Housing and Labor Market Vacancies and Beveridge Curves: Theoretical Framework and Illustrative Statistics [1]

Yannis M. Ioannides[*] and Jeffrey E. Zabel[**]

Department of Economics

Tufts University

Medford, MA 02155

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Abstract

The chapter presents a model of housing and labor markets in the DMP tradition. The model treats decisions about housing and labor supply as joint decisions of individuals, articulates how the renting and owner segments of housing markets interact and adjust through turnover flows. It also highlights the transitions across different discrete states in those markets, that is owner-to-owner, owner-to-renter, renter-to-owner, and renter-to-renter by unemployed or employed workers. It allows for vacancy rates in both the rental and owner segments of the housing market, and introduces a novel concept of "unemployment" in housing markets thus allowing for the definition of Beveridge curves for housing markets. The chapter documents the empirical significance of these concepts by means of data from the Panel Study of Income Dynamics for 1969–2015.

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1 Introduction

The basic model of markets with frictions developed by Diamond, Mortensen and Pissarides (DMP)[2] that formalizes the central role of the natural unemployment rate is the standard framework for studying labor markets with frictions. A similar approach for the housing market that gives rise to the natural vacancy rate in equilibrium initiated by Wheaton (1990) has not been as fully developed. Obviously, the housing and labor markets are linked and this became abundantly clear in the Great Recession of 2007–2009 in the United States. So, it makes sense to develop a joint model of the housing and labor markets.

This conceptual framework proposed by the article is firmly in the DMP style. It provides a simple conceptual framework for the collection of DMP-based models of the housing market included in the volume. It encompasses a theory that extends the model of the housing market with frictions developed by Head and Lloyd-Ellis (2012). One extension is a frictional labor market along the lines of Pissarides (1985; 2000). In contrast, the labor market in Head and Lloyd-Ellis (2012) is Walrasian. A second one is a frictional rental housing market, which gives rise to the natural vacancy rate in the rental segment of the housing market. The third one allows for rationing of renters who wish to enter the ownership market and of owners who wish to enter the rental market. The fourth uses these extensions to develop a novel concept of "unemployment" for the rental and ownership housing markets. This ushers in the development of Beveridge curves for housing markets. The framework incorporates spillovers across the Beveridge curves for the housing and labor markets.

These extensions are novel and critical in appreciating the sort of illustrative statistics provided herein and in structuring empirical investigations, such as Ioannides and Zabel (2017). Vacancy rates in both the housing and labor markets emerge naturally from search models. The link between them emanates from the same framework used in studying the behavior of individuals as employees in the labor market and as owners and renters in housing markets. That is, decisions by owners and renters rest on the same preference structure as those by individuals in the roles as prospective employees when they interact with employers.

An important contribution of the DMP framework for labor markets is a rigorous foundation for the Beveridge Curve [Beveridge (1944)]. Several recent papers have revisited the Beveridge Curve as a tool of business cycle analyses of labor markets.[3] The theoretical model in the present article develops the counterpart of the Beveridge Curve in the housing market. While vacant units in housing markets naturally correspond to job vacancies in the labor market, the concept of unemployment is difficult to translate in the housing market. Our proposed solution is motivated by intuitive similarities between housing and labor markets and by several sources of information on the cyclical dependence of housing turnover, as by the work by Bachmann and Cooper (2014) and in the evidence provided herein on the correlation between residential moves and job changes.

We posit that frictions affecting renters generate an "unfulfilled" demand for owner occupied housing (just as unemployment is the unfulfilled demand for employment). That is, in a frictionless world, some renters would rather own, given fundamentals, but are rationed out because they cannot get a mortgage or for other reasons (Henderson and Ioannides 1986). A similar concept holds for owners who would rather rent. For them to rent, they have to deal with the frictions associated with selling a home and moving. We believe that this is the first development of a housing market counterpart of the Beveridge curve.[4] Our model of housing and labor market vacancies originates from viewing housing and employment as joint decisions, and thus explicitly captures the interdependence of the two markets via the joint setting of tightness and wages, as we elaborate in detail further below. Empirically, vacancies in the housing market can shift the labor market Beveridge Curve and vice versa.[5]

We provide empirical support for the theory developed in this chapter by following Bachmann and Cooper (2014) and using PSID data for 1969-2015 to illustrate the numerical magnitude of turnover in the US housing market. We augment the housing statistics with similar information from the labor market and then analyze the joint distribution of housing and labor market transitions. The data show that around 70% of households neither move residences nor change their labor market status in a given year. For those who change both their labor and housing market statuses, the most common transition is job-to-job and rent-to-rent. Our results indicate that renters are more likely to change their job market status than owners and conditional on a change in labor market status, about one-quarter to one-third of households change residence.

We find evidence that housing turnover is pro-cyclical whereas labor market turnover is acyclical relative to both the housing and business cycles. Results show that while there is a mildly negative correlation between the turnover rates in the housing and labor markets, there is a much stronger relationship between the cycles in the two markets.

1.1 Literature Review

A number of papers in the literature employ search models in the empirical study of housing markets, though very few among them examine both the housing and labor market by means of the full complement of ideas proposed here. Coulson and Fisher (2009) and Rupert and Wasmer (2012) develop models of joint housing-labor search. Ioannides (1975) is the first paper to apply search theory to housing decisions.

Rupert and Wasmer (2012) develop a theory of the relationship between unemployment and housing market frictions that focuses on the trade-off between commuting time and location decisions within a single labor market. With job and housing vacancy searches being jointly indexed by commuting distance, the housing search process is subsumed into the job search. The housing market is not explicitly modeled. The spatial distribution of new and existing vacancies plays the role of housing supply, but demand is not rationed by housing price. In a notable study Limnios (2014) explores whether frictions in the rental housing market can help explain frictions in the labor market. Unlike Rupert and Wasmer, Head and Lloyd-Ellis (2012) focus on frictions in the housing market and the role of housing markets in generating frictions between labor markets. They do not, however, allow for frictions originating in the labor market, which they assume to be Walrasian. Head and Lloyd-Ellis do distinguish between homeownership and renting, with Bellman equations being defined separately for employed and unemployed renters and owners and are conditional on two different city types. The housing market is intermediated by real estate firms. Head and Lloyd-Ellis rely on the steady-state equilibrium values of the Bellman equations to establish that the rent differential across different city types is determined by unemployed renters who are assumed to move costlessly between cities. A key friction modelled by Head and Lloyd-Ellis pertains to the illiquidity of housing for homeownership. Their calibration of the model in order to match aggregate US statistics on mobility, housing, and labor flows predicts that the effect of homeownership on aggregate unemployment is small. When unemployment is high, however, changes in the rate of homeownership can have economically significant effects.

In a sequence of papers, Ngai and Sheedy (2013; 2015) focus on the frictions associated with buying and selling homes. Ngai and Sheedy (2015) emphasize, in particular, the dynamic consequences of the fact that the majority of housing purchase transactions involve households moving from one house to another, whereby they put their existing homes on the market and plan to buy new homes. This is motivated by households' desire to improve match quality, and consequently their decisions produce a cleansing effect on the quality distribution. Moving may be triggered by events, like a demographic shock to a household that causes a reassessment of its housing demand. Ngai and Sheedy (2013) emphasize sellers' decisions, namely when to put a house up for sale and when to agree to a sale. They do not take a position on the interdependence between residential moves and job changes.

Remarkably, there is relatively little literature on joint models of housing and labor markets jointly from a DMP perspective, with both segments of the housing market, that is rental and ownership markets, being jointly considered. Arnott (1989) and Igarachi (1991) involve rental housing markets and Wheaton (1990) ownership ones. None of the previous studies propose a Beveridge curve for housing markets, which is a key contribution of the present chapter.

Particularly relevant for our chapter are important facts reported by Bachmann and Cooper (2014), who use data from the 1969–2009 waves of the Panel Study of Income Dynamics (PSID). They report evidence on households' propensity to move and tenure choices and how such decisions correlate with aggregate economic activity. For example, 15.3% of households move each year, with roughly 55% of these moves being by renters moving to new rental dwellings, 20% by owners changing homes, 10% by owners moving to rent and 15% by renters moving to own. Only a small fraction of these moves generated net additions to the stock of owners. Bachmann and Cooper also report that whereas total housing turnover is very weakly contemporaneously correlated with the unemployment rate, it is quite strongly correlated with the growth rate of GDP (detrended by means of an HP-filter). The correlation of the unemployment rate with the owner-to-owner moving rate is substantial and negative (-0.52), with the renter-to-renter moving rate is substantial and positive (0.51) and with the renter-to-owner moving rate is absolutely smaller and negative ($-.32$). So moving in order to own is negatively correlated with the unemployment rate and owner-to-owner and renter-to-owner moves are positively correlated with output growth, 0.44 and 0.59, respectively. When leads and lags are included, owner-to-owner moves are contemporaneous with the business cycle, renter-to-owner and renter-to-renter moves lead it, and owner-to-renter moves are acyclical. Furthermore, turnover seems to lead house prices, especially renter-to-owner moves, which also lead aggregate economic activity. Bachmann and Cooper speculate that households start buying houses because of good news about economic activity and about the housing sector.

Housing search is often associated with, as well as prompted by, job change. Using data from the PSID for 1991-1993, Ioannides and Kan (1996) report that for 1974-1983, the proportion of moves combined with job changes was 6% for household heads, while the proportion of job changes was 15% per year, and that for residential moves was 15.6%. Thus, more than 40% of the movers also changed jobs, which implies a substantial correlation between moving and job change. Furthermore, nearly two-thirds of movers did so in order to rent, and one-third to own.[6]

A distinguishing feature of the housing market is the coexistence of tenure modes, renting and owning and the accordant household's choice of renting versus owner-occupancy, with rental and homeownership vacancy rates and the associated market-level variables being important determinants. We propose a joint model of frictional labor and housing markets that allows for tenure modes and use it to motivate empirical analyses of both types of vacancy rates. To the best of our knowledge, ours is the first paper to introduce a Beveridge Curve for housing markets in a manner that is consistent with the original definition for the labor market. We arrive at this result by extending Head and Lloyd-Ellis (2012) in order to account for frictional rental markets as well as frictional tenure choice. Furthermore, we examine the interdependence of labor and housing market vacancies by extending Head and Lloyd-Ellis (2012) to allow for frictional labor markets.

The remainder of this chapter is organized as follows. The first part introduces a theoretical framework. The second part illustrate the framework by means of statistics drawn from the Panel Study of Income Dynamics for 1969–2015. In this fashion, we extend in part and update those of Bachmann and Cooper (2014).

2 Theoretical Framework

2.1 Preferences

Let W^j, U^j, denote the conditional value functions, that is expected lifetime utility, conditional on being employed (W^j) and unemployed (U^j), for a renter (R) and a homeowner (H), $j = R, H$ respectively. These are expressed in real terms, and under the assumption of unrestricted borrowing or lending at a fixed rate of interest, ρ.[7] They are generated by flow of utility per unit of time, denoted by π^j, and defined in terms of non-housing consumption c^j, labor supply, l^j, and housing consumption, z^j, per unit of time. Following Head *et al.* (2014), Eq. (3), we let the flow of utility be linear in non-housing consumption, housing consumption, and leisure, $1 - l^j$:

$$\pi^j(c, l, z) = c^j - l^j + z^j, j = R, H. \tag{2.1}$$

We assume that a person is either employed and earning w^j, or unemployed and receiving $b < w^j$. Note also that we allow for the possibility that bargaining between firms and workers may lead to wage rates that are different between renters and owners, w^H, w^R, respectively. More on this below.

In defining the flow of indirect utility (2.1), we allow for housing costs to depend on housing tenure. Let non-housing consumption be the numeraire, with its price set equal to 1, and let κ be rent per unit of rental housing. Ignoring commuting costs, the quantity of housing consumed by renters in a particular area is given by rent expenditure divided by κ.

