MSAs and CMSAs in 1983, the 18 large metropolitan areas which are now CMSAs would seem to take an enormous jump in size. Therefore, we reassembled the 1990 data to fit the 1980 definitions (by county). We consequently have metropolitan area data from 1900 to 1990. The 1900 to 1940 data are constructed using the 1950 definitions of SMAs according to Bogue. The 1950 to 1980 data are consistent with the SMSA definitions in those years. The 1990 data are reconstructed using the 1980 definitions. We believe that this method, while not perfect, is the most consistent way to construct urban area sizes. Dobkins and Ioannides (1996) provides more details on the data.

This method highlights a critical issue, the number of metropolitan areas in each census year. If we adhere to the rough definition of metropolitan areas as having populations of more than 50,000 people, then we see a change in the number of cities each year. In 1900, there are 112 urban areas that qualify; they grow to 334 by 1990. We believe that the number of new, "entering" cities, as defined in each decade, is a key feature of the U.S. urban system, especially in its spatial aspects as well, with many entrants appearing in newly developed geographical areas. In the remainder of the paper we shall refer interchangeably to cities and metro areas, as defined here. Our task and method therefore contrasts with Eaton and Eckstein, who premiss their study on an assertion that the number of cities in Japan and France remain the same over the time periods involved. Of course, this broadened approach is not costless.

We measure schooling by means of the number of students enrolled in school as percentage of the 15-20 years of age grouping. Data unavailability has forced us to do with minor variations of the base cohort in certain years. See Dobkins and Ioannides (1996) for further details.

We measure wages in terms of mean wage in cities proper, which are available for years 1900–1930. In 1940, Census reported details on the frequency entire distribution, up to a maximum of $5,000. For 1950 and 1960, the median income is reported by SMSA's and smaller cities. Since 1970, median earnings are reported separately for male and female workers. We averaged those two numbers for each of the decennial years since then. We used the national CPI (1967 = 100) to deflate them. It would have been more appropriate to use city-specific deflators. Such information is available only for recent years when the CPI-U and other cost of living figures are reported for selected metropolitan areas. See *Statistical Abstract of the United States,* 1994, No. 749. Local indices show greater variability than the national index but their long-term trends are similar. Our perusal of these numbers suggests strong correlations between city size and real earnings (and personal income), if nominal amounts are deflated by the city-specific index for those areas for which they are readily available.

5 Empirics of the evolution of city size distributions

Since city sizes are the outcome of economic processes associated with interactions of thoroughly open economic entities, one would expect such interactions to be important. When all cities in an economy are sampled, their sizes exhibit extensive variation simultaneously in both the cross-section, that is, across i for given t, and the time-series, that is, across t for given i, dimensions. In most econometric time-series settings, one studies the dynamics of a vector of random variables, whose dimension is fairly small and fixed. Time-series analysis aims at understanding of the dynamic behavior of such a vector and patterns of interactions among its components. Time-series techniques utilize time averaging and other curve-fitting techniques, but do not involve averaging across components of a vector. Cross-section and panel-data analyses involve investigation of the behavior of the average (or representative) member of the each cross-section and deviation of each individual observation from the average across all cross section units.

As Quah (1993) has forcefully argued, typical cross-section or panel data techniques do not allow inference about patterns in the intertemporal evolution of the entire cross-section *distribution*. They do not allow us to consider the impact over time of one part of the distribution upon another, i.e., of the development of large cities as a group upon smaller cities. Making such inferences requires that one models directly the full dynamics of the entire distribution of cities. In contrast, typical panel data analyses involve efficient and consistent estimation of models where the error consists of components reflecting individual effects (random or fixed), time effects and purely random factors. The evolution of urbanization and suburbanization may affect individual cities so drastically as to render conventional methods of accounting for attrition totally inappropriate. As smaller urban units fuse to create larger ones,[10] and given the small number of time series observations, non-parametric or semi-parametric distributional approaches such as the one proposed here would be the only appropriate ones. In fact, these techniques are appropriate when the sample of interest is the entire distribution, and individual observations are used to recover information about the entire distribution.

We may elaborate further the process of evolution of the system of cities by considering alter-

[10] Such a process is aptly described by Simon (1955) and Krugman (1996). Similar phenomena are addressed by the literature in economies with interacting agents. See Ioannides (1997a).

native scenaria that articulate the spatial context. Consider first a situation where cities of uniform sizes are uniformly spread over space. Appearance of new cities that are randomly scattered over space is likely not to alter the pattern of uniformity. To the extent that geographical proximity leads invariably to agglomeration, this setting implies creation of larger cities of uniform sizes. Consider, alternatively, cities of uniform sizes scattered over space but in a way that exhibits clustering. Appearance of additional cities of uniform sizes makes it more likely that ever larger cities will be created through the agglomeration of existing ones. The availability of data are severely restricted both in the time and the cross-section dimensions: there are only ten cross-sections, one for each of the ten census years since 1900, with 112 metropolitan areas and 334 in 1990.

The paucity of the data naturally lends itself to techniques used by Quah (1993) and Eaton and Eckstein (1997). That is, one may construct from population data a fairly low-dimensional vector indicating the frequency of cities in each of a number of suitably defined intervals (cells).[11] Let f_t denote the frequency (density) distribution of P_{it} at time t. Eaton and Eckstein assume that f_t evolves according to a first-order autoregression (that applies to the entire distribution function (rather than scalars or vectors of numbers):

$$f_{t+1} = M \cdot f_t, \tag{20}$$

where M is a matrix of parameters. If F_t were restricted to be measures defined over a discrete set, then M in (20) is a Markov transition matrix.[12]

Absence of a random disturbance allows us to iterate (20) forward to get:

$$f_{t+s} = (M \cdot M \cdot \ldots \cdot M) \cdot f_t = M^S \cdot f_t. \tag{21}$$

We may characterize the long-run distribution of city sizes by taking the limit of (21) for $s \to \infty$.