Let p_h be the annual user cost of owner-occupied housing. This is defined as the annualized user cost of housing per unit of housing value [*c.f.* Poterba (1984); Henderson and Ioannides (1986)]: a dwelling unit of value V^H generates an annualized user cost of $p_h V^H$, and superscript H accounts for the fact that dwelling units for owner occupancy or renting are distinct. For the latter, the symbol V^R will be introduced later on. [8] The user cost of housing reflects the implications of the tax treatment of housing as well as its durability.[9] The respective quantity of housing consumed, that is, housing services, is given by $\frac{p_h V}{p}$, where p is a house price index. Suppose that there are no property or taxes, nor maintenance, depreciation, and appreciation, and an individual borrows at the real rate of interest ρ to finance living in a house of value V. She would thus incur housing costs per unit of time equal to the opportunity cost of housing of value V, ρV. Equivalently, since housing is durable, services from an actual housing stock V are given by $\rho \frac{V}{p}$.

Under the assumption of perfect capital markets, with individuals' being able to borrow against their expected future income or to save at rate ρ, the Bellman equations for the conditional value functions W^j, U^j, may be defined once we have defined the respective flows of real utility, π^H, π^R. For a homeowner, ignoring the disutility of work and allowing for institutional considerations to enter through the definition of p_h, flow utility according to (2.1) may be written as the sum of the flow of housing and non-housing consumption, defined as the real wage rate (or unemployment compensation, as appropriate) plus dissaving:

$$\pi^H(w^H) = \frac{p_h V^H}{p} + w^H - \rho V^H + \text{dissaving}, \tag{2.2}$$

where $-\rho V$ denotes the opportunity cost (dissaving) associated with holding (durable) housing stock of value V. For a renter, we have correspondingly:

$$\pi^R(w^R) = \frac{\text{rent expenditure}}{\kappa} + w^R - \text{rent expenditure} + \text{dissaving}. \tag{2.3}$$

For an unemployed individual, b takes the place of $w^j,\ j = H, R$, on the right hand sides of Eq.'s (2.2) and (2.3).

We now explore the implications of this formulation in the simplest possible case at the steady state, with renters and owners retaining their housing tenure status forever. Let δ

denote the exogenous job destruction rate and μ the job finding rate (which will be specified in section 2.4 below as a function of labor market tightness). The Bellman equations for the conditional value functions are, first for employed and unemployed owners:

$$\rho W^H = \pi^H(w^H) + \delta[U^H - W^H]; \tag{2.4}$$

$$\rho U^H = \pi^H(b) + \mu[W^H - U^H]; \tag{2.5}$$

and correspondingly for for employed and unemployed renters:

$$\rho W^R = \pi^R(w^R) + \delta[U^R - W^R]; \tag{2.6}$$

$$\rho U^R = \pi^R(b) + \mu[W^R - U^R]; \tag{2.7}$$

where the flow utilities π^H and π^R are specified in Eq.'s (2.2)-(2.3) above, except that the term dissaving is of course dropped when we integrate from the flow to the stock (to arrive at the respective value functions). From now on, we will use $\pi^j, j = H, R$, without the term dissaving.[10] Below we solve for the expressions for the conditional value functions under the assumption that renters transition to owners at their first opportunity. The model can accommodate tenure choice.[11]

The associated steady-state unemployment rate is given by: $\frac{\delta}{\delta+\mu}$. The job finding rate is typically specified in terms of the the job matching process and labor market tightness, to which we come further below. Housing spells of homeowners are initially assumed to last forever, if job market events and housing tenure events are independent. We assume that housing units for renters and owners are perfect substitutes.

2.2 Frictions in Housing Markets

Both housing and labor markets are subject to frictions. The individual (or household, the two terms will be used interchangeably) is subject to the risk of job loss: jobs break up at a Poisson rate δ, and the unemployed individual finds a job at a Poisson rate μ, per unit of time. Dwelling units, either for owner-occupancy or renting may be occupied or vacant. Frictions are present in the matching of dwelling units and individuals via search, which leads to the determination of vacancy rates for the ownership and rental housing markets. Suppose first, like Head and Lloyd-Ellis (2012), that rental units may be found instantaneously and thus frictionlessly, while units for owner-occupancy involve a matching process, i.e. frictions. Consequently, the values of vacant units as assets may differ from the transaction prices at which they change owners. We extend the model in section 2.2.5 below to allow for frictions in the rental housing market.

Specifically, let γ^H denote the rate at which new dwelling units sold by construction firms match with prospective homeowners. Head and Lloyd-Ellis specify γ^H as the product of the rate at which prospective homeowners match with dwelling units, $\bar{\lambda}^H$, times housing market tightness, ϕ^H:

$$\gamma^H = \bar{\lambda}^H \phi^H. \tag{2.8}$$

Housing market tightness is defined here as the ratio of prospective homeowners (the "demand") to vacant units in the homeownership (the "supply") segment of the market. This definition may be generalized by specifying, in the standard Pissarides fashion, a neoclassical matching function for individuals and vacant dwelling units.[12] It may also be generalized to account for the time it takes owner-occupied dwelling units to be transferred from one household to another, if turnover in owner-occupied units is allowed (as for example by Wheaton (1990)). Because matching in housing markets involves frictions, it renders housing to some extent illiquid; its value when vacant depends on how fast buyers may be found for dwelling units on the market. Our model highlights this feature.

The population consists of N individuals whose number is assumed to grow at a rate ν. Individuals may be found in one of four different discrete states, employed and unemployed homeowners, and employed and unemployed renters, whose stocks are denoted by $N^{WH}, N^{UH}, N^{WR}, N^{UR}$), respectively. Therefore:

$$N^{WR} + N^{UR} + N^{WH} + N^{UH} = N. \tag{2.9}$$

It is more convenient to work with the relative numbers of agents, that is their shares in different states. By using lower case n's, i.e. $n^{WH} = \frac{N^{WH}}{N}$, Eq. (2.9) becomes:

$$n^{WH} + n^{UH} + n^{WR} + n^{UR} = 1. \tag{2.10}$$

Let $\mathcal{R}, \mathcal{H}$ denote the total housing stock, in the rental and homeownership segments of the housing market, and let r, h denote their respective per capita values. Using the above notation, housing market tightness for the ownership market becomes: $\phi^H = \frac{N^{WR}+N^{UR}}{H-N^{WH}-N^{UH}}$.[13] Eq. (2.8) can be expressed as

$$\gamma^H = \bar{\lambda}^H \frac{n^{WR}+n^{UR}}{h-n^{WH}-n^{UH}}. \tag{2.11}$$

2.2.1 Housing Supply: The Rental Housing Market

Following Glaeser *et al.* (2014) and Head and Lloyd-Ellis, *op. cit.*, we assume free entry into the housing construction-real estate business and specify a supply equation for rental housing units: the present value of rents equals the asset value of their unit construction costs, that is:

$$\frac{\kappa}{\rho} = c_0 + c^R r,$$

where c_0 denotes fixed construction costs, and $c^R r$ variable costs that depend linearly on the rental housing stock per person, r, in order to express the cost of land due to congestion. We assume initially (but relax later) that the entire stock of rental units are occupied as soon as they are produced, that is, the rental housing market is not subject to frictions and rental vacancy rates are equal to 0. Since all rental units are occupied, $r = n^{WR} + n^{UR}$, the above equation may be rewritten instead in terms of n^{WR} and n^{UR} :

$$\frac{\kappa}{\rho} = c_0 + c^R \left(n^{WR} + n^{UR}\right). \tag{2.12}$$

Housing is assumed to last forever, and the rental price κ may be assumed to include maintenance costs. Under the assumption of free entry, we need not worry about the profits of owners of the rental housing stock. We modify Eq. (2.12) further below in section 2.2.5 in order to introduce frictions in the rental housing market.

2.2.2 Housing Supply: The Homeownership Market

The value of units for owner-occupancy when vacant must compensate their producers:

$$V^H = c_0 + c^H h, \tag{2.13}$$

where $c^H h$ denotes unit variable costs, that depend linearly on the per capita housing stock for homeownership, h, in order to express the cost of land due to congestion. It is also possible to allow for (costly) conversion of dwelling units from one mode of tenure to the other.

We note that the supply equations (2.12)-(2.13) link "prices," that is rents and values of vacant units, to their respective stocks relative to the numbers of individuals. Next, we relate supply equations to demand conditions by specifying the decision problems of individuals.

2.2.3 The Value of Vacant Housing in the Owner-Occupied Market

The value, V^H, of vacant dwellings in the home ownership market must, at asset equilibrium, reflect the fact that dwelling units may be purchased by either employed or unemployed renters, whose willingness to pay may be different. The return per unit of time to holding an asset of value V^H is equal to the probability per unit of time that it may be sold either to an employed renter, at price P^W, or an unemployed renter, at price P^U, whichever of the two bids is higher:

$$\rho V^H = \gamma^H \mathcal{E}\left[\max_j \left\{P^j - V^H\right\}\right], j = W, U. \tag{2.14}$$

The expectation on the rhs of Eq. (2.14), the arbitrage equation for V^H, may be written out by recognizing that a unit may either be purchased by an employed renter, if $P^W > P^U$, an event that occurs with probability equal to the proportion of those employed among all renters, $\alpha = \frac{N^{WR}}{\mathcal{R}}$; or by an unemployed renter, if $P^W \leq P^U$, an event that occurs with probability $1 - \alpha = \frac{N^{UR}}{\mathcal{R}}$.

Consistent with the literature of markets with frictions, a seller and a buyer of a dwelling unit who come into contact engage in Nash bargaining and split the surplus from the transaction, with a share σ of V^H going to the seller and $(1-\sigma)(W^H - W^R)$ to the buyer, if employed, or $(1 - \sigma(U^H - U^R)$, if unemployed. So, the prices paid by employed and unemployed households satisfy:

$$P^W = \sigma V^H + (1-\sigma)\left[W^H - W^R\right]; P^U = \sigma V^H + (1-\sigma)\left[U^H - U^R\right]. \tag{2.15}$$

Solving for V^H from Eq.'s (2.14) and (2.15) gives:

$$V^H = \frac{(1-\sigma)\gamma^H}{\rho + (1-\sigma)\gamma^H} \left[\alpha \left[W^H - W^R\right] + (1-\alpha)\left[U^H - U^R\right]\right]. \tag{2.16}$$

Recall that we have assumed that once renters purchase dwelling units and become homeowners they remain so forever. Their conditional value functions are given by (2.4)–(2.5) above. Renters, on the other hand, are faced with opportunities, at a rate γ^H, to purchase dwelling units and become homeowners. Thus, the respective Bellman equations, the counterparts of (2.6) and (2.7), for the conditional value functions become:

$$\rho W^R = \pi^R(w^R) + \delta[U^R - W^R] + \gamma^H \left[W^H - P^W - W^R\right]; \tag{2.17}$$

$$\rho U^R = \pi^R(b) + \mu[W^R - U^R] + \gamma^H \left[U^H - P^U - U^R\right]. \tag{2.18}$$

We may modify the model to allow for the interdependence between employment and housing tenure mode transitions, but so far, such transitions are assumed to be independent. However, conditions in the housing market have a profound effect on the conditional value functions.

Next, we use Eq. (2.16) in (2.15) in order to express the transaction prices, P^W, P^R, in terms of the conditional value functions, W^H, W^R, and U^H, U^R. We substitute back into the Bellman equations, (2.4-2.5) for owners, and (2.17-2.18) for renters, and solve for the conditional value functions, namely for W^H, W^R, U^H, U^R, as functions of the real wage rate and unemployment compensation, on the one hand, and of labor market and housing market tightness, on the other. Labor market tightness enters the job finding rate for owners and renters, as we discuss in more detail in section 2.4 below.

2.2.4 Housing Market Flows and Conditional Value Functions

Given labor market magnitudes, that is wages, unemployment, and job vacancy rates, which are determined from the model of labor markets with frictions, we may proceed as follows. In view of the value of vacant units for sale from Eq. (2.16), the transaction prices for owner-occupied units P^W and P^R, may be expressed in terms of the four conditional value functions, W^H, W^R, U^H, U^R, that enter their definitions. There are seven unknowns, the per capita stocks for owner-occupancy and renting, h, r, the rent, κ, and the relative numbers of agents in the four different states, $(n^{WR}, n^{UR}, n^{WH}, n^{UH})$. By solving Eq's (2.4), (2.5), (2.17), and (2.18) for the conditional value functions, we can express the value of a vacant home, V^H, in terms of the four unknown relative numbers of agents in different states, the unknown rental price, κ, the wage rates w^H and w^R, and the unemployment compensation rate, b.

It is more convenient to think of the model in a steady state, with the number of individuals growing at an exogenous rate ν. Along the steady state, all stocks of agents grow at the same rate leaving the relative numbers of agents constant.[14] This leads to four relationships in terms of the relative numbers of agents. First, Eq. (2.10) expresses that all individuals may find themselves in one of the four states, so that their respective relative numbers sum up to 1. We derive next the other three flow equations that express transitions across different states. The four equations allow us to solve for $n^{WR}, n^{UR}, n^{WH}, n^{UH}$.

Second, the change in the number of employed renters in a given city, $\frac{dN^{WR}}{dt}$, equals the number of unemployed renters who become employed, μN^{UR}, minus the measure of employed renters whose jobs are destroyed, δN^{WR}, and minus those renters who become owners, $\bar{\lambda}^H N^{WR}$. That is:

$$\frac{dN^{WR}}{dt} = \mu N^{UR} - (\delta + \bar{\lambda}^H) N^{WR}.$$

Imposing the condition that for a steady state, $\frac{dN^{WR}}{dt} = \nu N^{WR}$, and rewriting the above condition in terms of relative numbers of agents yields:

$$(\nu + \delta + \bar{\lambda}^H) n^{WR} - \mu n^{UR} = 0. \tag{2.19}$$

Third, working in a like manner, the change in the relative number of unemployed homeowners, νn^{UH}, is equal to minus those unemployed homeowners who find jobs, μn^{UH}, plus the number of those employed homeowners who lose their jobs, plus the number of unemployed renters who become homeowners, $\bar{\lambda}^H n^{UR}$. Rewriting, we have:

$$(\mu + \nu) n^{UH} - \delta n^{WH} - \bar{\lambda}^H n^{UR} = 0. \tag{2.20}$$

Fourth, the increase in the number of employed homeowners, νn^{WH}, is equal to the number of unemployed homeowners finding jobs, μn^{UH}, plus the number of employed renters who become homeowners, $\bar{\lambda}^H n^{WR}$, minus those employed homeowners who become unemployed. Rewriting, we have:

$$\nu n^{WH} + \delta n^{WH} - \bar{\lambda}^H n^{WR} - \mu n^{UH} = 0. \tag{2.21}$$

Rewriting the above equations in matrix form gives:

$$\begin{bmatrix} \delta + \bar{\lambda}^H + \nu & -\mu & 0 & 0 \\ 0 & -\bar{\lambda}^H & -\delta & \mu + \nu \\ -\bar{\lambda}^H & 0 & \delta + \nu & -\mu \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} n^{WR} \\ n^{UR} \\ n^{WH} \\ n^{UH} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}. \tag{2.22}$$

The matrix on the l.h.s. of Eq. (2.22) depends on the parameters $(\delta, \bar{\lambda}^H, \mu, \nu)$ only. This yields the solution:

$$n^{WR} = \frac{\mu\nu}{(\bar{\lambda}^H + \nu)(\bar{\lambda}^H + \nu + \delta + \mu)}, n^{UR} = \frac{\nu(\bar{\lambda}^H + \nu + \delta)}{(\bar{\lambda}^H + \nu)(\bar{\lambda}^H + \nu + \delta + \mu)}; \tag{2.23}$$

$$n^{WH} = \frac{\bar{\lambda}^H \mu(\bar{\lambda}^H + \nu + \delta) + \bar{\lambda}^H \mu(\mu + \nu)}{(\bar{\lambda}^H + \nu)(\delta + \mu + \nu)(\bar{\lambda}^H + \nu + \delta + \mu)}, n^{UH} = \frac{\bar{\lambda}^H \delta(\bar{\lambda}^H + \nu + \delta + \mu) + \bar{\lambda}^H \nu(\delta + \bar{\lambda}^H + \nu)}{(\bar{\lambda}^H + \nu)(\delta + \mu + \nu)(\bar{\lambda}^H + \nu + \delta + \mu)}. \tag{2.24}$$

With these results, the share of employed renters, $\alpha = \frac{n^{WR}}{n^{WR} + n^{UR}}$, and the unit matching rate introduced in Eq. (2.11) are given by:

$$\alpha = \frac{\mu}{\bar{\lambda}^H + \nu + \delta + \mu}; \tag{2.25}$$

$$\gamma^H = \bar{\lambda}^H \frac{\frac{\nu}{\bar{\lambda}^H + \nu}}{h - \frac{\bar{\lambda}^H}{\bar{\lambda}^H + \nu}}. \tag{2.26}$$

In equilibrium, the denominator in Eq. (2.26) above must be positive. The model implies steady state equilibrium homeownership and rental rates, $(\mathrm{hr}, \mathrm{rr})$, given by:

$$\mathrm{hr} = n^{WH} + n^{UH} = \frac{\bar{\lambda}^H}{\bar{\lambda}^H + \nu}, \ \mathrm{rr} = \frac{\nu}{\bar{\lambda}^H + \nu}. \tag{2.27}$$

We note that these rates depend critically on the rate of growth of the population. Below, we derive the equilibrium unemployment rate in the presence of population growth, Eq. (2.48), which also depends on ν.

Clearly, we may arrive at a more general model by specifying churning within the housing market. We go some way further in this direction below in sections 2.3 and 2.3.1 where we relax the assumption that all renters seek to become homeowners, which boosts the equilibrium rental rate. We rework the system of equations in Eq. (2.22) accordingly.

The conditional value functions for homeowners may be obtained by solving Eq. (2.4) and (2.5). Thus, we have:

$$W^H = \frac{1}{\rho(\delta+\mu+\rho)}\left[(\rho+\mu)\pi^H(w^H)+\delta\pi^H(b)\right]. \tag{2.28}$$

$$U^H = \frac{1}{\rho(\delta+\mu+\rho)}\left[\mu\pi^H(w^H)+(\delta+\rho)\pi^H(b)\right]. \tag{2.29}$$

Recall that we have allowed for bargaining between firms and workers to lead to different wage rates for renters and owners, w^H and w^R, respectively, which is natural in the context of our model. Wage setting is defined in terms of the improvement in utility that an unemployed owner expects from accepting a job. By subtracting (2.29) from (2.28), we obtain:

$$W^H - U^H = \frac{w^H - b}{\delta+\mu+\rho}. \tag{2.30}$$

Under our assumptions, transitions from employment to unemployment and vice versa occur at a Poisson rate $\delta+\mu$. Thus, the expected discounted net benefit for a homeowner of moving from unemployment to employment is simply the increase in pay times the expected length of stay in employment, which when allowing for discounting yields (2.30).

We can solve for the conditional value functions for renters, Eq.'s (2.17) and (2.18), after we have expressed the transaction prices for vacant units in terms of the conditional value functions. Recall Eq. (2.15) which gives give transaction prices via Nash bargaining. From Eq.'s (2.15) – (2.18), we can solve for W^R and U^R. However, it is more directly useful that we solve instead for $W^R - U^R$, which is the quantity that enters the wage setting below. That is, by subtracting Eq. (2.18) from Eq. (2.17) we obtain an equation that contains $P^W - P^U$. From Eq. (2.15), by subtracting the solution for P^U from that for P^W we may express $P^W - P^U$ in terms of $W^H - U^H$ and $W^R - U^R$, and substituting into the equation for $W^R - U^R$, a solution readily follows:

$$W^R - U^R = \frac{w^R - b}{\delta+\mu+\rho+\gamma^H\sigma} + \frac{\gamma^H\sigma(w^H-b)}{(\delta+\mu+\rho)(\delta+\mu+\rho+\gamma^H\sigma)}. \tag{2.31}$$

For unemployed renters, there are two types of transitions: transition to employment while remaining a renter, with the expected length of stay being equal to $(\delta+\mu+\rho)^{-1}$, and the expected increase in pay given by $(\delta+\mu+\rho)(\delta+\mu+\rho+\gamma^H\sigma)^{-1}(w^R-b)$; and transition to employment and homeownership, with the expected length of stay being equal to $(\delta+\mu+\rho)^{-1}$, and the expected increase in pay given by $\gamma^H\sigma(\delta+\mu+\rho+\gamma^H\sigma)^{-1}(w^H-b)$.

As we discuss further below, if renters and owners are perfect substitutes in production and treated symmetrically in wage setting, then they receive equal wage rates, $w^H=w^R=w$, and Eq. (2.31) yields: $W^R-U^R=\frac{w-b}{\delta+\mu+\rho}=W^H-U^H$. That is, the improvement in expected lifetime utility resulting from becoming employed is equal for renters and owners. At the steady state equilibrium, individuals who are identical in production experience the same expected utility from becoming employed.

To summarize, the conditional value functions (W^H,U^H,W^R,U^R) have been solved in terms of the wage rates, w^H,w^R, the unit matching rate, γ^H, which in view of Eq. (2.26) depend on h, and the labor market tightness that enters via the employment rate, μ, as we see further below. From Eq. (2.12) and in view of Eq. (2.23), the rent κ is determined as a function of the share of renters $n^{WR}+n^{UR}=\frac{\nu}{\bar{\lambda}^H+\nu}$, and thus is exogenous. Finally, from Eq. (2.16), V may be expressed, via the conditional value functions, in terms of the wage rates, w^H,w^R, labor market tightness (via the job finding rate μ) and the unit matching rate $\gamma^H=\bar{\lambda}^H\frac{\frac{\bar{\lambda}^H}{\bar{\lambda}^H+\nu}}{h-\frac{\nu}{\bar{\lambda}^H+\nu}}$, which depends on h. These derivations when used in Eq. (2.13) yield an equation for the per capita stock of owner-occupied units, h. Finally, the equilibrium is fully determined once the wage rates are set. We turn to this below, which requires looking at the labor market with frictions, after we also introduce frictions in the rental housing market. Recall that Head and Lloyd-Ellis (2012) assume frictionless rental housing and labor markets. When the model is extended to allow transitions also from owning to renting, the equations for the conditional value functions and the above solutions have to be amended accordingly.

2.2.5 Allowing for Frictions in Rental Markets

The rental segment of the housing market is also subject to frictions, though to lesser degree than the ownership one. Rental housing units may be vacant, for reasons that are identical to those in the ownership market. In fact, data on rental market vacancies are also available and are generally higher than homeownership vacancy rates. The purpose of our generalization of Head and Lloyd-Ellis (2012) is to introduce rental market vacancies, which helps structure the use of such data in our empirical analysis. To the variables denoting the relative stocks of individuals in different labor market states, $(n^{WR}, n^{UR}, n^{WH}, n^{UH})$, and the per capita housing stocks for owner-occupancy and renting, h and r, we need to add as unknowns the stocks of vacant units, υ^H and υ^R, respectively.

We extend the model of housing market frictions by also allowing for rationing of owners, that is, given their circumstances some owners would rather be renting. Similarly, for renters, given their circumstances some renters would rather be owning. Both types of rationing may be due to financial and mobility frictions. Let the numbers of rationed individuals be $N_{u,rent}$ and $N_{u,own}$, unfulfilled owners and renters, respectively. Correspondingly, let the respective shares of mismatched renters, who would rather own, and mismatched owners, who would rather rent, denoted by msm^R and msm^H, respectively, be defined as follows:

$$\text{msm}^R = \frac{N_{u,rent}}{N^{WR} + N^{UR}}, \ \text{msm}^H = \frac{N_{u,own}}{N^{WH} + N^{UH}}, \tag{2.32}$$

If only rationed renters are allowed, $msm^R \neq 0$, and $msm^H = 0$, the first three equations in Eq. (2.22) continue to hold with the modification that instead of $\bar{\lambda}^H$, the rate at which the non-rationed renters find dwelling units, we now have $\lambda^H = \bar{\lambda}^H(1 - \text{msm}^R)$.