[11] All these studies use six cells, defined relative to the mean. Quah defines the end points within each distribution as the mean in the respective period times (0, .25, .50, 1, 2, ∞); Eaton and Eckstein define them as the mean times (0, .30, .50, .75, 1, 2, 20).

[12] More generally, instead of (20) we have:

$$\forall A \in \mathbb{R}: \; F_t(A) = \int_A M(x, A) F_{t-1}(dx), \text{ where } M : R \times \mathbb{R} \to [0,1]$$

maps the Cartesian product of the real line R with its Borel sets $\mathbb{R}$ to the unit interval, then M would be a mixed-discrete continuous analogue of a transition probability matrix. There is a fairly well-developed literature on the invariant distributions of such generalized Markov chains [See Futia (1982); Quah *op. cit.*]. Arthur (1994), pp. 33-48 and pp. 185-201, provides additional insight. That is, in a nonlinear version of (22), Arthur *et al.* show that only stable fixed points of M^* can serve as limit points of f. This fact is particularly interesting within the urban model, because city size is often not uniquely determined and, of course, not all solutions are stable.

Divergent, convergent or parallel growth may be ascertained by the properties of $f_\infty \equiv \lim_{t\to\infty} : f_t$. If a limit distribution f_∞ exists, then according to the Perron-Frobenius theorem it is given by the eigenvector corresponding to the unique unitary eigenvalue of M, the nonzero solution of $[M - I]f_\infty = 0]$, where 0] denotes a column vector of zeroes.

Parallel growth is understood to occur if f_∞ tends to a limit with non-zero probability over the entire support. Convergent growth would occur if f_∞ is a mass point, and divergent growth if f_∞ is a polarized or segmented distribution. Equ. (20) may be generalized to allow for a stochastic disturbance, U_t,

$$f_{t+1} = M^*(f_t, U_{t+1}), \tag{22}$$

where M^* is an operator that maps (f_t, U_{t+1}) to a probability measure. The random growth model in Simon (1955) may be considered as a special case of processes consistent with specification (22).

Quah (1993) uses (22) with data for the growth of countries, and conditions his non-parametric estimation on a number of "exogenous" variables. The transition dynamics are obtained for OLS residuals of pooled cross-section and time-series observations, with no individual effects being allowed for in those regressions. Individual effects, Quah argues, would remove, and thus leave unexplained, the very object of analysis, the relative growth rates of nations. Eaton and Eckstein (1997) do not allow for any conditioning and compute the long-run average transition probabilities. They estimate M by computing the average $M_{i,i+1}$ for all periods in the sample.

We adapt Equ. (22) in order to allow for new cities to enter according to a frequency distribution ε_t. If the number of entrants between t and $t+1$ is I_t^n, $I_{t+1} = I_t + I_t^n$, then

$$f_{t+1} = \frac{I_t}{I_{t+1}} M_t\, f_t + \frac{I_t^n}{I_{t+1}} \varepsilon_t. \tag{23}$$

If M_t and $\iota_t \equiv \frac{I_t^n}{I_{t+1}}$ are time-invariant, then the above equation is amenable to the standard treatment. Letting M and ι be the respective time-invariant values, we may iterate Equ. (23) backwards to get:

$$f_t = (1-\iota)^t M^t f_0 + \sum_{\tau=0}^{t} [(1-\iota)M]^{t-\tau} \iota \varepsilon_\tau, \tag{24}$$

where f_0 denotes the initial distribution of city sizes.

In general, if there are few or no entrants, $\iota \approx 0$, the homogeneous solution dominates: the invariant (ergodic) distribution is a useful measure of the state of the urban system in the long run. If, on the other hand, ι is non-negligible, then the particular solution may *not* be ignored. In fact, in that case, the magnitude of the largest eigenvalue of $(1-\iota)M$ is $(1-\iota)$, and the impact of the initial conditions would be less important the higher is ι, the number of new cities that have entered over the last decade as a proportion of the new total number of cities.

Our approach may be adapted to accommodate a number of different possibilities. One would be Simon's "random urban growth" model [Simon (1955)][13], which implies as its stationary solution a law, approximated by a family of skew distributions of the form $f(p;a,b,\beta)=\frac{a}{p^\beta}b^p$. This prediction is, in principle, testable. Alternatively, other models, including the hierarchical model sketched in subsection 3.3 above, would also imply laws that may be written as first-order autoregressions like (22).

In our data, the values of ι_t are as follows: $\iota_{1910}=.194$, $\iota_{1920}=.067$; $\iota_{1930}=.051$, $\iota_{1940}=.019$, $\iota_{1950}=.012$, $\iota_{1960}=.229$, $\iota_{1970}=.136$, $\iota_{1980}=.245$, and $\iota_{1990}=.036$. These numbers suggest a non-stationary series and the intertemporal variations in ι_t are interesting and worthy of special analysis. In general, the stochastic specifications of Equ. (23) are very complicated. E.g., the forces that cause growth and decline may operate quite differently at the upper level of the distribution than at the lower one. The distribution of new entrants has a lot more mass at the lower end and may reflect very different forces. Furthermore, the results of Arthur *et al.*, [Arthur (1994)] have bearing in the long run, whereas what we observe is clearly not a steady state process. It is for these reasons that in the remainder of this paper we eschew a full analysis of the determinants of M and concentrate instead on an approximate treatment of certain key aspects of the evolution of city size distributions. We pursue further issues of entry of new cities in Dobkins and Ioannides (1997).

[13]Briefly, Simon's model considers that a new city, a lump, may either, with probability ϖ, locate on its own, or, with probability $1-\varpi$, attach itself to a "clump", an existing agglomeration. The probability that a lump will join an existing agglomeration is assumed to be proportional to the clump's size (measured in lumps).