By definition, the rental housing stock may be occupied by employed or unemployed renters or be vacant. Rental housing per capita thus satisfies

$$r = n^{WR} + n^{UR} + \frac{\upsilon^R}{\mathcal{R}} r. \tag{2.33}$$

This allows us to rewrite Eq. (2.12), the supply equation for rental housing stock, to account for the expected value of a vacant rental unit, V^R,

$$V^R = c_0 + c^R \left(n^{WR} + n^{UR} + \frac{\upsilon^R}{\mathcal{R}} r \right). \tag{2.34}$$

The matching model for rental housing units, to be developed shortly, allows us to obtain an expression for the "demand" for rental housing units.

2.3 Owner Rationing

Introducing owner rationing (alternatively, unfulfilled renters), $msm^H \neq 0$, requires a greater modification of the model. That is, there are now transitions of owners, unemployed and employed, into renters. This needs to be accounted for in the Bellman equations and in the system of equations Eq. (2.22) that determine the equilibrium distribution of agents across states. The number of employed and unemployed renters must account for inflow from employed and unemployed owners who are mismatched and would rather be renters.

The algebra is straightforward. Specifically, it involves two steps. First, in Eq.'s (2.19)–(2.21), $\bar{\lambda}^H$ is replaced by $\lambda^H \equiv \bar{\lambda}^H(1-\text{msm}^R)$. Second, the term $\lambda^R n^{WH}$, where $\lambda^R \equiv \bar{\lambda}^R(1-\text{msm}^H)$, is added to the lhs of Eq. (2.19), the term $\lambda^R n^{UH}$ is added to the lhs of Eq. (2.20), and the term $-\lambda^R n^{WH}$ is added to the rhs of Eq. (2.21), where $\bar{\lambda}^R$ denotes the rate at which owners make contacts with dwelling units for renting. The resulting counterpart of Eq. (2.22) is more complicated but still linear in the n's.[15] This extended model has the advantage that owning is no longer an absorbing state and a nonzero probability of renting is possible even if $\nu = 0$, which removes a drawback of the previous model.

These definitions allow us to complete the determination of the value functions for rental housing units, vacant and occupied, V_v^R and V_o^R, respectively. In equilibrium, vacant and occupied rental units must earn the market return, that is:

$$\rho V_v^R = \text{maint} + \gamma^R \left[V_o^R - V_v^R\right]; \tag{2.35}$$

$$\rho V_o^R = \kappa + \bar{\lambda}^R \frac{\text{msm}^R(N^{WR}+N^{UR})}{\mathcal{H} - N^{WH} - N^{UH}} \left[V_v^R - V_o^R\right], \tag{2.36}$$

where the matching rates with dwelling units of prospective renters and prospective owners, γ^R and γ^H, are now defined as:

$$\gamma^R = \bar{\lambda}^R \frac{(1-\text{msm}^H)(n^{WH}+n^{UH})}{r - n^{WR} - n^{UR}}; \ \gamma^H = \bar{\lambda}^H \frac{(1-\text{msm}^R)(n^{WR}+n^{UR})}{h - n^{WH} - n^{UH}}. \tag{2.37}$$

By solving the system of linear equations (2.35)– (2.36) in terms of (V_v^R, V_o^R) we obtain an expression for the expected value of a vacant rental unit, V^R, from the demand side:

$$V^R = \frac{\upsilon^R}{\mathcal{R}} V_v^R + \left(1 - \frac{\upsilon^R}{\mathcal{R}}\right) V_o^R. \tag{2.38}$$

By equating V^R from Eq. (2.34) and Eq. (2.38), and in view of Eq. (2.33), which expresses the allocation of the per capita rental stock into employed and unemployed renters and vacant units, along with the system of Eq.'s (2.35) and (2.36), the remaining endogenous variables, the vacancy rate and the rental capital stock per capita, $(\frac{\upsilon^R}{\mathcal{R}}, \frac{\mathcal{R}}{N})$, are determined. Here we take κ, the housing rent as given.[16] Noting that msm^H, msm^R, and maint are given, the solutions for $(n^{WR}, n^{UR}, n^{WH}, n^{UH})$ are obtained from the augmented Eq. (2.22), now eq. (5.1) in footnote 15, explicitly for the vacancy rate in the rental housing market, $\frac{\upsilon^R}{\mathcal{R}}$.

2.3.1 Housing Beveridge Curves

The Beveridge Curve for labor markets is a widely researched concept. See Beveridge (1944), Pissarides (1985; 1986) and Blanchard and Diamond (1989). The conceptual similarities between housing and labor markets motivates us to develop a Beveridge Curve for housing. Indeed, it is remarkable that this has not been done to date. Analogous to vacancies in labor markets, which is unsatisfied demand for workers by firms, there correspond prospective buyers and prospective renters in housing markets, which is unsatisfied demand for individuals, by owners and landlords. Analogous to unemployed individuals, which is unsatisfied demand for employment by individuals, there are unsatisfied renters who wish to own, and unsatisfied owners who wish to rent. They are prevented from doing so by frictions. Our development of Beveridge Curves for housing markets is adapted to the institutional features of housing markets, where there are owners and renters, and adheres to the notion of the Beveridge Curve as an accounting relationship at the steady state.

We work first with the homeownership market; the vacancy rate, vown, is defined as:

$$\text{vown} = \frac{\upsilon^H}{\mathcal{H}} = \frac{\mathcal{H} - N^{WH} - N^{UH}}{\mathcal{H}} = 1 - \frac{1}{h}\left(n^{WH} + n^{UH}\right). \tag{2.39}$$

We next express the vacancy rate in terms of a new concept which serves as the unemployment counterpart in ownership housing markets. Allowing for mismatch for renters gives rise to unsatisfied homeownership demand. The respective solutions for n^{WH} and n^{UH} depend on $\bar{\lambda}^H(1 - \text{msm}^R)$ instead of just $\bar{\lambda}^H$ and $\bar{\lambda}^R$, and thus on the incidence of mismatch. Working with the solution from Eq. (2.27) for the homeownership rate and assuming that $\text{msm}^H = 0$, we have that the equilibrium homeownership rate is now:

$$hr = n^{WH} + n^{UH} = \frac{\bar{\lambda}^H(1 - \mathrm{msm}^R)}{\bar{\lambda}^H(1 - \mathrm{msm}^R) + \nu}. \tag{2.40}$$

The equilibrium homeownership rate decreases with the probability of mismatch.[17]

In developing a Beveridge Curve for the homeownership market, we propose the concept of the unfulfilled homeownership rate as the counterpart of the unemployment rate and normalize it appropriately. We start with the definition of the unfulfilled homeownership rate

$$\mathrm{uhr} = \frac{N_{u,rent}}{N_{u,rent} + N^{WH} + N^{UH}}.$$

This quantity is at most equal to the rental rate, and therefore normalizing it by the rental rate yields the relative unfulfilled homeownership rate,

$$\mathrm{ur}^H = \frac{\mathrm{uhr}}{n^{WR} + n^{UR}}. \tag{2.41}$$

This serves as our analog of the unemployment rate for the ownership market. It ranges between 0 and 1, if all renters wish to become owners, which Head and Lloyd-Ellis assume.

We can express ur^H in terms of the n's:

$$\mathrm{ur}^H = \frac{\mathrm{uhr}}{n^{WR} + n^{UR}} = \frac{\mathrm{msm}^R}{\mathrm{msm}^R(n^{WR} + n^{UR}) + n^{WH} + n^{UH}}.$$

Solving the flow equations using Eq. (2.40), and expressing msm^R in term of the n's gives the Beveridge curve for the homeownership market:

$$\mathrm{vown} = 1 - \frac{1}{h} + \frac{1}{h}\frac{\nu}{\bar{\lambda}^H(1 - \mathrm{msm}^R) + \nu}\,\frac{1}{\mathrm{ur}^H}. \tag{2.42}$$

Thus, the Beveridge Curve for the homeownership market is a decreasing function of ur^H, the homeownership analog of the "unemployment rate," a result that agrees with the Beveridge Curve for labor markets. In this expression, the owner-occupied housing stock per capita, h, is endogenous, which may cause the Beveridge Curve to shift and tilt by the cyclical variation in h. From Eq. (2.42), it follows that when $\bar{\lambda}^H$, the matching rate of prospective homeowners with dwelling units increases, as during an upswing in the housing cycle, the housing Beveridge Curve shifts downwards, implying greater efficiency in the housing markets (just as with the labor market Beveridge Curve).

Turning next to the rental market, we propose the concept of the unfulfilled rental rate as the analog of the unemployment rate for the rental market. We start with the definition of the unfulfilled rental rate:

$$\mathrm{urr} = \frac{N_{u,own}}{N_{u,own} + N^{WR} + N^{UR}},$$

which is at most equal to the home ownership rate, if all owners wish to be renters: $N_{u,own} = N^{WH} + N^{UH}$, and therefore normalizing it by the home ownership rate yields the unfulfilled rental rate,

$$\mathrm{ur}^R = \frac{\mathrm{urr}}{n^{WH} + n^{UH}}. \tag{2.43}$$

This serves as the unemployment rate for the rental market: ur^R ranges between 0, which is the assumption made by Head and Lloyd-Ellis namely that owners never leave that mode, and 1, which would mean that all owners wish to become renters. By manipulating the definitions we may express ur^R in terms of the n's. That is:

$$\mathrm{ur}^R = \frac{\mathrm{msm}^H}{\mathrm{msm}^H(n^{WH} + n^{UH}) + n^{WR} + n^{UR}},$$

From the solution of the flow equations (2.22) we have expressions for the n's in terms of parameters, including the imputed shares of mismatched renters and owners, msm^R and msm^H. Unlike in the case of the homeownership and rental rates when there is mismatch of owners only, the case with renter mismatch as well leads (as noted above) to a more complicated modification of the flow equations. We can obtain an expression for the Beveridge Curve for the rental housing market starting from the expression for the vacancy rate:

$$\mathrm{vrent} = 1 - \frac{1}{r} + \frac{1}{r}(n^{WR} + n^{UR}). \tag{2.44}$$

From solving the generalized flow equations and by expressing the rental vacancy rate in terms of the rental unemployment rate, both msm^R and msm^H enter the expressions for the housing unemployment rates, ur^R and ur^H, and thus enter the expressions for both vacancy rates, Eq.'s (2.39) and (2.44), as well. As with the vacancy rate in the homeownership market, the rental vacancy rate depends on r, the per capita rental housing stock, which is endogenous and varies procyclically, thus shifting and tilting the rental Beveridge Curve. Because the two housing vacancy rates share common determinants, in the most general case, it is appropriate to treat them as a system when estimating these equations. To the best of our knowledge, Fig. 6, in Ioannides and Zabel (2017), provides the first empirical illustration of Beveridge curves for housing markets.

2.4 The Labor Market with Frictions

So far we have taken the wage rate and the employment rate as given. The treatment that follows completes the analysis by employing the same preference structure to examine symmetrically the labor market with frictions. Since housing market magnitudes enter the analysis, it follows that housing market outcomes show up as determinants of wages and the unemployment rate. That is, we embed the above model of individuals into a DMP model, by following Pissarides (1985), as presented in Pissarides (2000), the canonical model of equilibrium unemployment. By doing so, we also extend Head and Lloyd-Ellis (2012) by adding a frictional labor market and a frictional rental housing market.

2.4.1 Labor market flows

Consider a labor market in a steady state with a fixed number of labor force participants, L who are either employed or unemployed. Time is continuous and agents have infinite time horizons. Recalling the basic details, jobs are destroyed at the exogenous rate δ, all employed workers enter unemployment at the same rate, and unemployed workers enter employment at the rate μ, which is endogenously determined, as we see shortly below. Frictions in the labor market are modeled by a matching function[18] of the form

$$M = \mathcal{M}(uN, \upsilon N), \tag{2.45}$$

where uN, the number of unemployed workers, and υN, the number of job vacancies, are both stocks. The matching function is taken as increasing in both arguments, concave and exhibiting constant returns to scale.

Unemployed workers find jobs at the rate

$$\mu = \frac{\mathcal{M}(uN, \upsilon N)}{uN} = \mu(\theta),$$

where $\theta \equiv \frac{v}{u}$ is labor market tightness. It follows that firms fill vacancies at the rate

$$q = \frac{\mathcal{M}(uN, vN)}{vN} = \mathcal{M}\left(\left(\frac{v}{u}\right)^{-1}, 1\right) = q\left(\frac{v}{u}\right) = q(\theta). \tag{2.46}$$

It is straightforward to show, using the concavity of the matching function, that:

$$\mu'(\theta) > 0,\ q'(\theta) < 0. \tag{2.47}$$

The intuition is straightforward: the tighter the labor market, the easier it is for workers to find a job, and the more difficult it is for firms to fill a vacancy. A steady state in the labor market requires that the unemployment rate is constant over time. This occurs when the inflow from employment into unemployment, $\delta(1-u)N$, equals the outflow from unemployment to employment, $\mu(\theta)uN$. The steady-state unemployment rate[19] is thus given as:

$$u = \frac{\delta + \nu}{\delta + \mu(\theta) + \nu}. \tag{2.48}$$

Since $\mu(\theta)$ is increasing in its argument, Eq. (2.48) also implies a negative relationship, at the *steady state*, between unemployment and vacancies known as the Beveridge Curve, typically depicted in an unemployment rate – vacancy rate, (u, v) space. In an important sense, this is a mechanical accounting relationship, the consequences of flow balance. It is this feature that we sought to emulate in Section 2.3.1 above in defining Beveridge Curves for housing markets.

A deterioration of matching efficiency, i.e., a decline in job finding given the level of tightness, results in an outward shift of the Beveridge curve in the (u, v) space. An increase in the job destruction rate, possibly induced by faster sectoral reallocation of jobs, is also associated with an outward shift of the Beveridge curve. The Beveridge curve, computed using U.S. monthly data on unemployment and vacancies, is regularly reported by the BLS and is based on its Job Openings and Labor Turnover Survey (JOLTS) [`www.bls.gov/ljt`]. Monthly observations on the unemployment rate, u, measured here as unemployment divided by the labor force, and in the job openings (vacancy) rate, v, measured here as openings divided by employment plus openings, are typically used to track the business cycle. See Ioannides and Zabel (2017), Figures 4 and 5.

During the Great Recession of 2007–2009, a marked outward shift in the Beveridge Curve was observed. Earlier recessions were also associated with such shifts, though not as pronounced.[20] The reasons for this shift are not yet fully understood. However, it is clear that the curve is pivoting, exactly as predicted by Pissarides' theory. We come to that shortly below. This feature of the observed Beveridge Curve has consequences for the housing market, and it is one of the aims of the present chapter to explore it fully.

2.4.2 Hiring by Firms and the Job Creation Condition

Jobs are created by firms that decide to open new positions. Job creation involves some costs and firms care about the expected present value of profits, net of hiring costs. The unit price of a firm's output is p_g, which for consistency with the earlier part of the chapter can be set equal to 1, as the good is the numeraire. Assume, as is standard in this literature, that firms are small, in the sense that each firm has only one job that is either vacant or occupied by a worker. There is a flow cost, associated with a vacancy, defined in terms of the value of the output, $p_g c$, per unit of time. Let V_u denote the expected present value of having a vacancy unfilled and V_{fH} and V_{fR}, the corresponding values of having a vacancy filled, by a worker who is an owner and a renter. Although owners and renters are perfect substitutes in production, the logic of the bargaining model suggests that their tenure status be taken into consideration. A job vacancy is an asset from which the firm expects to earn profit. A job vacancy is filled with an owner or a renter, at the rate $(n^{WH} + n^{UH})q(\theta)$ or $(n^{WR} + n^{UR})q(\theta)$, respectively, whereas an occupied job is destroyed at the rate δ. The value functions associated with a vacancy and a filled job satisfy, respectively, the following equations:

$$\rho V_u = -p_g c + (n^{WH} + n^{UH})q(\theta)(V_{fH} - V_u) + (n^{WR} + n^{UR})q(\theta)(V_{fR} - V_u). \tag{2.49}$$

$$\rho V_{fH} = p_g - w^H + \delta(V_u - V_{fH}), \text{ and } \rho V_{fR} = p_g - w^R + \delta(V_u - V_{fR}). \tag{2.50}$$

The l.h.s. of Eq. (2.49) is the opportunity cost per unit of time of a vacancy. Its r.h.s. is the expected return, when costs are incurred per unit of time, $p_g c$, plus the expected capital gain from a job vacancy being filled by an owner, $(n^{WH} + n^{UH})q(\theta)(V_{fH} - V_u)$, or a renter, $(n^{WR}+n^{UR})q(\theta)(V_{fR}-V_u)$. Similarly, the l.h.s.'s of the equations in (2.50) are the opportunity cost per unit of time of a filled vacancy, ρV_{fj}; their r.h.s.'s are the expected return, which consist of output minus the wage rate, profit per unit of time, $p_g - w^j$, plus the expected capital gain from a job becoming vacant, $\delta(V_u - V_{fj}),\ j = H, R$.

Hiring by firms is done indirectly by opening vacancies. Firms open vacancies as long as it is profitable to do so. As firms open up vacancies, the value of a vacancy decreases. At the free entry equilibrium, $V_u = 0$. Using this in Eq. (2.50), and solving for V_{fH} and V_{fR} yields:

$$V_{fH} = \frac{p_g - w^H}{\rho+\delta},\ \text{and}\ V_{fR} = \frac{p_g - w^R}{\rho+\delta}, \tag{2.51}$$

Substituting into Eq. (2.49) yields

$$(n^{WH}+n^{UH})w^H + (n^{WR}+n^{UR})w^R = p_g - (\rho+\delta)\frac{p_g c}{q(\theta)}. \tag{2.52}$$

Once filled, each job produces a unit of output per unit of time. It is equal to the expected wage rate plus the capitalized value of the firm's hiring cost. A vacancy once created is expected to last for $q(\theta)^{-1}$ periods of time, generating costs $\frac{pc}{q(\theta)}$. Each vacancy is created with probability δ per unit of time and the hiring cost incurs an interest cost at a rate ρ. The capitalized value of the firm's hiring cost is given by $(\rho+\delta)\frac{p_g c}{q(\theta)}$. From this relationship, since q is decreasing in labor market tightness, θ, the higher the expected wage rate, $(n^{WH}+n^{UH})w^H + (n^{WR}+n^{UR})w^R$, the lower the labor market tightness.

Equ. (2.52) will be referred to as the *job creation* condition. It plays the role of the demand for labor in the standard model of a labor market without frictions, where the quantity of labor is represented by labor market tightness, $\theta = \frac{v}{u}$, the ratio of the vacancy rate to the unemployment rate. Note, in equilibrium, from Equ. (2.52), that given p_g and the expected wage rate, the incentive to create vacancies is reduced by a higher real interest rate, a higher job destruction rate and a higher vacancy cost. Vacancy creation is encouraged by improved matching efficiency that exogenously increases the rate at which the firm meets job searchers.

2.5 Wage bargaining

The main approach that has been used by the markets with frictions literature assumes that bargaining between the employer and the worker, which takes place in bilateral meetings, determines the wage rate. In the remainder of this section we first lay out the logic of our model which requires that we distinguish between homeowners and renters in their bargaining with employers.[21] We then examine the implications of assuming that wage bargaining is not conditional on housing tenure.

2.6 Wage bargaining distinguishing owners and renters

The logic of our model suggests that if firms could distinguish between homeowners and renters, then we would expect that wage bargaining would be conditional on individuals' mode of housing tenure. This delivers, as we see shortly, a more general model allowing for richer interactions between the labor market and the rental and homeownership housing markets. After we have developed the model we discuss how this outcome may be sustained in the light of the fact that homeowners and renters as workers are perfect substitutes in production.

2.6.1 Homeowners' bargaining and labor market equilibrium

The expected capital gain for an unemployed homeowner from becoming employed is equal to $W^H - U^H$. A firm, on the other hand, gives up $V_u = 0$, in order to gain V_{fH}. Following generalized Nash bargaining, the wage rate is determined so as to split the total surplus,

$$\text{Total Surplus}^H = W^H - U^H + V_{fH} - V_u, \tag{2.53}$$

in order to

$$\max_{w^H} : \left(W^H - U^H\right)^{1-\sigma_L} \left(V_{fH} - V_u\right)^{\sigma_L}, \tag{2.54}$$

where $1 - \sigma_L$ is a measure of the worker's relative bargaining power. With free entry of vacancies, $V_u = 0$, and thus: $V_{fH} = \frac{p_g - w^H}{\rho + \delta}$. Note that the threat points in Nash bargaining are taken to be what the worker and the firm would receive upon separation from each other.

As Hall and Milgrom (2008) note, the job-seeker then returns to the market and the employer waits for another applicant. A consequence is that the bargained wage is a weighted average of the applicant's productivity on the job and the value of unemployment. That latter value, in turn, depends on the wages offered by other jobs.[22]

The first-order condition for the maximization of the total surplus is:

$$W^H - U^H = (1-\sigma_L)\left[W^H - U^H + V_{fH}\right],$$

which yields $\sigma_L(W^H - U^H) = (1-\sigma_L)V_{fH} = (1-\sigma_L)\frac{p_g - w^H}{\rho+\delta}$. From Eq. (2.30) we have $W^H - U^H = \frac{w^H - b}{\delta+\rho+\mu}$, which along with the first order condition above allows to solve for w^H :

$$w^H = \frac{\delta+\rho}{\delta+\rho+(1-\sigma_L)\mu(\theta)}\sigma_L b + \frac{\delta+\rho+\mu}{\delta+\rho+(1-\sigma_L)\mu(\theta)}(1-\sigma_L)p_g, \tag{2.55}$$

the *wage curve* for owners. Not surprisingly, it does not depend on housing market conditions, if (as in the original model) once individuals become homeowners, they stay as homeowners and do not move. Of course, this would no longer be the case were we to modify the model and allow for turnover for homeowners, while staying either in the owneship mode or transiting to the rental mode. It can be verified that the r.h.s of Eq. (2.55) is increasing in θ, labor market tightness.

2.6.2 Renters' bargaining and labor market equilibrium

Working in a like manner, we formulate the bargaining problem for renters in order to obtain the wage curve for renters. Because renting is a transitional state, the wage curve reflects conditions both for renters and owners. The bargaining model is defined as maximizing

$$\max_{w^R} : \left(W^R - U^R\right)^{1-\sigma_L}\left(V_{fR} - V_u\right)^{\sigma_L},$$

subject to a total surplus condition, like Eq. (2.53), which yields:

$$\sigma_L(W^R - U^R) = (1-\sigma_L)V_{fR} = (1-\sigma_L)\frac{p_g - w^R}{\rho+\delta}. \tag{2.56}$$

where we used Eq. (2.50) to solve for V_{fR}. By substituting the solution for $W^R - U^R$ from Eq. (2.31) into Eq. (2.56) we obtain the wage curve for renters:

$$\frac{\delta+\rho+(1-\sigma_L)\mu+(1-\sigma_L)\gamma^H\sigma}{\delta+\mu+\rho+\gamma^H\sigma}(w^R - b) + \frac{\sigma_L\gamma^H\sigma}{(\delta+\mu+\rho)(\delta+\mu+\rho+\gamma^H\sigma)}(w^H - b) = \frac{1-\sigma_L}{\rho+\delta}p_g. \tag{2.57}$$

In contrast to Eq. (2.55), the wage curve for owners, the wage curve for renters exhibits spillovers from the labor market for owners. This solution holds even if we allow for owner rationing, that is, if $\mathrm{msm}^R \neq 0$. As we discussed earlier, a greater modification of the model is called for if we also introduce renter rationing, that is, if $\mathrm{msm}^H \neq 0$. In that case, the wage curve for owners would also reflect the fact that there are transitions from ownership to renting, which makes both wage rates and labor market tightness to be simultaneously determined, and would thus reflect both housing and labor market parameters. The resulting solutions enter the determination of the relative stocks of individuals in different states and therefore the expressions for vacancy rates in the ownership and rental markets, Eq.'s (2.39) and (2.44) in the main text. It is for this reason that we use the housing vacancy rates in the regressions for the job vacancy rate.