6 Results

6.1 The Rank Size Rule Revisited

We start our study of the determinants of city size distribution by first returning to the methods of Singer (1936).[14] A random variable p is said to be Pareto distributed with parameters p_0, ς if it has a density function given by $f(p; p_0, \varsigma) \equiv (\frac{\varsigma}{p_0})(\frac{p_0}{p})^{\varsigma+1}$, with support $[p_0, +\infty)$ and $\varsigma > 0$. The mean exists and is given by $E(p) = \frac{\varsigma}{\varsigma-1}p_0$, if $\varsigma > 1$; the variance exists and is given by $\text{Var}(p) = \frac{\varsigma}{(\varsigma-1)^2(\varsigma-2)}p_0^2$, if $\varsigma > 2$. [Spanos (1989), pp. 339-340.] The corresponding cumulative distribution function is given by $F(p; p_0, \varsigma) = 1-(\frac{p}{p_0})^{-\varsigma}$, the countercumulative probability function by $1 - F(p; p_0, \varsigma) \equiv (\frac{p}{p_0})^{-\varsigma}$, which by taking logarithms of both sides yields: $\ell n[1 - F(p; p_0, \varsigma)] = \varsigma \ell n p_0 - \varsigma \ell n p$.

When we apply the Pareto law with data and hold constant the parameter p_0, in effect the size that defines a metro area, then we would expect that as $E(p)$ increases over time, the estimated ς would decrease. Consequently this rather mechanically produced decline in the estimated parameter ς, for economies where metro area populations increase over time, should not be interpreted as evidence against (or for) power laws.

We estimate the parameters of a Pareto distribution for each cross-section of cities in the ten census years in our sample. The estimation is based on two versions of the equation:

$$\ell n[1 - F_{it}] = A_t - \varsigma_t \, \ell n \, p_{it} \ + \epsilon_{it} \, , i = 1, \ldots, I_t; \ t = 1, \ldots, T; \tag{25}$$

where $[1 - F_{it}]$ is the empirical countercumulative distribution of X_{it}, the proportion of cities with population greater than or equal to X_{it} at time t, and ϵ_{it} is a random variable that is identically normally distributed across I for every t. We allow for A_t and ς_t to possibly be time-varying parameters. This is, of course, a more general version of the equation used in the rank size rule literature, and implies, as a special case, ς_t equal to unity.[15] Finally, we note that when ϵ_{it} is

[14] Recently, geographers have become disenchanted somewhat with the Pareto law and its infinite upper tail, and have estimated constrained Pareto distributions with a finite upper bound. See Roehner (1995).

[15] The usual configuration of this relationship in the rank size literature is to simply define y_{it} to be the rank of city size P_{it}, or alternatively expressed, the *number* of cities with size greater than or equal to P_{it}. As explained above, we define y_{it} to be the *proportion* of cities with size greater than or equal to P_{it} because we derive our Equ. (25) from the countercumulative probability function of the Pareto distribution. Our formulation, in comparison to the standard formulation, will leave the critical ς_t coefficient unchanged. However, our A_t differs. Alperovich (1984) insists that A_t should be the logarithm of the size of the largest city, if the strict rank size rule is to hold. However,

assumed to be normally distributed, estimations according to (25) are associated with a pseudo-Pareto distribution. Our estimation results are reported in columns 3–6 of Table 3.

We have also estimated this equation by means of a semi-parametric method using generalized cross validation (GCV) [Härdle (1990), p. 61]. For brevity, only the results for 1990 are juxtaposed graphically on Figure 4, where the actual data are indicated with circles, the GVC-kernel estimate by squares connected with a curve, and the OLS fitted line according to Equ. (25) above. Not surprisingly, the GVC kernel estimate clearly fits the data much better than OLS. Its shape is largely concave, and thus in sharp contrast to the convexity of the Pareto countercumulative.

Estimation of a true Pareto distribution is also possible, except that p_0 must be set externally. In our data, there is the obvious choice, namely $p_0 = 50,000$. Its consequence for the maximum likelihood estimate of ς is straightforward, as the latter is available in closed form: $\hat{\varsigma}_t = \frac{N_t}{\sum_i \ell n(p_{it}/p_0)}$. The larger is p_0 the larger is $\hat{\varsigma}_t$, as a larger exponent is necessary for convergence.

The results reported on Table 3 show that the estimated ς_t's are generally lower for the true Pareto distribution, in which case they vary from .953 to .556, in 1900 and 1990 respectively, than for the pseudo Pareto, in which case they vary from 1.044 to .993. Column 8 reports results for the true Pareto distribution applied to the upper one-half of the sample, which vary from 1.212 to .993. They are higher than when the entire sample is used and all but one, the one for 1990, lie a bit above 1, thus confirming findings by the previous literature, including, most recently, Krugman (1996).

These results imply an increasing concentration in the upper tail over time, which is to say that more cities are getting larger relative to an increasing mean. The values of the constant reflect the changing proportion of "rank" to the number of cities over time. The estimated ς's offer indications of subtle change in the U.S. urban structure. Chow tests suggest that each year's ς coefficient significantly differs from the succeeding year's coefficient at the .01 level except for the 1950 to 1960 period (presumably because of the large increase in the number of cities). Obviously, there is significant difference over longer time periods as well, including 1950 to 1990 and 1960 to 1990, 1900 to 1930 and 1900 to 1990.

The jump from 1900 to 1910 may reflect heavy foreign immigration during the first decade of

our A_t is the logarithm of a city size minus the logarithm of the number of cities in the sample. The A_t values are reported along with the ς_t coefficients in Table 3.

the century. The stagnation of the 1930s is suggested in the Table as is the resurgence of economic activity in the 1950s. One of the most obvious movements is the much touted counterurbanization of the 1970s.[16] A more subtle reflection may be the move from manufacturing to service industries in U.S. cities, a trend that begins in the 1950s and accelerates in the 1960s.

We see that the estimated ς's are much smaller when we use the entire sample than when we use only the upper tail. This is not surprising, given that the sample mean is increasing and the estimates depend on setting x_0 externally and holding it equal to 50,000. Still, the results cast additional doubt upon the relevance of the Pareto distribution as a stylized fact for city size distributions when applied to the entire distribution. We performed one additional estimation, that is by means of the truncated Pareto distribution, as proposed by Roehner (1995), where the largest city in sample is used to truncate the distribution. As expected the estimate tracked, but were larger than, those of the true Pareto distribution applied to the entire sample.