2.7 Linking Housing and Labor Market Vacancies

Our theory suggests that the labor market determines wage rates, conditional on tenure status, (w^H, w^R), and labor market tightness, θ, which in turn determines the employment rate, $\mu(\theta)$, and the unemployment rate at the steady state, $u = \frac{\delta+\nu}{\delta+\mu(\theta)+\nu}$. This implies a solution for the job vacancy rate. The wage and employment rates then enter the conditional value functions, which allows us to solve for the per capita rental and owner-occupied housing stocks, r, h. Finally, the housing vacancy rates, vown and vrent, defined in Eq.'s (2.39) and (2.44), respectively, follow. Despite the many details the derivations are quite elementary.

Specifically, in view of Eq.'s (2.30) and (2.31), Eq. (2.16) is simplified to yield:

$$V^H = \frac{(1-\sigma)\gamma^H}{\rho + (1-\sigma)\gamma^H}[U^H - U^R].$$

Using this expression on the rhs of (2.15), substituting for P^W and P^R in Eq.'s (2.17) and (2.18), and using Eq.'s (2.28) and (2.29) allows us to solve for V^H. This solution for V^H contains h. Substituting into the lhs of Eq. (2.13) gives an equation for h, the per capita owner-occupied housing stock.

Working in a like manner we can solve for the per capita rental housing stock, r. Specifically, from the Bellman equations for the conditional value functions for rental units, Eq.'s

(2.35) and (2.36), we can solve for V_v^R and V_o^R, and by plugging into Eq. (2.38) we obtain an expression for the expected value of a vacant rental unit, V^R. This expression includes both per capita housing stocks, r and h. By substituting into the lhs of the supply equation for rental housing stock, Eq. (2.34), we obtain an equation for r, which includes h, and the rental vacancy rate, $\frac{v^R}{\mathcal{R}}$. Finally, this equation together with Eq. (2.33) as a simultaneous system determine the per capita rental housing stock, r, and the rental vacancy rate, $\frac{v^R}{R}$. This solution takes the rental rate, κ, as given. Instead of this simplifying assumption we could, following the logic of the DMP model, assume that the housing rental rate is determined by bargaining between a prospective tenant and a landlord. The bargaining model would introduce an additional equation which would determine κ. We think that for our purposes it would be unnecessary to complicate the model even further.

2.7.1 An Augmented Beveridge Curve

The job creation condition, Eq. (2.52), which equates the expected wage rate to the net expected benefit to the firm from hiring, along with the definitions of owner and rental vacancy rates, Eq. (2.39) and (2.44), respectively, takes the form of a relationship between labor market tightness and the housing vacancy rates. Specifically, by solving for the homeownership rate, $n^{WH} + n^{UH}$, from Eq. (2.39), and for the rental rate, $n^{WR} + n^{UR}$, from Eq. (2.44), and by substituting into Eq. (2.52), the resulting equation that follows expresses labor market tightness as a function of the owner and rental vacancy rates. By substituting into this structural relationship for (w^H, w^R) from Eq.'s (2.55) and (2.57), the wage curves for owners and renters, we obtain a reduced form which we may take to the data.

That is, the job creation condition, Eq. (2.52), may be written as,

$$h(1 - \text{vown})w^H + r(1 - \text{vrent})w^R = p_g - (\rho + \delta)\frac{p_g c}{q(\theta)}. \tag{2.58}$$

Labor market tightness enters the r.h.s. as well as the l.h.s. of Eq. (2.58) via $\mu(\theta)$, which enters the wage curves, Eq.'s (2.55) and (2.57). The intuition of Eq. (2.58) is straightforward. In posting vacancies, firms recognize that they may attract either unemployed renters or unemployed owners. Since wage rates are set via firm-worker bargaining, they depend on the workers' housing tenure status. Therefore, firms' equating the expected contribution to the profit from an additional unit of employment to the expected wage naturally generates a dependence between labor market tightness and housing market vacancy rates. It is this spillover between the labor and housing Beveridge curves that is highlighted in the empirical analysis reported by Ioannides and Zabel (2017).

3 Statistics on Housing and Labor Market Turnover

We follow the pioneering work of Bachmann and Cooper (2014), henceforth BC, and use PSID data for 1969–2015 to illustrate the numerical magnitude of turnover in the US housing market. PSID data are collected via interviews that have been conducted annually from its inception and until 1997, when they became biennial. In the remainder of the chapter we first replicate the work by Bachmann and Cooper for 1969–2009, but extend the calculations to all available data through 2015. Like BC, we distinguish four transitions: owner-to-owner (O2O), owner-to-renter (O2R), renter-to-owner (R2O), and renter-to-renter (R2R).

We augment the housing statistics with similar information from the labor market and then analyze the joint distribution of housing and labor market transitions. In the labor market, we consider the following seven distinct transitions: employed-to-employed (E2E), unemployed-to-employed (U2E), employed-to-unemployed (E2U), unemployed-to-out of the labor market (U2OLM), employed-to-out of the labor market (E2OLM), out of the labor market-to-unemployed (OLM2U), and out of the labor market-to-employed (OLM2E).

An important transition that is often not considered when analyzing labor market transitions is the job-to-job (E2E) transition. First, the household head had to be employed in consecutive waves. Then the definition of this transition varies depending on the answer to the relevant question asked. For 1969–1975, the question was "How long have you had this job?" The answer we used to determine an E2E transition was "Under 12 months." For 1976–1987 the question was either "How long have you worked for your present employer?" or "How many years altogether have you (HEAD) worked for your present employer?" The answer was given in the actual number of months so we designated an E2E transition if the answer was 1-11 months. Starting in 1988 the question was "In what month and year did you start working for your present employer?" The answer was given in years and we chose the current or previous year. That is, if the question was asked in the spring then the current year would only include a few months. Starting in 1999, when the survey was only conducted every two years, we defined an E2E transition to include starting a job in the current year or the previous two years.

Tables 1 and 2 give the bivariate distributions for transitions in the housing and labor markets for the mean annual (1970–1977) and biennial (1999–2015) frequencies. The marginal distributions are also reported. Note that the bottom row for both tables essentially replicates Table 1 in BC; the mean transition rates for the housing market. Figure 1[23] provides housing market transition rates by transition type and year. This is similar to Figure 2 in BC.

Labor market transition rates by transition type and wave are given in Figure 2. The most frequent transition is the E2E transition. There is quite a bit of variability in this transition rate which is probably due to the change in the question asked and the possible answer choices and the likely measurement error in the answer. It is also curious that the biennial E2E transition rates are lower than the annual rates and this might reflect greater recall bias since the question is asked every two years.

Another interesting result is that the working-to-not working transition rate, E2U/OLM is higher than the not working-to-working transition rate, U/OLM2E. The mean labor market transitions are given in the last columns of Table 1 and 2. Adding OLM2E and U2E gives the transition rate from not working to working; 2.93% and 3.91% for the annual and biennial frequencies, respectively. Adding E2OLM and E2U gives the working-to-not working transition rate; 4.23% and 6.73% for the annual and biennial frequencies, respectively. These results using the biennial data are consistent with the fact that both the percentage of individuals out of the labor market and the unemployment rate (national data) exhibited positive trends over this period.

Calculations for the incidence of household mobility, separately for owners and renters, confirm that renters move more frequently than owners. For owners, 6.7% moved since the spring of the preceding year, while 36.0% of renters did so (annual data). Table 3 gives the housing transition rates conditional on specific labor market transitions. What is clear from these results is that conditional on a labor market transition, the most frequent transition in the housing market is rent-to-rent. And while this is also true unconditionally (see the Bottom row of Tables 1 and 2), it is even more prevalent when conditioning on labor market transitions.

The top panel in Table 4 gives the frequencies of annual labor market transitions conditional on initial tenancy (own versus rent). What stands out from this table is there is a significantly higher labor market transition rate for renters than for owners. The relative annual transition rate from not working to working (OLM2E + U2E) is more than twice as high for renters (4.80%) than for owners (2.14%). The relative transition rate from working to not working (E2OLM + E2U) is also higher for renters (5.00%) than for owners (3.92%). Finally, the annual job-to-job transition rate is considerably higher for renters (15.89) versus owners (6.70). Similar results hold for the biennial data (bottom panel of Table 4). While the higher labor market transition rates for renters could reflect the higher costs of moving for owners, it is also the case that renters tend to be more mobile to begin with (i.e. they are younger, are more likely to be single and with no children).

Table 5 gives the labor market transition rates conditional on specific housing market transitions. Given any transition in the housing market, around one-third also involve a labor market transition and the vast majority are job-to-job transitions. Not surprisingly, conditional on a rent-to-rent transition, close to one-half involve a labor market transition (annual data).

Next, we analyze how housing and labor market transition rates compare to the business and housing cycles. We use real annual U.S. GDP to capture the business cycle and the real annual Case-Shiller house price index (CS_hpi) to capture the housing cycle. The latter is available starting in 1975. So we focus on the data from 1975-1997. As in BC, we include vacancy rates for the homeownership and rental markets. We also include the jobs vacancy rate to complete this analysis. Data on housing vacancies come from the Census Bureau's Housing Vacancy Survey (HVS). The HVS is a regular part of the Current Population Survey

(CPS). Units that are found to be vacant or were otherwise not interviewed are included in the HVS. Data on job vacancies come from the Help-Wanted Index; these are an aggregate of ads carried by the press that is provided by the Conference Board.

We detrend the housing and employment transition rates with a Hodrick-Prescott filter with parameter equal to 400. We de-trend the GDP, CS_hpi, and the three vacancy rates with a Hodrick-Prescott filter with parameter equal to 6.25. One issue is whether the levels or the growth rates of US GDP and CS_hpi are the relevant indicators of the business and housing cycles. So we include both and hence the data now cover the years 1976–1997. Summary statistics for the raw data are given in Table 6.

Correlations between the variables are given in Table 7. Note that the housing and business cycles, whether measured in levels or in growth rates are positively correlated. On the other hand, the jobs vacancy rate is negatively correlated with the homeownership vacancy rate but negatively correlated with the rental vacancy rate.

First we see that the housing and employment turnover rates are negatively related. The housing turnover rate is positively correlated with both the level and growth rate for GDP indicating it is pro-cyclical. Interestingly it is uncorrelated with HPI but is positively correlated with the growth rate in HPI. The latter is probably a better measure of the pro-cyclicality in regard to the housing market. The housing turnover rate, TOR-H, is also positively correlated with the jobs vacancy rate. This could well be the result of households moving to new jobs. Finally, TOR-H is negatively correlated with both the homeownership and rental vacancy rates though the correlation is relatively low.

Next, we find that the employment turnover rate, TOR-E, is uncorrelated with both the level and growth rate for GDP indicating it is acyclical. It is mildly positively correlated with the housing price index but uncorrelated with its growth rate. TOR-E is uncorrelated with the jobs vacancy rate but it is strongly negatively correlated with both the homeownership and rental vacancy rates.

To summarize, we have extended the analysis of housing market transition rates first carried out by Bachmann and Cooper (2014) by including the labor market transition rates.