All in all, we conclude that the estimate of the Pareto exponent is clearly close to 1. Having said that, we note that the fact that the 1990 estimate is below should raise doubts about the validity of strict rank-size rule. Our juxtaposition of the OLS and kernel estimates also contribute to our reservations.

6.2 Empirical transition matrices

By coding the position of each city relative to the others within the distribution, we are able to see whether or not specific cities move up or down in the distribution over time.[17] We constructed transition matrices, presented in Appendix A, in which each cell gives the proportion of cities which start in a given quantile (column) in a particular year (representing 1900 in the first matrix) which move to a particular quantile (row) in the next year (representing 1910 in the next census). Entries in the diagonal indicate that cities are staying in the same category as in the previous time period. Our categories are defined in terms of two alternative sets of intervals, namely one based on .30, .50, .75, 1.00, 2.00, and 20.00 times the contemporaneous mean, and a second based on deciles. The former facilitates a comparison with Eaton and Eckstein, whereas the second provides more

[16] Mills and Lubuele (1995) examine population (and employment) growth among the *quartiles* of the U.S. city size distribution. They find that the most rapid growth during the 1970s was in the third and fourth quartiles; in the 1980s, the fastest growth occurred in the top three quartiles.

[17] De Vries (1984), Ch. 7, appears to be the originator of this device in the study of urbanization.

detail.

Both sets of empirical transition matrices suggest that concentration at the upper end of the distribution becomes more pronounced over time: the diagonal entries are higher for higher percentiles. Another observation that follows is that most movements are to nearby cells, with very few big jumps. In interpreting these findings, one must bear in mind that the mean is changing over time.

We have averaged these decennial movements to get an average transition matrix for comparison to the Eaton and Eckstein results. It is presented in the Appendix. As we might expect in the U.S. data, there is somewhat more movement off the diagonal (compared to the French and Japanese data). Most of that movement is toward greater concentration in the time period from 1900 to 1990.

We have also computed but do not report here the invariant distribution which is associated with the average transition matrix (C.f. matrix M in (20)) for 1900 to 1990. The result confirms an increasing concentration at the upper end of the distribution of city sizes. This is in great contrast to the computed invariant distributions reported by Eaton and Eckstein, *op. cit.*, for France and Japan. So, it is not just an upward trend in the mean city size, but an overall, and sharp, tendency of the city size distribution that we see.

We note that the stationary distribution, associated with the average computed transition matrix for 1900-1990 (Table 3), contains most of its mass at the upper portion of its support. However, at any point in time, the actual distribution contains the influence of the newly entering cities, most of which (but not all) enter via the lower end of the support.

However, these transition matrices have limitations. They do not pick up the full effect of "entering" cities and they do not offer us any more insight into why such changes might occur. There are undoubtedly other variables that might impact on city size distribution. Collecting these data is often constrained either by the number of cities involved or by the time range. In order to give a sense of the change over the century, Table 1 presents descriptive statistics for decennial years for the total U.S. population, mean and median city sizes, real Gross National Product, real interest rate, the percentage of total employment in manufacturing, the average real value of agricultural land and buildings, education and earnings. Unfortunately, the availability of only ten years of time series data prevents us from examining the impact of variables which vary only with time.

6.3 Dynamics of City Population

We now discuss the dynamics of city populations and growth rates. Understanding the possible explanations for changing city sizes may shed some additional light on our question of evolving distributions. Our regression equation is:

$$\ell n\, P_{it} \;= a_i + a_t + b\ell n\, P_{i-1,t} + \epsilon_{it}, i = 1, \ldots, I_t, I_t = 1, \ldots, 334;\; t \;=\; 1910, \ldots, 1990, \qquad (26)$$

where P_{it} is the population of city i in time t, the random variable ϵ_{it} is independently, identically normally distributed for all i and t, the a_t's as time effects, reflecting the total effect of time-varying variables, and the a_i's as individual effects. Our setting suggests that fixed effects are more appropriate, although assumption of random effects may occasionally be convenient.

We account for the possibility of regional effects, by coding each city in the sample by the nine Census regions: New England, Middle Atlantic, South Atlantic, East North Central, East South Central, West North Central, West South Central, Mountain and Pacific. Figure 5 is a map indicating the boundaries of the nine regions and the number of cities located in each. Table 2 reports the descriptive statistics for all variables used in our regressions. The results of our regressions according to Equ. (26) are reported in columns 1–3 of Table 4. The results suggest strong individual effects, either in the form of random or of fixed effects.

The estimated coefficient of the lagged value of the dependent variable is very significant and close to but less than 1, especially when fixed effects are assumed. Inclusion of time dummies is very significant but does not alter this picture substantially. As for the regional dummy variables, we see highly significant, positive impact for cities being located in the South Atlantic, East and West South Central, and Pacific regions (in reference to the West North Central region).

The proximity of the estimate of the coefficient of the lagged value of the dependent variable is suggestive of a unit root. While unit roots have attracted particular interest by the macroeconometric time series literature, several authors, including notably Quah (1994), have drawn attention to special aspects of unit root inference in data structures resembling random fields, that is, where the cross-section and time dimensions are comparable.[18] However, in our case $T = 10$, and clearly

[18] Quah (1994) finds that the unit regression coefficient estimator is distributed neither (unbiased) normal at rate

an order of magnitude smaller than I_t, which ranges from 112 to 334, making those methods inapplicable. We instead ran a number of OLS and panel regressions where we "impose" a unit root by restricting the coefficient of the lagged value of the dependent variable to 1. Those restrictions are rejected comfortably in terms of the likelihood ratio test at the one percent level of significance.

Several authors have used regressions along the lines of (26) to test for divergent vs. convergent growth across national economies. Column 4 of Table 4 reports estimation results for the growth rate of city population, defined as $\ell n P_{t+1} - \ell n P_t$, as a function of the logarithm of contemporaneous population and of individual effects. Here the contemporaneous city population is treated as predetermined. The availability of only ten time periods makes it difficult to perform the battery of tests necessary to determine convergence or divergence. $\ell n P_t$ has a negative and significant coefficient. We have also worked with the growth rate using the standard definition instead of its logarithmic approximation and obtained similar results.

6.4 Urban Labor Productivity and City Size

An indirect test of our theoretical model is provided by regressing labor productivity, measured by earnings, against human capital, measured by education, and city size. The results are reported in columns 5 and 6 of Table 4, where we have controlled for individual and time effects and for regional effects. They suggest an important and highly significant effect of city size on earnings. A 10 per cent increase in population is associated with 1 percent increase in productivity. The presence of education is also highly significant and in agreement with a fair amount of recent research, e.g., Ciccone and Hall (1996), Glaeser *et al.* (1995), and Rauch (1993). As our estimated exponent of the Pareto tail decrease over time, our results imply that urbanization is associated with increasing inequality of earned incomes.

$O_p(N^{-\frac{1}{2}})$, as one might expect from standard panel data analysis, nor standard Dickey-Fuller at rate $O_p(^{-1})$, as one might expect from standard time-series analysis. Instead, the estimator is consistent and asymptotically normal, but with a non-vanishing bias in the asymptotic distribution.

7 Conclusions

This paper discusses the implications for the theory of city size distribution of increasing returns to scale leading to monopolistic competition. It proposes a rudimentary model of an urban hierarchy, in which the tendency of monopolistic competition models to generate increasing returns associated with product diversity is reinforced when the productivity of labor is augmented by having successive, vertically arranged set of sectors produce increasingly skilled labor. Broad implications of the model are then tested by means of a new data set.

Our data set is constructed to reflect changing definitions of Standard Metropolitan Statistical Areas from 1900 to 1990. While no ideal data set for cities exists, we feel fairly confident that the one we have constructed anew reflects changing numbers of cities, changing city sizes and the changing distribution. Our methods are designed to reflect the complexities of our panel of observations, which vary across both time and cities.

We address in this paper city size distributions in the United States in the twentieth century and the determinants of those distributions. The estimated Pareto distributions indicate small but significant movement toward increasing inequality, based on a declining exponent of the Pareto tail and increasing means.

We study in a non-parametric manner transitions over size distributions across each of the last ten decades for the U.S. In contrast to Eaton and Eckstein's findings for France and Japan, we find increasing concentration toward the upper end of the distribution for the U.S. over time, in spite of considerable entry of new cities. When we regress the population of metro areas against their own lagged values as well as variables reflecting individual and regional effects the results suggest that lagged own population has a significant coefficient which is close to but less than 1. Growth rate regressions also suggest convergent growth.

There are undoubtedly other variables which might be influential. We would like to be able to say something about commuting costs, because standard urban theory suggests that the changing technology of commuting has contributed to urban spread. We will continue to search for a broader set of measurable factors that might have an impact. Nevertheless, we have prescribed a method and described results which we feel shed light on the shifting distribution of city sizes in the United States to date and its implications for earnings inequality. Our work in progress on spatial interaction is also promising.

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Table 1

DESCRIPTIVE STATISTICS: DECENNIAL DATA
1900 – 1990[1]

1	2	3	4	5	6	7	8	9	10
Year	U.S. Pop. (000)	Mean Size	Median Size	GNP billion $	Interest rate (%)	Manuf. %	Education %	Agric. land value	Earnings $
1900	75,995	259952	121830	71.2	5.95	36	27.8	81.58	1770
1910	91,972	286861	121900	107.5	3.82	36.1	27.6	134.62	1939
1920	105,711	338954	144130	135.9	-.50	39	25.4	105.98	1875
1930	122,775	411641	167140	184.8	6.39	32.5	38.3	98.32	2542
1940	131,669	432911	181490	229.2	1.76	33.9	44.4	72.20	1983
1950	150,697	526422	234720	354.9	7.45	33.7	53.9	81.01	2827
1960	179,323	534936	238340	497.0	1.35	31	63.2	107.07	4108
1970	203,302	574628	259919	747.6	5.12	27.4	74.2	139.65	4763
1980	226,542	526997	232000	963.0	7.97	22.4	70.0	271.29	3520
1990	248,710	577359	243000	1277.8	5.71	17.4	81.1	154.54	3842

All figures are taken from *Historical Statistics of the United States from Colonial Times to 1970*, Volumes 1 and 2, and *Statistical Abstract of the United States, 1993.*
Columns 5, 6 and 9: GNP, interest rates and land values adjusted by the implicit price deflator constructed from sources above; 1958=100.
Column 7: "Manuf." indicates manufacturing employment as percentage of the total employment for each year.
Column 8: Mean percent of 15-20 years of age cohort across all cities.
Column 10. Mean real annual earnings, by city proper or metro area, in dollars, deflated by the consumer price index; 1967=100.

Table 2

DESCRIPTIVE STATISTICS: METRO AREAS
1900 – 1990: 1990 Observations

Variable	Mean	Std. Dev.	Skewness	Kurtosis	Min	Max
Population (000)	479.5	1001.5	6.6	58.8	50.7	9,372.0
Log(Population)	12.4028	0.9895	1.0	4.1	10.8343	16.374
Growth Rate (%)	10.58	41.99				
New England	.0879	.2833	2.9	9.5	0.00	1.00
Mid Atlantic	0.1276	0.3338	2.2	6.0	0.00	1.00
South Atlantic	0.1673	0.3734	1.8	4.2	0.00	1.00
East North Central	0.2030	0.4023	1.5	3.2	0.00	1.00
East South Central	0.0663	0.2489	3.5	13.1	0.00	1.00
West North Central	0.0910	0.2876	2.8	9.1	0.00	1.00
West South Central	0.1221	0.3275	2.3	6.3	0.00	1.00
Mountain	0.0462	0.2100	4.3	19.7	0.00	1.00
Pacific	0.0884	0.2840	2.9	9.4	0.00	1.00
Education (%)	57.1085	20.9284	-0.4	1.8	11.80	92.73
Real Wage ($)	3197.92	1132.37	0.2	2.3	1020.00	7311.00

Table 3

ESTIMATES OF THE PARETO DISTRIBUTION FOR CITY SIZES

This table gives the number of cities available for each year, the mean size, and the estimated parameters using three methods: the pseudo-Pareto distribution, according to Eq. (25), in columns 3–6, the true Pareto distribution, holding p_0 constant and equal to 10.81978 (corresponding to 50,000), column 7, and the true Pareto distribution using the upper one-half of the sample, column 8.