We find that the most frequent transition is moving from one job to another. For the joint distribution of the housing and employment markets, we find that around 70% of households neither move residences nor change their labor market status in a year. For those that change both their labor and housing market statuses, the most common transition is job-to-job and rent-rent. This likely reflects the activities of younger individuals who are more mobile both in terms of residences and employment.

We find that renters are more likely to change their job market status than owners and conditional on a change in labor market status, about one-quarter to one-third of households change residence with the most common being rent-to rent. Clearly there is a lot of churn in the rental market that often accompanies a change in the labor market status with the job-to-job transition being the most common.

Finally, we find evidence that housing turnover is pro-cyclical relative to both the housing and business cycles. Whereas the labor market turnover rate is acyclical relative to both the housing and business cycles. While there is a mildly negative correlation between the turnover rates in the two markets, there is a much stronger relationship between the cycles in the two markets as compared to their turnover rates. This is displayed in Figure 1 and in more detail in Figure 3, where the red dashed lines denote GDP, detrended with a Hodrick-Prescott filter with parameter equal to 6.25, and the blue solid lines denote the cyclical components of respective annual[24] disaggregated housing turnover rates, O2O, O2R, R2O, and R2R, obtained from the weighted non-SEO sample of the PSID, detrended using with a Hodrick-Prescott filter with parameter equal to 400. The NBER recession dates, 1970-2015, are indicated as shaded areas. Generally, O2O and R2O positively correlated with GDP, R2R likewise but weakly correlated with GDP, and O2R negatively correlated with GDP. As indicated on Table 7, the overall housing turnover is positively correlated with the GDP growth rate, with a correlation coefficient of 0.48. This accords with intuition since during the study period the ownership segment of the housing market is about two-thirds.

4 Conclusion

This chapter explores the interdependence between the housing and labor markets by means of a Diamond-Mortensen-Pissarides (DMP)-type model. The model treats housing decisions and labor supply as joint decisions of individuals, articulates how the renting and owner segments of the housing markets adjust through turnover flows and highlights the transitions across different discrete states in those markets, that is owner-to-owner, owner-to-renter, renter-to-owner, and renter-to-renter. The model gives rise naturally to equilibrium vacancy rates in housing and labor markets. The labor market model with frictions produces as an outcome the Beveridge Curve. We use the model to develop Beveridge Curves for the homeownership and rental markets. We do so by proposing a housing market counterpart for the concept of unemployment, namely the unfulfilled homeownership and rental rates. Finally, we use data from the Panel Study of Income Dynamics for 1969–2015 to illustrate the empirical significance of these concepts.

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Notes

[1]The theoretical part of this chapter was first circulated as an appendix to Ioannides and Zabel (2017); the empirical part is new.

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* 617-627-3294 Yannis.ioannides@tufts.edu ** 617-627-2318 jeff.zabel@tufts.edu

[2]The Nobel Prize citation is best description of the DMP approach. See also Pissarides (1985, 1986, 2000, 2011).

[3]See Diamond and Şahin (2015) for a discussion of the significance of shifts in the Beveridge Curve and Elsby et al. (2015) for the latest survey of the literature.

[4]As the present chapter was being posted, we became aware of Gabrovski and Ortega-Marti (2018), which proposes an entirely different concept of a Beveridge curve for housing. That chapter rests on a concept of unemployment for housing as the number of buyers. Our concept is more firmly rooted in the specifics of housing markets with frictions, as we detail further below.

[5]We recognize that according to Bachmann and Cooper, total housing turnover is positively but weakly correlated with and leads the rental vacancy rate, while it is positively but weakly correlated with and lags the owner vacancy rate. However, those calculations are based on HP-filter detrended data. Detrending is, of course, critical for understanding the cyclical patterns but interdependence in the raw data is of interest in its own right, especially when we allow for geographic detail in the data.

[6] These facts agree with data from the CPS for 2004: Ioannides and Zanella (2008), Table 1, report that

17% of residential moves occur for work-related reasons and 52.7% for housing- and neighborhood- related reasons.

[7]Wasmer and Weil (2004) extend the Pissarides model to account for credit frictions and Petrosky-Nadeau and Wasmer (2017) extend it further to account for credit multipliers.

[8] This definition maintains consistency between the rental and the homeownership sectors. However, this could be modified so as to be based on transactions values instead of the vacant unit value. Also property tax rates, maintenance and depreciation rates as well as housing price appreciation rates may be area-specific.

[9]Following Poterba (1984) and Henderson and Ioannides (1986; 1987) the user cost of housing reflects mortgage payments at a rate of interest ι, times the portion of the value of a dwelling unit that is financed, $1-$ equity, and adjusted for the tax deductability of mortgage interest associated with the portion of the value of owner-occupied housing that is leveraged, by multiplying by 1 minus the marginal US income tax rate, $1-\tau$. Property taxes, denoted by rate τ_p here, are also deductible for US income tax purposes. In addition, allowing for maintenance and depreciation, at rates maint and depr, respectively, and deducting the rate of expected housing price appreciation, appr^e, yields the annual user cost of housing as:

$$p_h = \left[(1-\tau)[\iota(1-\text{equity})+\tau_p] + \text{depr} + \text{maint} - \text{appr}^e\right].$$

This definition maintains consistency between the rental and the homeownership sectors. However, this could be modified so as to be based on transactions values instead of the vacant unit value. All these quantities may be defined per appropriate unit of time.

[10] In contrast to Head and Lloyd-Ellis, our definition of $\pi^H(w)$ in (Eq. 2.2) above makes it dependent on V^H, in general, which is endogenous. We will ignore this endogeneity from now on, when we derive the equilibrium value of V below. However, V cancels out of the expressions for the $\pi^j, j = H, R$ if we assume that $p = 1$, and $p_h = \rho$.

[11]Indeed, Ioannides and Zabel (2017) use tenure choice estimation results to impute the probabilities of unfulfilled renters and owners, which are actually employed in sections 2.3 and 2.3.1 below.

[12] Let $I_{b,t}, I_{s,t}$, denote the stock of buyers searching for houses and the stock of sellers searching for buyers, respectively. Let the matching process be specified in the standard fashion in terms of the Poisson rate of contacts generated, denoted by $\Gamma_t = \Gamma\left(I_{b,t}, I_{s,t}\right)$. So, in general, the rate of arrivals of contacts to the typical dwelling unit in Metro Area i is: $\gamma = \frac{1}{I_{s,t}}\Gamma\left(I_{b,t}, I_{s,t}\right)$, which under the assumption of constant returns to scale this may be written as:

$$\gamma = \Gamma(\phi, 1) = \Gamma\left(\frac{I_{b,t}}{I_{s,t}}, 1\right).$$

This differs from the Head and Lloyd-Ellis assumption, (2.8) above, only because of the nonlinearity of Γ, but is consistent with the assumptions typically made about matching models. Parameter λ is subsumed in this formulation.

[13]Although it looks as if this definition, which follows that of Head and Lloyd-Ellis, is in contrast to its counterpart for the labor market. However, it is not. Vacancies in the owner market as the "supply" of housing corresponds to unemployed workers, is in the denominator, and the number of renters, who are aspiring to become home owners, the "demand", is in the numerator. This is consistent with the definition of housing market tightness for the rental market, see (2.37) below.

[14]This treatment ignores, for simplicity how new agents enter the market.

[15] System (2.22) becomes:

$$\begin{bmatrix} \delta+\lambda^R+\nu & -\mu & \lambda^H & 0 \\ 0 & -\lambda^R & -\delta & \mu+\nu+\lambda^H \\ -\lambda^R & 0 & \delta+\nu-\lambda^H & -\mu \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} n^{WR} \\ n^{UR} \\ n^{WH} \\ n^{UH} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}. \tag{5.1}$$

[16]Rental housing transactions involve contact between landlords and prospective tenants. For symmetry with our treatment of the ownership market, we could specify the determination of the rent, κ, by means of bargaining between landlords and prospective tenants. But the agreements typically lead to spells of stay which are shorter than ownership spells [Henderson and Ioannides (1987)], and it is thus appropriate to assume that κ is determined competitively.

[17]In view of the generalization of the matching model in footnote 3 above, the rate at which buyers contact dwelling units, λ, may be written in terms of the matching function $\Gamma(.,.)$, and the ratio of potential buyers to vacant units, ϕ. That is:

$$\lambda = \Gamma(1, \phi^{-1}).$$

[18]See Stevens (2007) for a microfoundation of the matching function.

[19]The full dynamic equation for the unemployment rate readily follows: At any point in time, $(1-u)N$ people are employed. Of these people, per unit of time, $(1-u)N\delta dt$ people lose their jobs and enter unemployment. And during time dt, $uN\mu(\theta)\,dt$ people find jobs, thus reducing the ranks of the unemployed and $\nu N dt$ people enter the economy and become unemployed. Consequently,

$$d(un) = Ndu + udN = (1-u)N\delta dt - uN\mu(\theta)\,dt + \nu N dt.$$

Using the fact that $\frac{dN}{N} = \nu dt$ and rewriting this as a differential equation we have:

$$N\dot{u} + u\nu N = (1-u)N\delta - uN\mu(\theta) + \nu N.$$

The Beveridge Curve follows if we impose the condition that the unemployment rate remains constant, equilibrium unemployment $\dot{u} = 0$.

[20]For recent discussions of shifts in the Beveridge Curve for labor markets, see Elsby *et al.* (2014), Pissarides (2011) and Diamond and Sahin (2015).

[21]Our approach to both the housing and labor markets is based on the original formulation of labor markets with frictions due to Pissarides (1985). It can be extended by means of competitive search models, along the lines of Diaz and Jerez (2013), which is applied to housing markets, and Moen (1997), which focuses on job market applications.

[22]Some researchers have made alternative assumptions about the threat points. Hall and Milgrom (2008) assume that the threat point is to delay and postpone bargaining and agreement instead of threatening to walk out of the deal, as Pissarides does. "The bargainers have a joint surplus, arising from search friction, that glues them together." Hall and Milgrom (2008). They assume that the threats are to extend bargaining rather than to terminate it. The result is to loosen the tight connection between wages and outside conditions of the Mortensen–Pissarides model. When the labor market is hit with productivity shocks, the Hall–Milgrom bargaining model delivers greater variation in employer surplus, employer recruiting efforts, and employment than does the Nash bargaining model.

[23]All figures are drawn using data from the Panel Study of Income Dynamics.

[24] When only biennial turnover rates are provided for later years, the same value is applied for both years.