1	2	3	4	5	6	7	8
Year	Obs	Constant	Minimum	ς (s.e.)	R^2	ς (s.e.)	ς (s.e)
1900	112	11.419	91,035	1.044 (0.010)	0.990	.953 (.107)	1.212 (.170)
1910	139	11.106	66,569	1.014 (0.009)	0.989	.919 (.093)	1.120 (.149)
1920	149	11.214	74,161	1.010 (0.009)	0.990	.799 (.088)	1.108 (.143)
1930	157	11.075	64,537	0.985 (0.010)	0.983	.709 (.084)	1.082 (.136)
1940	160	11.263	77,886	0.995 (0.011)	0.982	.677 (.085)	1.131 (.136)
1950	162	11.523	101,013	0.999 (0.012)	0.978	.589 (.084)	1.154 (.138)
1960	210	11.278	79,063	0.977 (0.011)	0.974	.579 (.072)	1.106 (.121)
1970	243	10.986	59,042	0.949 (0.012)	0.963	.558 (.065)	1.096 (.112)
1980	322	11.378	87,378	0.985 (0.010)	0.970	.576 (.058)	1.048 (.098)
1990	334	11.977	159,054	0.949 (0.010)	0.964	.556 (.055)	.993 (.095)

Source: 1900-1950: D. Bogue (1953); 1960-1990: U.S. Census Bureau publications.

Table 4

DYNAMICS OF CITY POPULATION AND EARNINGS

Column 1: OLS regression, logarithm of city population

Column 2: OLS regression with fixed effects, logarithm city population

Column 3: GLS regression with random effects and time dummies, logarithm of city population

Column 4: GLS regression with fixed effects and time effects, ten-year first-difference of logarithms of city population

Column 5: GLS regression with random effects and period effects, logarithm of average city annual earnings

Column 6: GLS regression with random effects and time time dummies, logarithm of average city annual earnings

$t-$ statistics in parentheses.

	1	2	3	4	5	6
VARIABLE	b (t)	b (t)	b (t)	b (t)	b (t)	b (t)
Constant	.372 (6.75)		.052 (.970)	2.294 (11.51)	5.19 (54.85)	4.56 (71.36)
LogP_{t-1}	.984 (221.2)	.888 (127.5)	.998 (237.4)	-.202 (6.61)		
New England			-.036 (2.00)			.112 (3.34)
Middle Atlantic			-.028 (1.71)			.007 (.21)
South Atlantic			.010 (6.37)			-.025 (.86)
East North Central			.019 (1.29)			.083 (2.85)
East South Central			.057 (2.95)			-.054 (1.5)
West South Central			.119 (7.14)			-.300 (.98)
Mountain			.163 (7.31)			.055 (1.48)
Pacific			.187 (10.20)			.048 (1.49)
LogEducation$_t$					.398 (27.6)	.492 (49.3)
LogP_t				-.177 (10.98)	.1107 (20.9)	.1158 (22.57)
Observations	1657	1657	1657	1656	1990	1990
LLF	512.4	1101.6	748.0	316.5		905.3
χ^2 p		.0000		.0000		
R^2	.9673	.980	.974	.9597	.464	.824
F		254.6	3819	95.6		564.7

APPENDIX

AVERAGE TRANSITION MATRIX
U.S. CITIES 1900 - 1990

The cells in this matrix are identified by the upper endpoints of the categories, as explained in the text; that is, .3, .5, .75, 1, 2, and 20 times the mean. Entries in the cells are the averages over nine matrices that define decade to decade changes. See Appendix A for the decade matrices. The total cities column and row give the actual distributions for 1900 (summing to 112) and 1990 (summing to 322).

1990/1900	0.3	0.5	0.75	1	2	20	Total cities
0.3	79.84	6.90	0.41	0	0	0	110
0.5	19.79	66.36	9.07	0	0	0	64
0.75	0.37	25.66	62.54	7.62	2.53	0	51
1	0	1.09	21.48	53.47	4.48	0	23
2	0	0	6.49	38.91	79.23	.85	38
20	0	0	0	0	13.76	98.76	36
Total cities	24	31	15	14	15	13	322/112
Stationary distribution	0.15	0.38	1.00	1.27	8.03	89.17	

APPENDIX

TRANSITION MATRICES, DECADE BY DECADE

Each cell in these matrices represents the number of cities in the respective category in year *t+1* (rows) compared year *t* (columns), and the associated frequency. For example, in the first matrix, the proportion of cities belonging to the smallest category in 1900 which move to the next category (between the tenth and twentieth percentiles) in 1910 was 58.33 %; 7 cities did so. As explained in the text, these matrices do not show the entry of new cities. The first matrix picks up only the 112 ciities that meet our criteria for 1900.

TRANSITIONS: 112 cities; 1900 to 1910.

1910/1900	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00	Total	Enter 1910
0.10	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	14
0.20	7 58.33	1 9.09	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	8 7.14	6
0.30	3 25.00	3 27.27	3 27.27	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	9 8.04	5
0. 40	0 0.00	5 45.45	5 45.45	2 18.18	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	12 10.71	2
0. 50	1 8.33	0 0.00	0 0.00	0 0.00	0 0.00	2 16.67	1 9.09	3 27.27	4 36.36	3 27.27	14 12.50	0
0.60	0 0.00	1 9.09	0 0.00	5 45.45	6 54.55	2 16.67	0 0.00	0 0.00	0 0.00	0 0.00	14 12.50	0
0.70	0 0.00	0 0.00	0 0.00	0 0.00	1 9.09	6 50.00	7 63.64	0 0.00	0 0.00	0 0.00	14 12.50	0
0.80	0 0.00	0 0.00	0 0.00	0 0.00	1 9.09	2 16.67	4 36.36	7 63.64	0 0.00	0 0.00	14 12.50	0
0.90	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	1 8.33	0 0.00	4 36.36	9 81.82	0 0.00	14 12.50	0
1.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	2 18.18	11 100.00	13 11.61	0
Total	12 10.71	11 9.82	11 9.82	11 9.82	11 9.82	12 10.71	11 9.82	11 9.82	11 9.82	11 9.82	112 100.00	27

TRANSITIONS: 139 cities; 1910 to 1920.