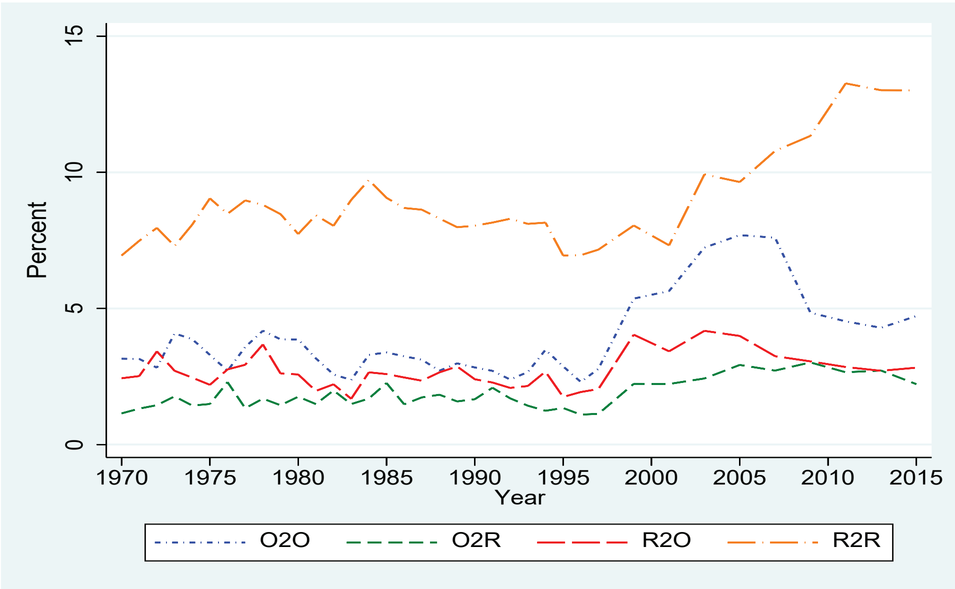

Figure 1: Housing Market Transition Rates
Source: PSID and authors' calculations. Note: Annual rates for 1970-1995 and biennial rates for 1997-2015

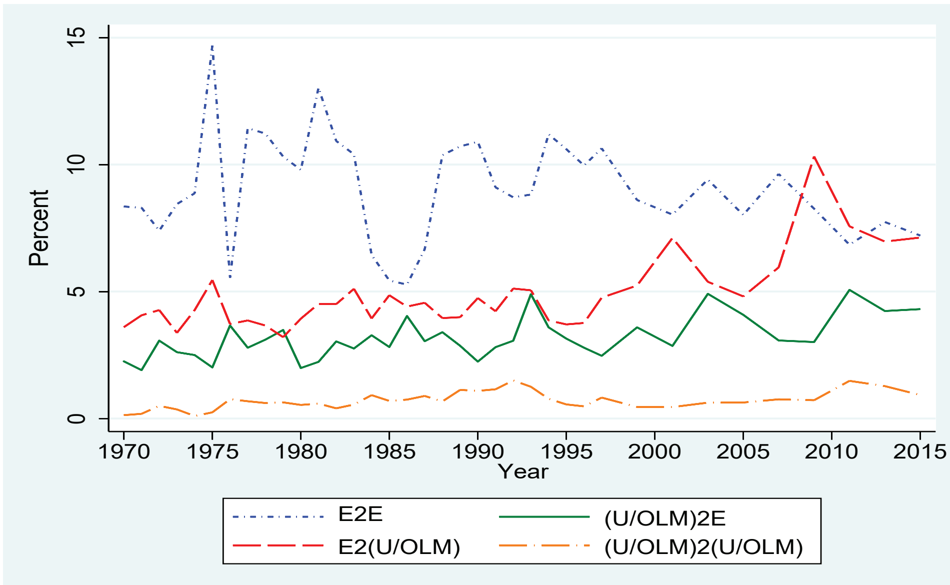

Figure 2: Labor Market Transition Rates
Source: PSID and authors' calculations. Note: Annual rates for 1970-1995 and biennial rates for 1997-2015

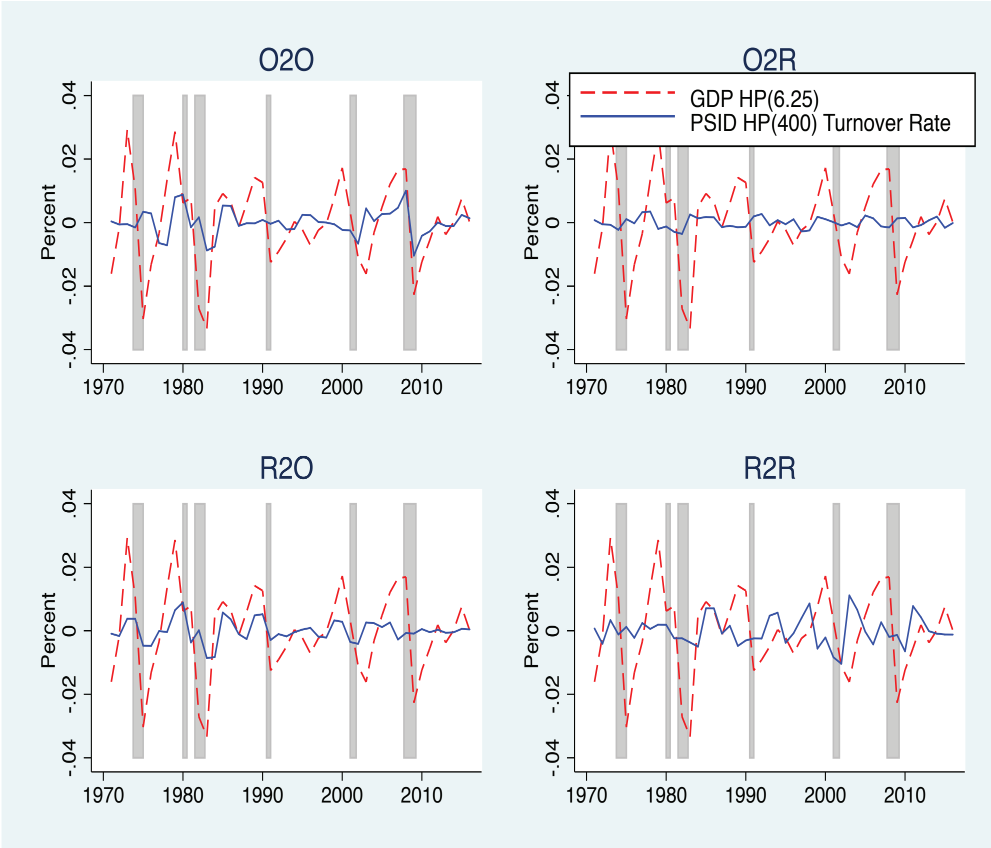

Figure 3: Cyclical Components of the Disaggregated Housing Turnover Rates

Table 1: **Bivariate Distribution, Annual Frequency: 1970-1997**

Transitions	**No Change**	**O2O**	**O2R**	**R2O**	**R2R**	**Employ**
No Change	72.95	2.36	1.03	1.78	4.63	82.74
OLM2E	1.13	0.04	0.03	0.06	0.29	1.56
OLM2U	0.22	0.00	0.01	0.00	0.06	0.29
U2E	0.93	0.04	0.05	0.03	0.32	1.37
U2OLM	0.30	0.01	0.00	0.01	0.07	0.39
E2U	1.02	0.05	0.05	0.05	0.32	1.48
E2OLM	2.26	0.13	0.06	0.05	0.25	2.75
E2E	5.84	0.50	0.36	0.48	2.24	9.42
Housing	84.65	3.12	1.59	2.47	8.17	100.00

Table 2: **Bivariate Distribution, Biennial Frequency: 1999-2015**

Transition	**No Ch**	**O2O**	**O2R**	**R2O**	**R2R**	**Employ**
No Change	65.02	4.46	1.86	2.33	6.68	80.36
OLM2E	1.54	0.08	0.04	0.08	0.40	2.15
OLM2U	0.14	0.02	0.01	0.01	0.08	0.26
U2E	0.99	0.08	0.11	0.08	0.50	1.76
U2OLM	0.36	0.02	0.02	0.02	0.15	0.56
E2U	1.18	0.11	0.07	0.08	0.52	1.96
E2OLM	3.73	0.30	0.13	0.15	0.45	4.76
E2E	4.61	0.72	0.33	0.61	1.92	8.20
Housing	77.59	5.77	2.57	3.37	10.71	100.00

Table 3: **Conditional Housing Market Transitions**

	TOR	**O2O**	**O2R**	**R2O**	**R2R**
Conditional on Employed to Employed					
Annual	37.98	5.27	3.87	5.09	23.75
Relative		13.87	10.18	13.41	62.55
Biennial	28.16	3.50	1.88	3.94	18.84
Relative		12.43	6.69	13.99	66.89
Conditional on Not Employed to Employed					
Annual	29.49	2.77	2.53	3.17	21.02
Relative		9.39	8.57	10.76	71.28
Biennial	38.70	4.27	3.05	3.28	28.09
Relative		11.04	7.89	8.48	72.59
Conditional on Employed to Not Employed					
Annual	22.54	4.28	2.62	2.35	13.28
Relative		19.00	11.64	10.42	58.94
Biennial	35.61	7.83	3.58	5.89	18.32
Relative		21.97	10.04	16.55	51.44
Conditional on Unemployed to Out of Labor Market (and vice versa)					
Annual	23.57	1.08	1.50	1.73	19.25
Relative		4.59	6.38	7.35	81.67
Biennial	41.50	4.92	4.81	4.53	27.24
Relative		11.86	11.60	10.91	65.63

Table 4: **Labor Market Transition Rates Conditional on Housing Tenure**

Transition	Own		Rent	
	Marginal	Relative	Marginal	Relative
		Annual		
No Change	61.22	86.86	21.52	72.91
OLM2E	0.88	1.24	0.68	2.31
OLM2U	0.10	0.14	0.19	0.64
U2E	0.63	0.90	0.74	2.49
U2OLM	0.17	0.24	0.22	0.75
E2U	0.72	1.03	0.76	2.58
E2OLM	2.04	2.89	0.72	2.42
E2E	4.73	6.70	4.69	15.89
Total	70.48	100.00	29.52	100.00
		Biennial		
No Change	61.02	83.85	19.34	71.00
OLM2E	1.39	1.90	0.76	2.80
OLM2U	0.11	0.15	0.15	0.55
U2E	0.83	1.14	0.93	3.40
U2OLM	0.27	0.37	0.29	1.05
E2U	1.01	1.39	0.96	3.52
E2OLM	3.70	5.08	1.06	3.89
E2E	4.45	6.11	3.75	13.78
Total	72.77	100.00	27.23	100.00

Table 5: **Conditional Labor Market Transitions**

State	O2O	O2R	R2O	R2R	Any Transition
	Annual—				
No Change	75.49	64.72	72.28	56.63	63.82
OLM2E	1.41	1.76	2.41	3.59	2.77
OLM2U	0.07	0.34	0.12	0.76	0.48
U2E	1.19	2.90	1.35	3.94	2.85
U2OLM	0.17	0.31	0.35	0.84	0.57
E2U	1.57	2.94	1.95	3.86	2.99
E2OLM	4.24	4.07	2.09	3.02	3.23
E2E	15.87	22.96	19.44	27.36	23.30
	Biennial—				
No Change	77.34	72.48	69.02	62.44	68.41
OLM2E	1.30	1.57	2.51	3.78	2.70
OLM2U	0.30	0.36	0.21	0.76	0.52
U2E	1.31	4.17	2.50	4.64	3.41
U2OLM	0.30	0.61	0.59	1.38	0.89
E2U	1.87	2.80	2.50	4.83	3.49
E2OLM	5.15	5.14	4.50	4.19	4.59
E2E	12.43	12.87	18.17	17.98	15.99

Table 6: **Summary Statistics: 1976-1997**

Variable	Mean	S.D.	Min.	Max.
Housing Turnover Rate (TOR-H)	15.38	1.47	12.29	18.35
Employment Turnover Rate (TOR-E)	17.56	2.01	13.71	20.37
Real GDP	8.03	1.60	5.66	10.98
Seasonally Adjusted (SA) **HPI**	53.85	3.25	48.07	60.73
Real GDP Growth Rate (**GDP gr**)	3.28	2.09	-1.18	7.69
SA HPI Growth Rate (**HPI gr**)	0.52	3.40	-6.13	6.40
Jobs Vacancy Rate (**VJobs**)	7.63	1.50	5.28	10.55
Homeowner Vacancy Rate (**VHouse**)	1.50	0.21	1.00	1.77
Rental Vacancy Rate (**VRental**)	6.59	1.08	5.03	7.85

Table 7: **Correlations: Housing, Employment, and Business Cycle Variables**

	TOR-H	TOR-E	GDP	GDP gr	HPI	HPI gr	VJobs	VHouse
TOR-E	-0.24							
GDP	0.52	0.01						
GDP gr	0.48	-0.06	0.47					
HPI	0.02	0.25	0.67	-0.03				
HPI gr	0.39	-0.01	0.51	0.39	0.41			
VJobs	0.53	-0.04	0.87	0.38	0.67	0.68		
VHouse	-0.20	-0.42	-0.20	-0.32	-0.22	-0.64	-0.36	
VRental	-0.20	-0.47	-0.03	-0.33	0.35	0.30	0.19	-0.05