1920/1910	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00	Total	Enter 1920
0.10	6 42.86	3 21.43	1 7.14	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	10 7.19	5
0.20	6 42.86	5 35.71	1 7.14	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	12 8.63	3
0.30	1 7.14	4 28.57	8 57.14	1 7.14	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	14 10.07	1
0.40	1 7.14	0 0.00	2 14.29	9 64.29	2 14.29	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	14 10.07	1
0.50	0 0.00	2 14.29	2 14.29	4 28.57	5 35.71	2 14.29	0 0.00	0 0.00	0 0.00	0 0.00	15 10.79	0
0.60	0 0.00	0 0.00	0 0.00	0 0.00	4 28.57	10 71.43	1 7.14	0 0.00	0 0.00	0 0.00	15 10.79	0
0.70	0 0.00	0 0.00	0 0.00	0 0.00	2 14.29	2 14.29	10 71.43	1 7.14	0 0.00	0 0.00	15 10.79	0
0.80	0 0.00	0 0.00	0 0.00	0 0.00	1 7.14	0 0.00	3 21.43	10 71.43	1 7.14	0 0.00	15 10.79	0
0.90	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	3 21.43	12 85.71	0 0.00	15 10.79	0
1.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	1 7.14	13 100.00	14 10.07	0
Total	14 10.07	14 10.07	14 10.07	14 10.07	14 10.07	14 10.07	14 10.07	14 10.07	14 10.07	13 9.35	139 100.00	10

TRANSITIONS: 149 cities; 1920 to 1930.

1930/1920	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00	Total	Enter 1930
0.10	9 60.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	9 6.04	7
0.20	6 40.00	8 53.33	1 6.67	1 6.67	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	16 10.74	0
0.30	0 0.00	3 20.00	7 46.67	6 40.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	16 10.74	0
0.40	0 0.00	4 26.67	5 33.33	4 26.67	2 13.33	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	15 10.07	0
0.50	0 0.00	0 0.00	2 13.33	3 20.00	8 53.33	2 13.33	0 0.00	0 0.00	0 0.00	0 0.00	15 10.07	1
0.60	0 0.00	0 0.00	0 0.00	1 6.67	3 20.00	11 73.33	1 6.67	0 0.00	0 0.00	0 0.00	16 10.74	0
0.70	0 0.00	0 0.00	0 0.00	0 0.00	2 13.33	2 13.33	9 60.00	2 13.33	0 0.00	0 0.00	15 10.07	0
0.80	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	5 33.33	9 60.00	2 13.33	0 0.00	16 10.74	0
0.90	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	4 26.67	12 80.00	0 0.00	16 10.74	0
1.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	1 6.67	14 100.00	15 10.07	0
Total	15 10.07	15 10.07	15 10.07	15 10.07	15 10.07	15 10.07	15 10.07	15 10.07	15 10.07	14 9.40	149 100.00	8

TRANSITIONS: 157 cities; 1930 to 1940.

1940/1930	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00	Total	Enter 1940
0.10	12 75.00	1 6.25	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	13 8.28	3
0.20	4 25.00	9 56.25	3 18.75	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	16 10.19	0
0.30	0 0.00	5 31.25	9 56.25	2 13.33	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	16 10.19	0
0.40	0 0.00	1 0.64	4 2.55	8 5.10	3 1.91	0 10.19 0.00	0 0.00	0 0.00	0 0.00	0 0.00	16 10.19	0
0.50	0 0.00	0 0.00	0 0.00	5 33.33	9 56.25	2 12.50	0 0.00	0 0.00	0 0.00	0 0.00	16 10.19	0
0.60	0 0.00	0 0.00	0 0.00	0 0.00	3 18.75	12 75.00	1 6.67	0 0.00	0 0.00	0 0.00	16 10.19	0
0.70	0 0.00	0 0.00	0 0.00	0 0.00	1 6.25	2 12.50	13 86.67	0 0.00	0 0.00	0 0.00	16 10.19	0
0.80	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	1 6.67	14 87.50	1 6.25	0 0.00	16 10.19	0
0.90	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	2 12.50	14 87.50	0 0.00	16 10.19	0
1.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	1 6.25	15 100.00	16 10.19	0
Total	16 10.19	16 10.19	16 10.19	15 9.55	16 10.19	16 10.19	15 9.55	16 10.19	16 10.19	15 9.55	157 100.00	3

TRANSITIONS: 160 cities; 1940 to 1950.

1950/1940	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00	Total	Enter 1950
0.10	12 75.00	3 18.75	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	15 9.38	2
0.20	3 18.75	10 62.50	3 18.75	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	16 10.00	0
0.30	1 6.25	1 6.25	11 68.75	3 18.75	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	16 10.00	0
0.40	0 0.00	2 12.50	2 12.50	8 50.00	4 25.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	16 10.00	0
0.50	0 0.00	0 0.00	0 0.00	5 31.25	8 50.00	3 18.75	0 0.00	0 0.00	0 0.00	0 0.00	16 10.00	0
0.60	0 0.00	0 0.00	0 0.00	0 0.00	3 18.75	10 62.50	3 18.75	1 6.25	0 0.00	0 0.00	17 10.63	0
0.70	0 0.00	0 0.00	0 0.00	0 0.00	1 6.25	3 18.75	9 56.25	3 18.75	0 0.00	0 0.00	16 10.00	0
0.80	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	3 18.75	10 62.50	3 18.75	0 0.00	16 10.00	0
0.90	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	1 6.25	2 12.50	13 81.25	0 0.00	16 10.00	0
1.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	16 100.00	16 10.00	0
Total	16 10.00	16 10.00	16 10.00	16 10.00	16 10.00	16 10.00	16 10.00	16 10.00	16 10.00	16 10.00	160 100.00	2

TRANSITIONS: 162 cities; 1950 to 1960.

1960/1950	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00	Total	Enter 1960
0.10	5 29.41	0 0.00	1 6.25	1 6.25	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	7 4.32	14
0.20	9 52.94	4 25.00	0 0.00	1 6.25	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	14 8.64	7
0.30	3 17.65	7 43.75	2 12.50	0 0.00	1 6.25	0 0.00	1 6.25	0 0.00	0 0.00	0 0.00	14 8.64	7
0.40	0 0.00	4 25.00	7 43.75	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	11 6.79	10
0.50	0 0.00	0 0.00	4 25.00	12 75.00	0 0.00	1 5.88	1 6.25	0 0.00	0 0.00	0 0.00	18 11.11	3
0.60	0 0.00	0 0.00	2 12.50	1 6.25	11 68.75	4 23.53	2 12.50	0 0.00	0 0.00	0 0.00	20 12.35	1
0.70	0 0.00	1 6.25	0 0.00	1 6.25	4 25.00	7 41.18	3 18.75	2 12.50	2 12.50	0 0.00	20 12.35	1
0.80	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	4 23.53	7 43.75	7 43.75	1 6.25	0 0.00	19 11.73	2
0.90	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	1 5.88	2 12.50	7 43.75	10 62.50	0 0.00	20 12.35	1
1.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	3 18.75	16 100.00	19 11.73	2
Total	17 10.49	16 9.88	16 9.88	16 9.88	16 9.88 10.49	17 9.88	16 9.88	16 9.88	16 9.88	16 100.00	162	48

TRANSITIONS: 210 cities; 1960 to 1970.

1970/1960	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00	Total	Enter 1970
0.10	15 71.43	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	15 7.14	10
0.20	5 23.81	11 52.38	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	16 7.62	8
0.30	0 0.00	6 28.57	12 57.14	2 9.52	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	20 9.52	4
0.40	1 4.76	3 14.29	7 33.33	9 42.86	1 4.76	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	21 10.00	4
0.50	0 0.00	1 4.76	1 4.76	8 38.10	9 42.86	2 9.52	0 0.00	0 0.00	0 0.00	0 0.00	21 10.00	3
0.60	0 0.00	0 0.00	1 4.76	2 9.52	10 47.62	10 47.62	0 0.00	0 0.00	0 0.00	0 0.00	23 10.95	1
0.70	0 0.00	0 0.00	0 0.00	0 0.00	1 4.76	9 42.86	13 61.90	0 0.00	0 0.00	0 0.00	23 10.95	2
0.80	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	8 38.10	15 71.43	1 4.76	0 0.00	24 11.43	0
0.90	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	6 28.57	18 85.71	0 0.00	24 11.43	0
1.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	2 9.52	21 100.00	23 10.95	1
Total	21 10.00	21 10.00	21 10.00	21 10.00	21 10.00	21 10.00	21 10.00	21 10.00	21 10.00	21 10.00	210 100.00	33

TRANSITIONS: 243 cities; 1970 to 1980.

1980/1970	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00	Total	Enter 1980
0.10	18 7.41	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	18 7.41	16
0.20	5 71.43	6 25.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	11 4.53	21
0.30	1 14.29	10 41.67	8 33.33	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	19 7.82	13
0.40	1 14.29	7 29.17	6 25.00	5 20.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	19 7.82	12
0.50	0 0.00	1 4.17	10 41.67	11 44.00	3 12.50	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	25 10.29	7
0.60	0 0.00	0 0.00	0 0.00	7 28.00	14 58.33	7 29.17	0 0.00	0 0.00	0 0.00	0 0.00	28 11.52	5
0.70	0 0.00	0 0.00	0 0.00	2 8.00	7 29.17	12 50.00	10 40.00	0 0.00	0 0.00	0 0.00	31 11.52	1
0.80	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	5 20.83	14 56.00	10 41.67	0 0.00	1 4.17	30 12.35	2
0.90	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	1 4.00	14 58.33	16 66.67	0 0.00	31 12.76	1
1.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	8 33.33	23 95.83	31 12.76	1
Total	25 10.29	24 9.88	24 9.88	25 10.29	24 9.88	24 9.88	25 10.29	24 9.88	24 9.88	24 9.88	243 100.00	79

TRANSITIONS: 322 cities; 1980 to 1990.

1990/1980	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00	Total	Enter 1990
0.10	28 82.35	3 9.38	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	31 9.63	4
0.20	4 11.76	16 50.00	11 34.38	0 0.00	1 3.13	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	32 9.94	1
0.30	2 5.88	10 31.25	12 37.50	4 12.90	3 9.38	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	31 9.63	3
0.40	0 0.00	2 6.25	8 25.00	12 38.71	7 21.88	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	29 9.01	3
0.50	0 0.00	1 3.13	1 3.13	15 48.39	12 37.50	3 9.09	1 3.13	0 0.00	0 0.00	0 0.00	33 10.25	0
0.60	0 0.00	0 0.00	0 0.00	0 0.00	8 25.00	20 60.61	5 15.63	0 0.00	0 0.00	0 0.00	33 10.25	1
0.70	0 0.00	0 0.00	0 0.00	0 0.00	1 3.13	10 30.30	20 62.50	2 6.25	0 0.00	0 0.00	33 10.25	0
0.80	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	6 18.75	24 75.00	4 12.50	0 0.00	34 10.25	0
0.90	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	6 18.75	26 81.25	1 3.13	33 10.25	0
1.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	0 0.00	2 6.25	31 96.88	33 10.25	0
Total	34 10 56	32 9.94	32 9.94	31 9.63	32 9.94	33 10.25	32 9.94	32 9.94	32 9.94	32 9.94	322 100.00	12

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